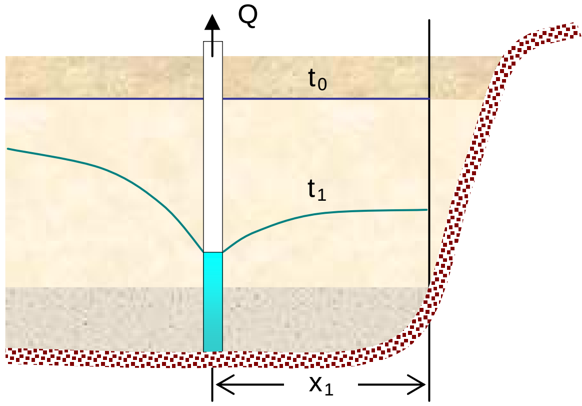
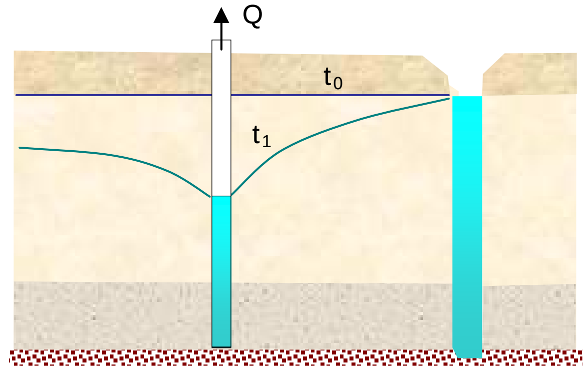
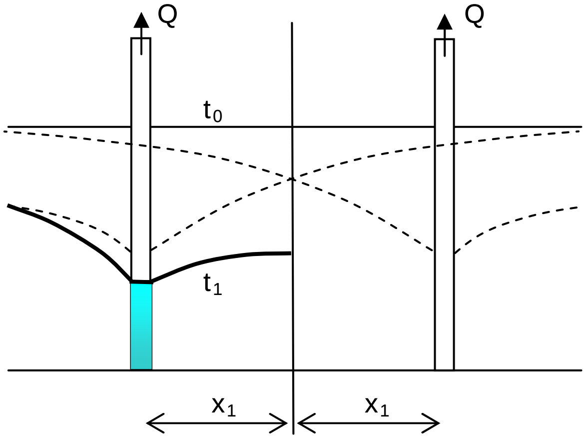
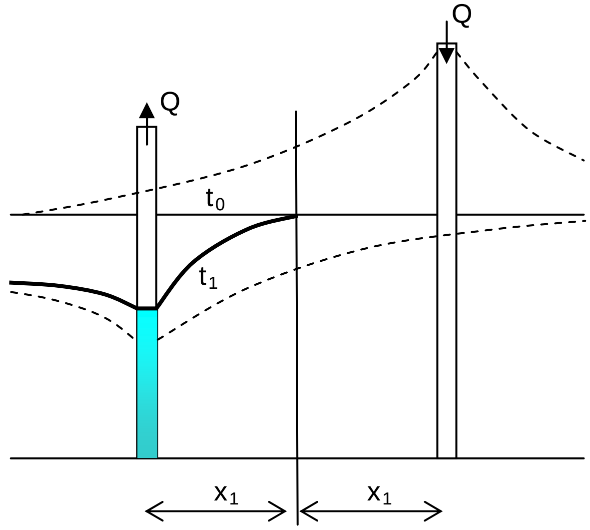
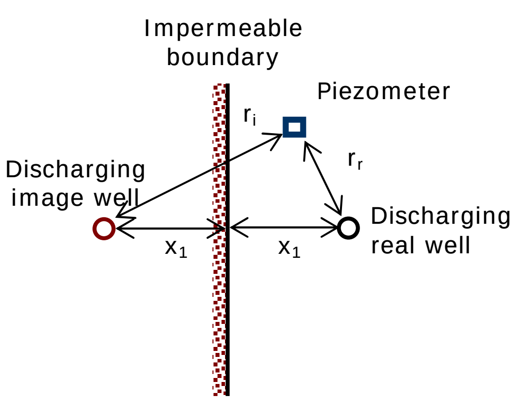
1.72, Groundwater Hydrology
Prof. Charles Harvey
Lecture Packet #10: Superposition

