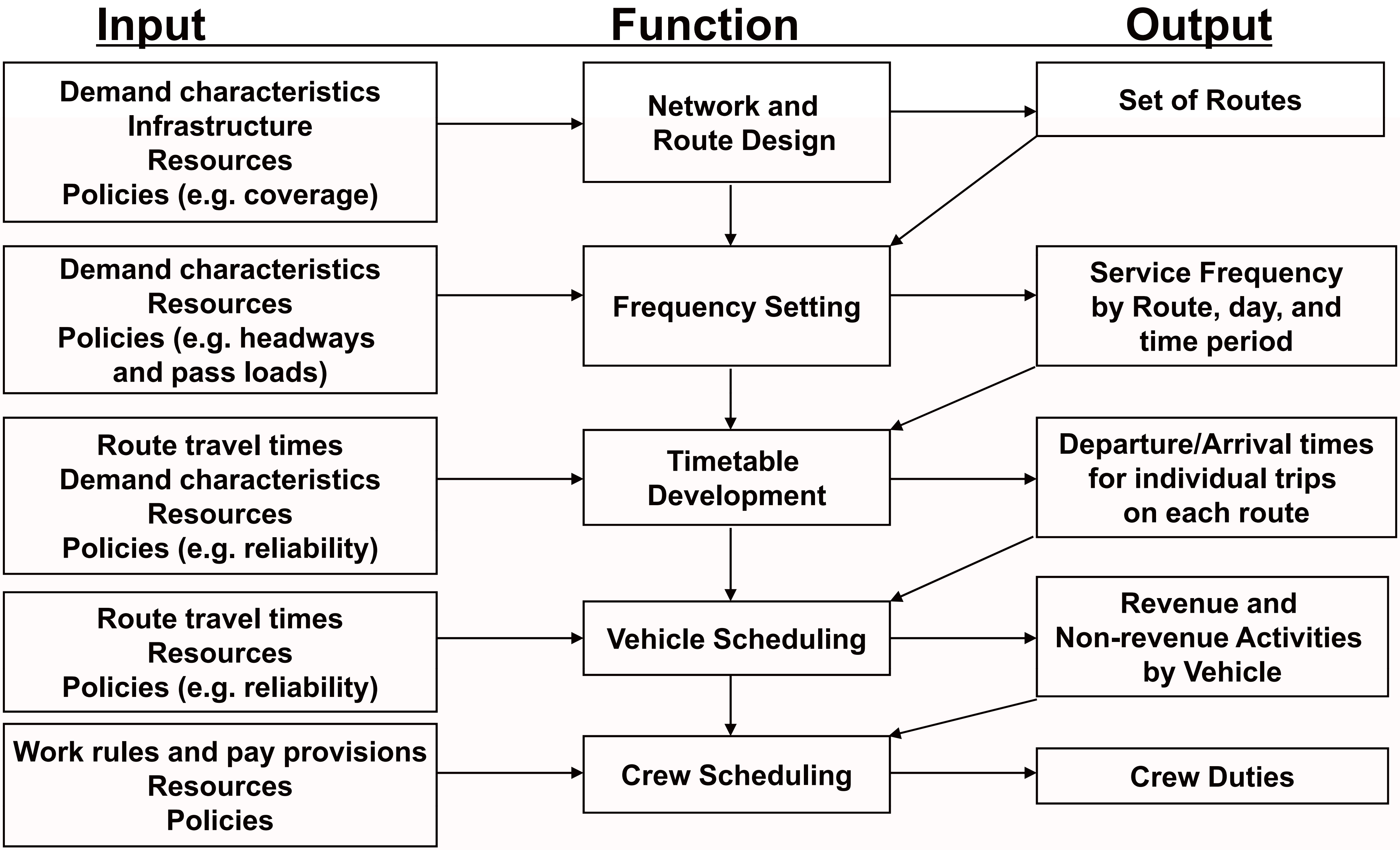
Frequency Determination

Outline

1. Service Planning Hierarchy
2. Introduction to Scheduling
3. Frequency Determination
   - current practice
   - simple rules
4. Optimization formulation for independent routes
5. Heuristic for complex overlapping networks

