Lecture Notes on Fluid Dynmics
(1.63J/2.21J)
by Chiang C. Mei, MIT

3-4press-grad.tex

3.4 The effects of pressure gradient

Let us follow the incoming flow from infinity A towards the stagnation point O, and then along the body. The flow first decelerates from A to O, accelrates from O to E, then decelerates again from E to F. In the inviscid flow outside the boundary layer, the pressure variation can be estimated by Bernoulli's theorem. The pressure must increase from A to O, decrease from O to E and decrease from E to F again as shown in Figure 3.4.1. Inside

accaxx

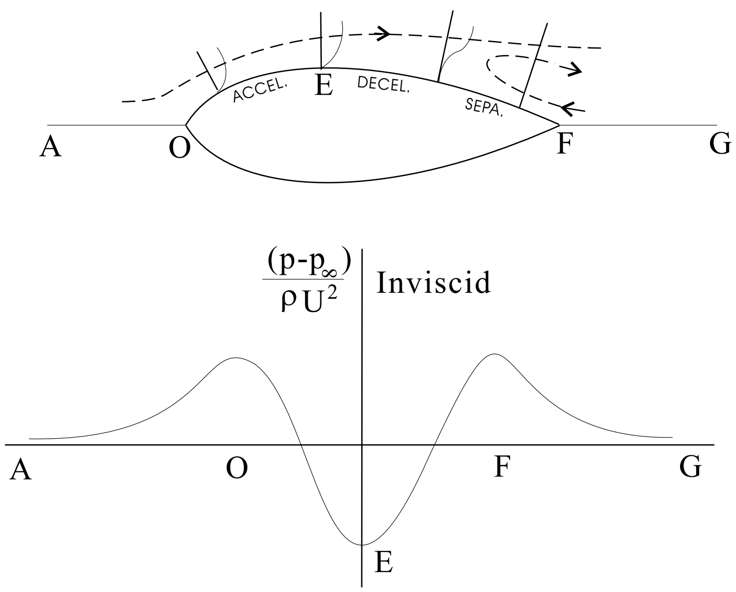

Figure 3.4.1: Pressure variation along the surface of a a blunt body

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the boundary layer the pressure variation must be the same.

Let us denote the tangential coordinate by x and the transverse coordinate by y, and recall the boundary layer equation

$$uu_x + vu_y = UU_x + \nu u_{yy} \tag{3.4.1}$$

On the body surface, the no slip condition leads to

$$UU_x + \nu u_{yy} = 0 \tag{3.4.2}$$

which implies that

$$u_{yy} > 0, \quad \text{if} \quad U_x < 0, \quad \text{deceleration} \tag{3.4.3}$$

$$u_{yy} = 0, \quad \text{if} \quad U_x = 0, \quad \text{no acceleration} \tag{3.4.4}$$

$$u_{yy} < 0, \quad \text{if} \quad U_x > 0, \quad \text{acceleration} \tag{3.4.5}$$

Let us differentiate (3.4.1),

$$uu_{xy} + vu_{yy} + u_y(U_x + v_y) = \nu u_{yyy}$$

On the body surface the no slip condition and continuity require that

$$u_{yyy} = 0$$

therefore the curvature of the velocity profile u_{yy} is an extremum. Now let us examine the implied profiles of shear and velocity by integration, sketched in Figure 3.4.2. In the stage of acceleration, $u_{yy} < 0$, hence u_y (shear) decreases in y. At the point of no acceleration, $u_{yy} = 0$, hence u_y (shear) must have a negative maxmum at some height $y > 0$. In the stage of deceleration, $u_{yy} > 0$, there must be a point where $u_{yy} = 0$ at some ehight $y = 0$ where shear is the greatest, u has a point of inflection.

If the flow external flow further decreases, the shear stress at the boundary vanishes, the velocity profile becomes tangential to the y axis. Still further deceleration cause flow reversal. The flow separates!!

Remark: Pressure gradient and vorticity: As another physical insight, let us consider the same flow arond the blunt body, see Figure 3.4.3, Since the vorticity in the boundary layer is dominated by $\zeta = u_y$, the total vorticity in the boundary layer is

$$\int_0^\delta \zeta dy = \int_0^\delta u_y\, dy = U(y = \delta)$$

which is the inviscid velocity at the outer edge of the boundary layer. Now the total rate of flux of vorticity is

$$\int_0^\delta u\zeta dy = \int_0^\delta uu_y\, dy = \frac{U^2}{2}$$

hence the mean speed of vorticity transport is $U/2$. The spatial variation of vorticity transport is

$$\frac{d}{dx}\left(\frac{U^2}{2}\right) = UU_x = -\frac{1}{\rho}p_x$$

This vorticity increase or decrease, which is forced by the external pressure gradient, must come from the body surface. Thus UU_x is the vorticity source strength per unit length of the surface. From O to A, $U_x > 0$ vorticity is generated. From A to B, $U_x < 0$, vorticity is destroyed.