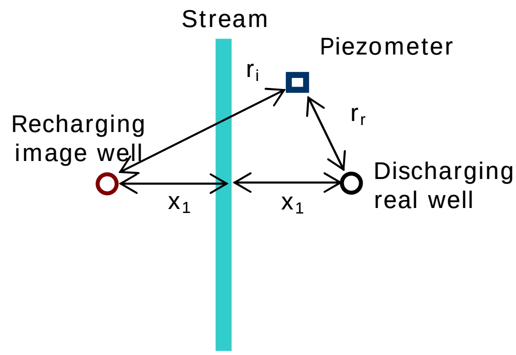












Superposition

Aquifer flow equation:

$$\frac{\partial T \partial H}{\partial x^2} + \frac{\partial T \partial H}{\partial y^2} + Q = S\frac{\partial H}{\partial t}$$

Subject to:

H = A, and/or $\frac{\partial H}{\partial \eta} = 0$ along some boundary

H = A at t = 0

Suppose the system has constant head boundaries (H = A, where A is uniform) and no-flow boundaries, then we can convert the GW equation to be in terms of drawdown rather than head.

d = A – H ⟹ H = A – d

$$\frac{\partial T \partial d}{\partial x^2} + \frac{\partial T \partial d}{\partial y^2} - Q = S\frac{\partial d}{\partial t}$$

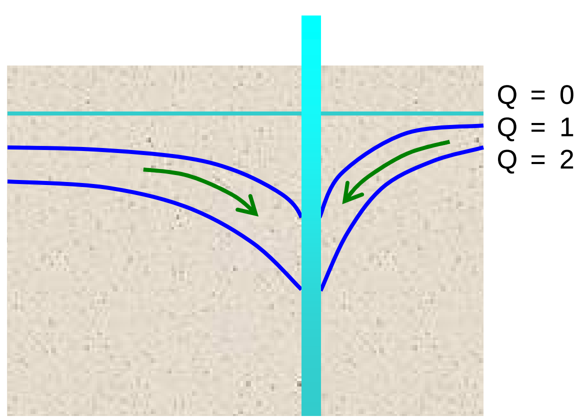
d = 0, and/or $\frac{\partial d}{\partial \eta} = 0$ along some boundary

Now we can investigate the effects on increasing the pumping rate, by multiplying the whole equation by a constant:

$$\frac{\partial T \partial 2d}{\partial x^2} + \frac{\partial T \partial 2d}{\partial y^2} - 2Q = S\frac{\partial 2d}{\partial t}$$

2d = 0, and/or $\frac{\partial 2d}{\partial \eta} = 0$ along some boundary

Thus the solution is the same, except all the drawdowns have been increased by the increase in the pumping rate. We did not make any assumptions about the geometry of the domain, the spatial pattern of T, or the location of the well.



Now we suppose we have two wells at different locations. We can add the two equations to get one equation:

$$\frac{\partial T \partial d_1}{\partial x^2} + \frac{\partial T \partial d_1}{\partial y^2} = Q_1$$

$$\frac{\partial T \partial d_2}{\partial x^2} + \frac{\partial T \partial d_2}{\partial y^2} = Q_2$$

$$\frac{\partial T \partial (d_1 + d_2)}{\partial x^2} + \frac{\partial T \partial (d_1 + d_2)}{\partial y^2} = Q_1 + Q_2$$