Service Planning Hierarchy



Service Planning Hierarchy

Infrequent Decisions | Service Considerations Dominate | Judgement & Manual Analysis Dominate

- Network Design
- Frequency Setting
- Timetable Development
- Vehicle Scheduling
- Crew Scheduling

Frequent Decisions | Cost Considerations Dominate | Computer-Based Analysis Dominates

Introduction to Scheduling

Sequence of steps:

1. Determine Running Times and Layovers based on:
   - running time data
   - desired reliability levels
2. Determine Frequencies by route and time period
3. Determine # of vehicles by time period, focusing on:
   - policies affecting integer constraints
   - revise step 1 and 2 decisions as needed
   - focus on transition periods

Introduction to Scheduling

Sequence of steps:

4. Determine timetable, typically:
   - start at peak load point
   - generate start and end times
5. Chain vehicle trips together to form vehicle “blocks”
6. Cut and combine vehicle blocks to form crew duties or runs.

Common Issues

A. Integrality constraints:

- **If book times are 26 mins each way, recovery time is 5 mins at each terminus, and desired frequency is 10 per hour:**

$$Min\ \#\ of\ vehicles = \left\lceil \frac{31*2}{6} \right\rceil^{+} = \left\lceil 10.3 \right\rceil^{+} = 11$$

  Trade-off between shortening cycle time by 2 mins to save 1 vehicle, or not?

- **In a similar case, but if desired frequency is 1 per hour, choice is to:**
  - **shorten cycle time by 2 mins, or**
  - **interline with another route having cycle time of 58 mins or less**

Common Issues

B. Marginal cost of additional trips:

- a single trip for a vehicle/crew in peak period is typically uneconomic, so choice is between:
  - eliminating the single trip and saving the vehicle/crew costs
  - adding additional trips to make a minimum sized "piece of work"
- where you add extra trips will affect the costs -- outer shoulders of peak tend to be most expensive.

C. Hard constraints:

- contract terms include hard constraints which determine feasibility or infeasibility

Frequency Determination: Current Practice

Frequencies typically based on:

- **Policy headways - vary by time of day and route type**
- **Maximum loads - vary by time of day and route type**

These represent constraints rather than decision algorithms

Simple Rules for Frequency Determination

A. Constant max load factor at a level below official max load factor

- may vary by time period

B. Constant average occupancy level subject to capacity constraint

- may also be subject to a max time for loads above a specified level

Importance of Frequency Determination

- Major short-range planning decision
- Affects service quality through wait time and crowding
- Affects transit path selection (assignment) in complex networks

Two different contexts:

- North American city:
  - ridership sensitive to service quality
  - sparse network, little transit path choice
  - maximum acceptable crowding levels specified
  - defined level of subsidy available
- Less developed country city:
  - ridership constrained by capacity
  - crowding levels very high
  - dense network, significant transit path choice

North American Frequency Determination Problem*

Decision variables:

- headway on each route for each time period

Objective function:

- maximize: consumer surplus + social ridership benefit

  ≡ *a* • wait time savings + *b* • ridership

Subject to constraints on:

- total subsidy is exhausted
- total fleet size is not exceeded
- headway meets policy maximums and loading maximums

* *Furth, P.G. and N.H.M. Wilson, "Setting Frequencies on Bus Routes: Theory and Practice," Transportation Research Record 818, 1981, pp 1-7*

Maximize Social Surplus (multiple routes problem)

Context

Given a fixed fleet size and subsidy,

Determine: Optimal allocation of this fleet to the various routes (thus setting the frequencies on the routes)