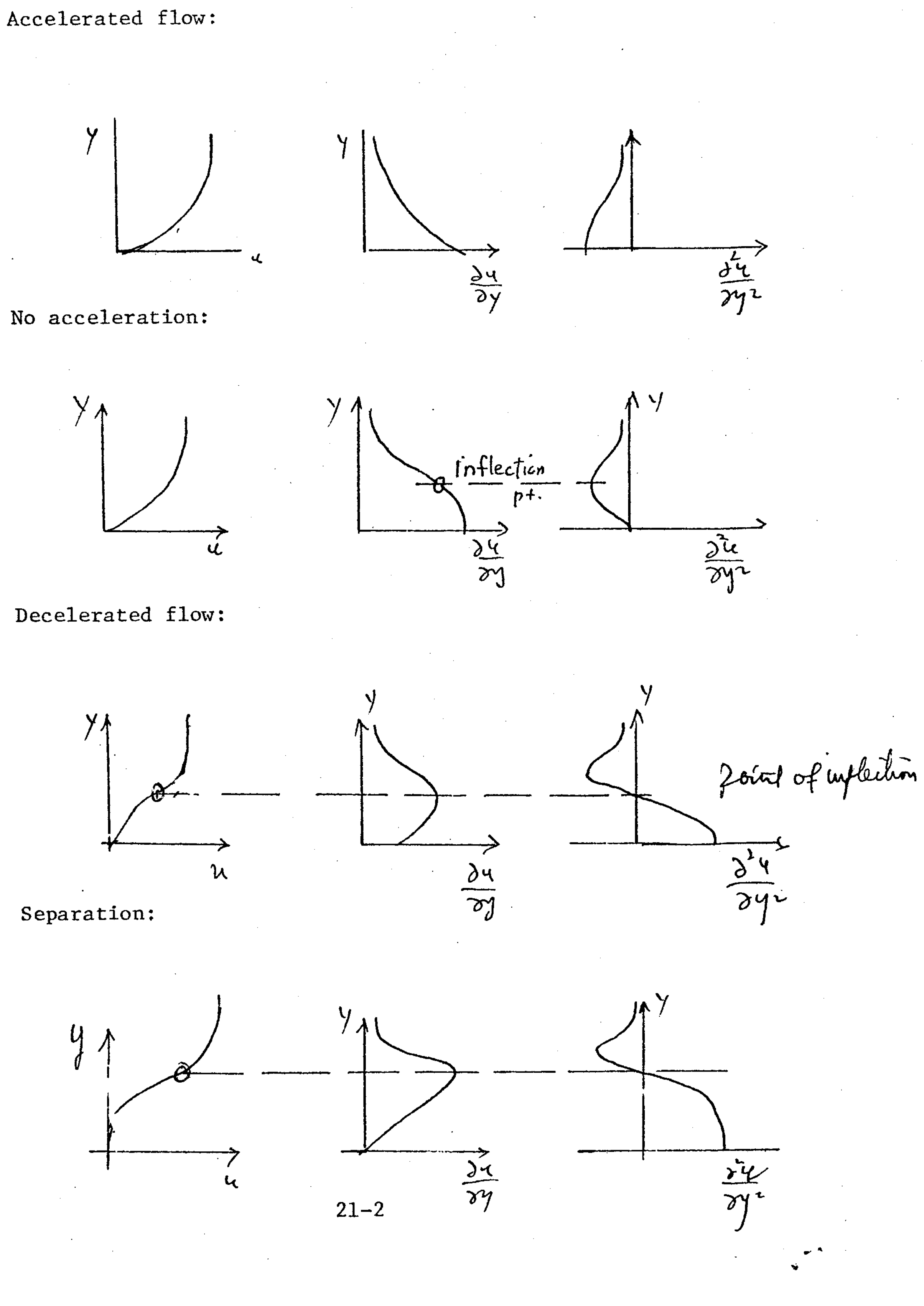

Figure 3.4.2: Effects of pressure gradient on velocity shear.

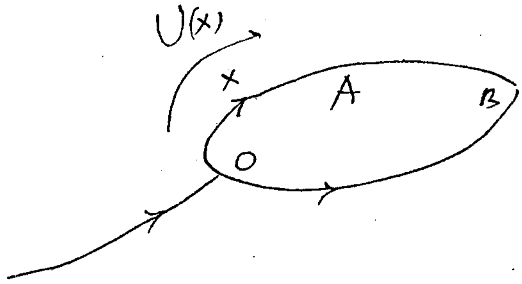

Figure 3.4.3: Vorticity and pressure gradient

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