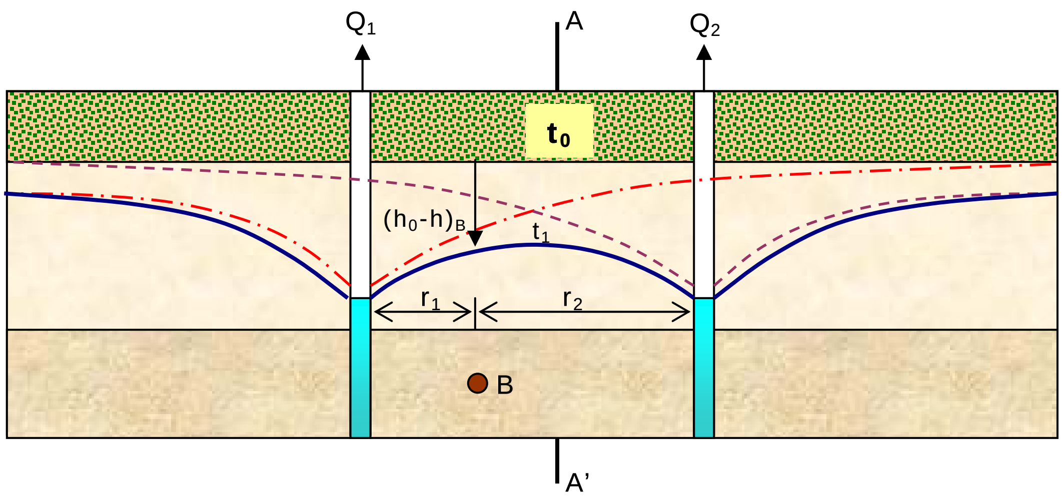
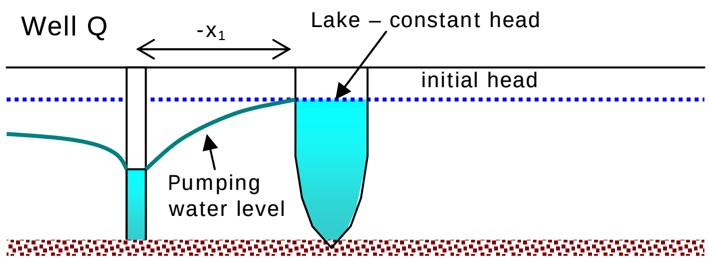
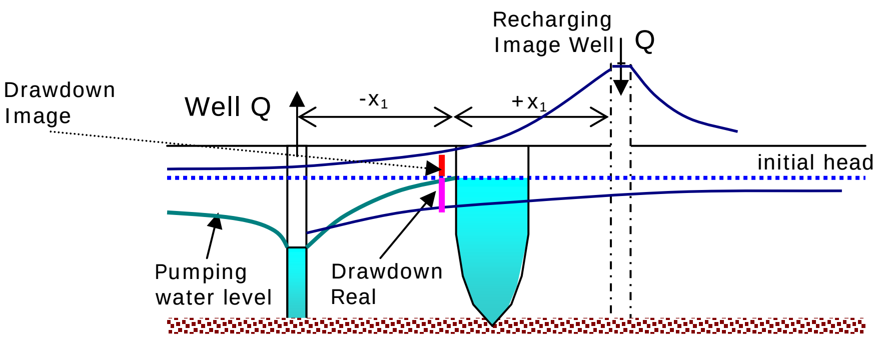


Image Well Theory & Superposition

Concept: To predict aquifer behavior in the presence of boundaries, introduce imaginary wells such that the response at the boundary is made true.