Formulation

Maximize: $\sum_{routes}$ social surplus

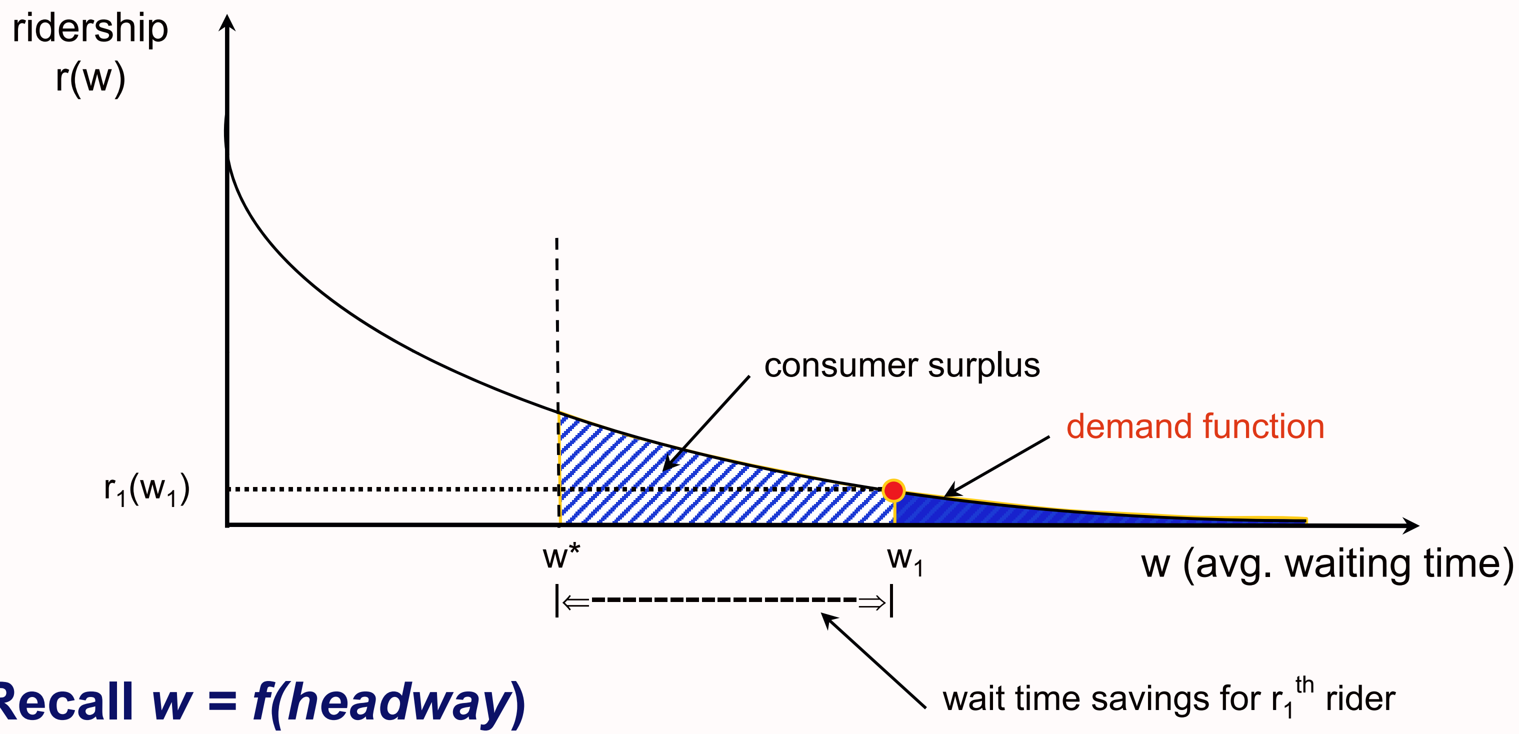
Subject to: subsidy

fleet size

acceptable LOS

Max Social Surplus (cont'd)

- **Social surplus**
  - **(i) consumer surplus**



Recall *w = f(headway)*

Max Social Surplus (cont'd)

For a given headway h^*, $w = f(h^*) = w^*$

consumer surplus:

$$CS = b\int_{w^*}^{\infty} r(w)\, dw$$

where

	b	=	**monetary value of waiting time**
∴	CS	=	**savings in wait time cost that accrues to system riders who would have been prepared to ride at higher waiting times**

Max Social Surplus (cont'd)

(ii) Social benefits (of transit)

- -- mobility for non-auto owners
- -- reduced congestion
- -- reduced pollution
- -- reduced energy consumption
- -- positive land use effects

All of these benefits are highly associated with ridership:

SB for a route = $a \cdot r(w)$

where, a = monetary value of social benefit associated with an additional rider <u>less</u> the fare

Max Social Surplus (cont'd)