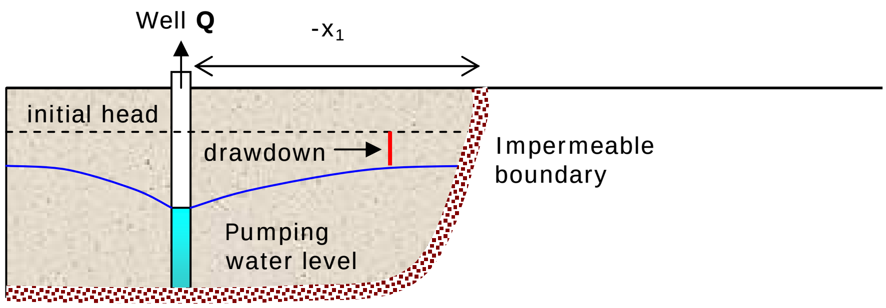


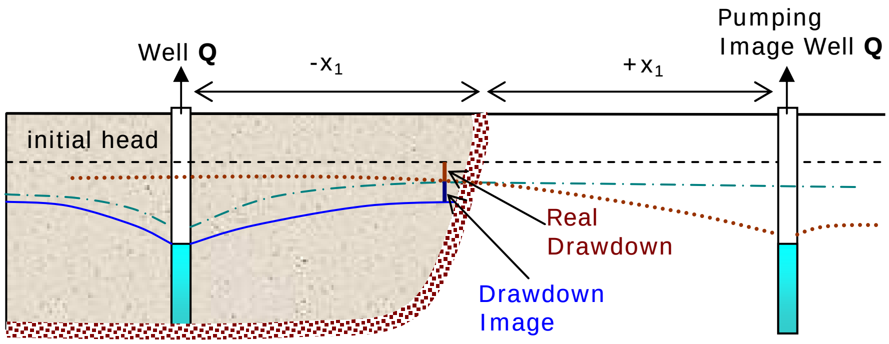
To maintain constant head at lake

- Introduce an image well
- It recharges (artificially)
- It creates a cone of impression
- Resultant cone is due to pumping well and recharge well

Consider an impermeable boundary

- Drawdown is enhanced due to reflection

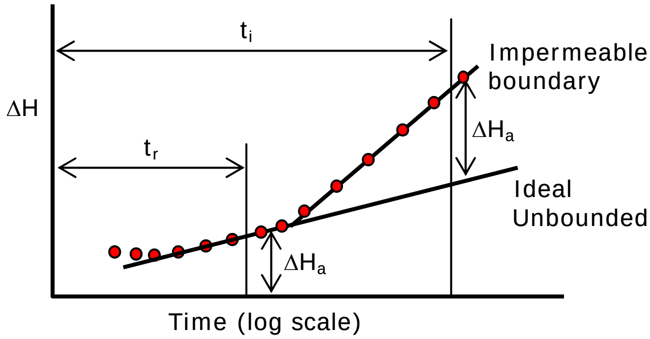


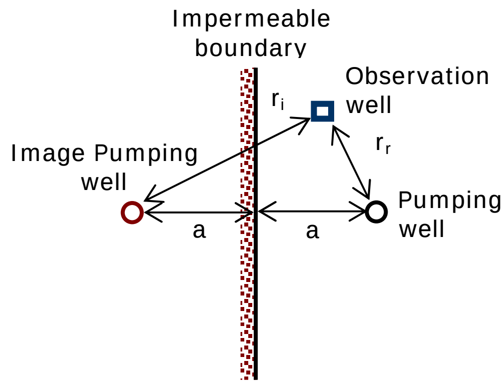


- A pumping image well creates the effect of the boundary

Two things you can do with superposition or image wells.

1) Find the hidden boundary
   For a semi-log plot one might obtain:





(applies to any linear system – can work with Theis equation as well)

$$[h_0 - h(r,t)]_r = \frac{2.30Q}{4\pi T}[\log_{10}]\frac{2.25Tt}{r^2 S}$$

$$[h_0 - h(r,t)]_i = \frac{2.30Q}{4\pi T}[\log_{10}]\frac{2.25Tt}{r_i^{\;2}}$$

Drawdown due to real well is set to drawdown due to image pumping well (at different times ← f(boundary))

$[h_0 - h(r,t)]_r = [h_0 - h(r,t)]_i$ which gives

$$\frac{r_i^{\;2}}{t_i} = \frac{r_r^{\;2}}{t_r} \rightarrow r_i = r_r\sqrt{\frac{t_i}{t_r}}$$

- Slope of line on semi-log plot depends on Q and T
- If an impermeable boundary is present, drawdown will double under the influence of the image well at some point in time
- Select an arbitrary drawdown ΔH_a and the time for this drawdown to occur under the real pumping well
- Find the same drawdown to be produced by the image well and its corresponding time
- Knowing drawdowns and times one can determine distance to boundary – uniqueness?
  multiple observation wells are needed

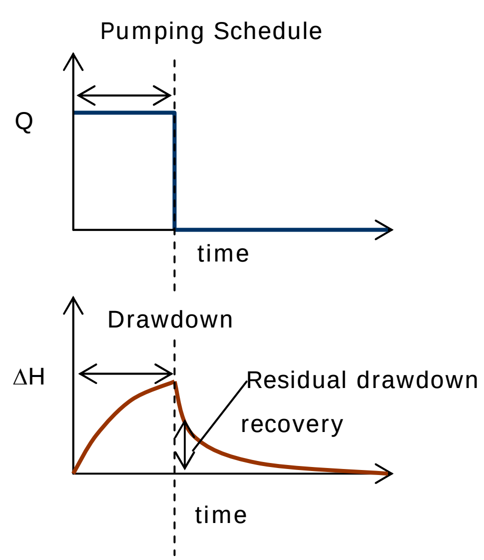
2) Analysis of water level recovery test data

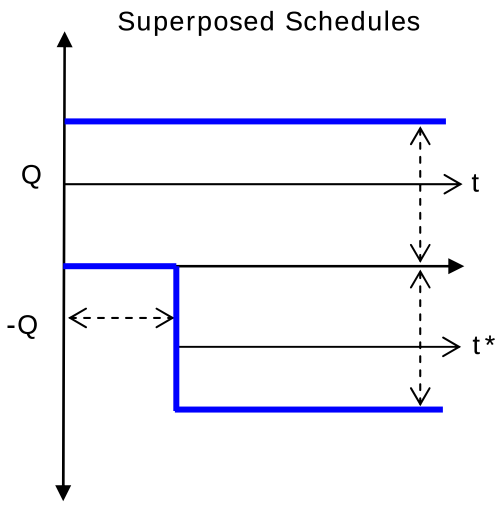
At end of pumping test, pump is stopped and water levels recover, this is called **recovery**.

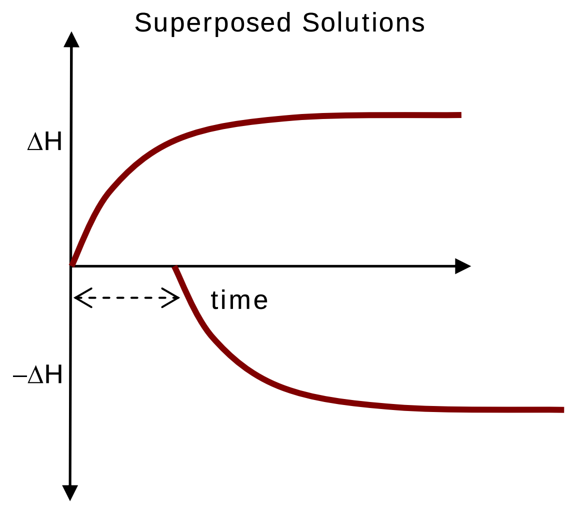
Water level below the original head during recover is called **residual drawdown**.

Advantages of recovery test – inexpensive and “pumping” rate is nearly constant.

What is drawdown from a pulse input?







residual drawdown

$$= \Delta h^* = \frac{Q}{4\pi T} W(u) - \frac{Q}{4\pi T} W(u^*) \quad \text{where}$$

$$u = \frac{r^2 S}{4Tt} \text{ and } u^* = \frac{r^2 S}{4Tt^*}$$

For small r and large t*, we get $\Delta h^* = \frac{2.30Q}{4\pi T} \log \frac{t}{t^*}$

for a plot of Δh vs. log t/t* over one log cycle, we get T as

$$T = \frac{2.30Q}{4\pi \Delta h^*}$$