We know $w = f(h)$

∴ **derive the function** $r(h)$ **from** $r(w)$
i.e., $r(h) = r(f(h))$

(iii) Total social surplus to maximize:

$$CS + SB - \sum_{routes\,(i)} \left[b \int_{h_i *}^{\infty} r(h)\,dh + a r(h_i *) \right]$$

where h^* **is the headway on route** i **whose optimal value is to be determined (decision variable)**

Max Social Surplus (cont'd)

- **Constraints**

  (i) subsidy

$$\sum_{routes}[\text{operating cost - fare revenue}] = \text{subsidy limit}$$

$$\sum_{routes\ (i)}[c(h_i^*) - F \cdot r(h_i^*)] = S_o$$

fare

  (ii) fleet size

$$\sum_{routes\ (i)} \frac{\text{round - trip time}}{h_i^*} \le \text{Fleet size, M}$$

\# of buses needed for each route

Max Social Surplus (cont'd)

(iii) Level of service

$h^* < h_o$

vehicle load $< l_o$

→ $g(h^*)$

h_o, l_o = headway and load standards

Max Social Surplus (cont'd)

Critical Assumptions/Limitations:

- independence across routes

  i.e., ridership on a route depends only on the headway of that route

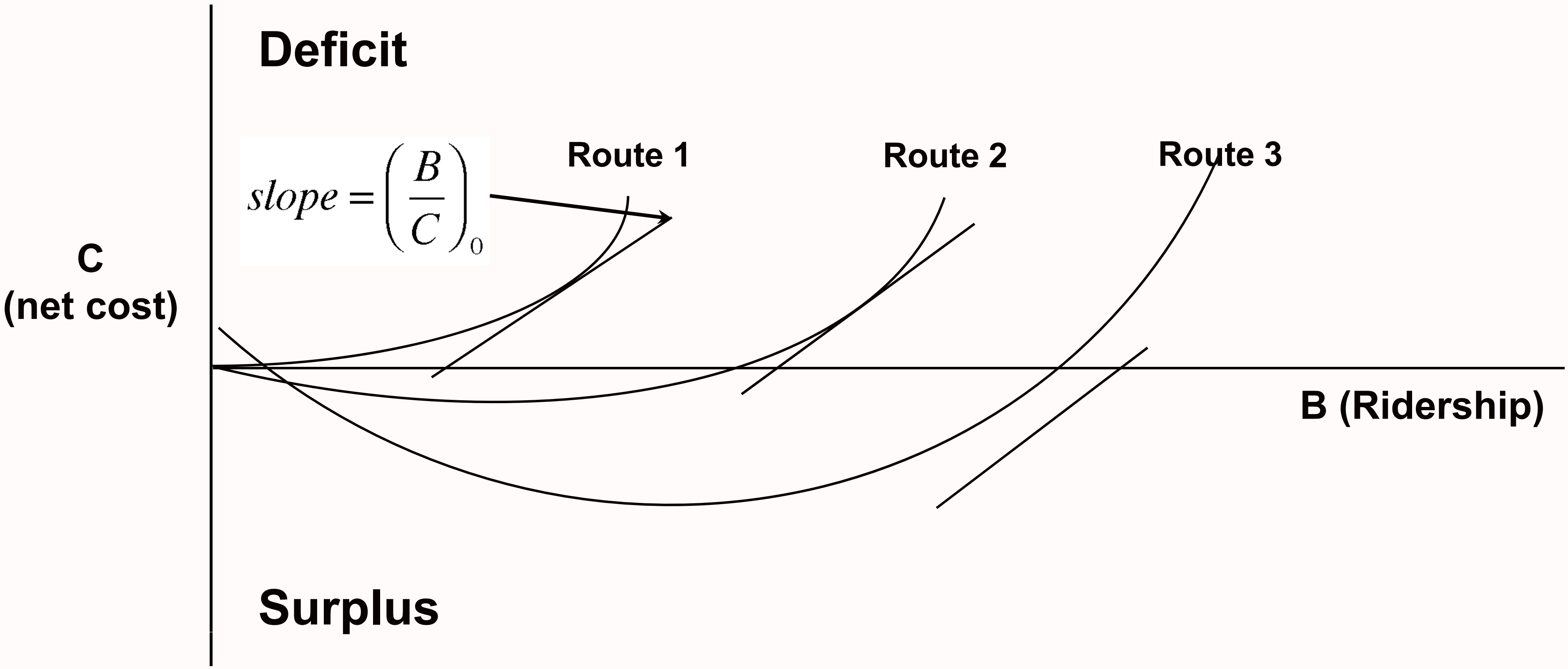
  BUT, in general, ridership also depends on headways on competing routes <u>and</u> complementary routes (transfers)
- network design is not considered

Advantages:

- ridership = *f* (frequency)
- captures trade-offs across routes
- introduces system wide budget constraint

Efficiency in Subsidy Allocation

This is a resource allocation problem: for optimality, allocate enough resources to each route so that Marginal Benefit/Cost Ratio is same on each route.



Conclusions

- square root rule is valid where constraints are not binding
- problem can be solved using Lagrangian relaxation and single variable search techniques -- not very complex
- existing scheduling practice over allocates service to peak and to long, high ridership routes
- minimizing wait time assuming fixed demand gives similar solutions to more complex objective and variable demand
- best allocation of resources is quite robust with respect to objectives and parameters assumed

Developing Country/City Frequency Determination Problem

Objectives:

- minimize crowding levels
- minimize waiting times

Subject to constraints on:

- loading feasibility
- passenger assignment
- total fleet size

Passenger Assignment Heuristic Approach

1. **Classify flow into:**
   - "captive flow" (CF) -- any O-D pair with only one feasible path
   - "variable flow" (VF) -- O-D pairs with more than one feasible path

2. **Assign VF in proportion to frequency share on acceptable routes**
   - consistent with random bus arrival process

$$\frac{D_i}{\sum_{j \in J} D_j} = \frac{F_i}{\sum_{j \in J} F_j}$$

where

D_i = demand assigned to route *i* for specific O-D pair

F_i = frequency offered on route *i*

J = set of acceptable routes

Models

A. Normative Model

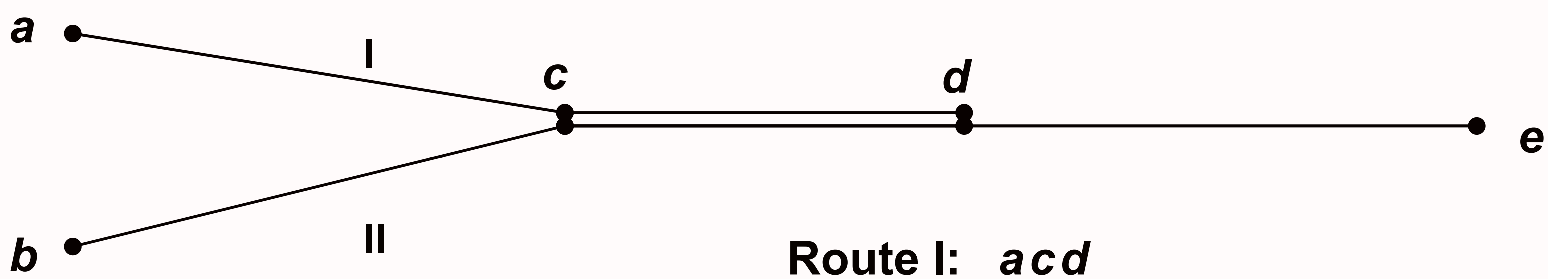
- "assign" passenger flows to routes with minimum round trip vehicle time among all acceptable paths
- compute frequency and fleet size required on this assignment basis

B. Descriptive Model

- assign passengers to alternative acceptable paths in proportion to frequency share in an iterative process

The difference in the total fleet sizes from the normative and descriptive models indicates the extent of inefficiency resulting from the overlapping route structure.

Simple Example of Overlapping Routes



Route I: *acd*
Route II: *bcde*

- **O-D pair *cd* is VF, all other pairs are CF**
- **ideally would like to assign *cd* flow to route I, which is shorter, but these passengers will take route I or II depending on which arrives first.**
- **some *ce* passengers may be forced to board route I buses, then make a transfer at *d* to route II**

MIT OpenCourseWare
http://ocw.mit.edu

1.258J / 11.541J / ESD.226J Public Transportation Systems
Spring 2010

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.