MACRO DESIGN MODELS FOR A SINGLE ROUTE

Outline

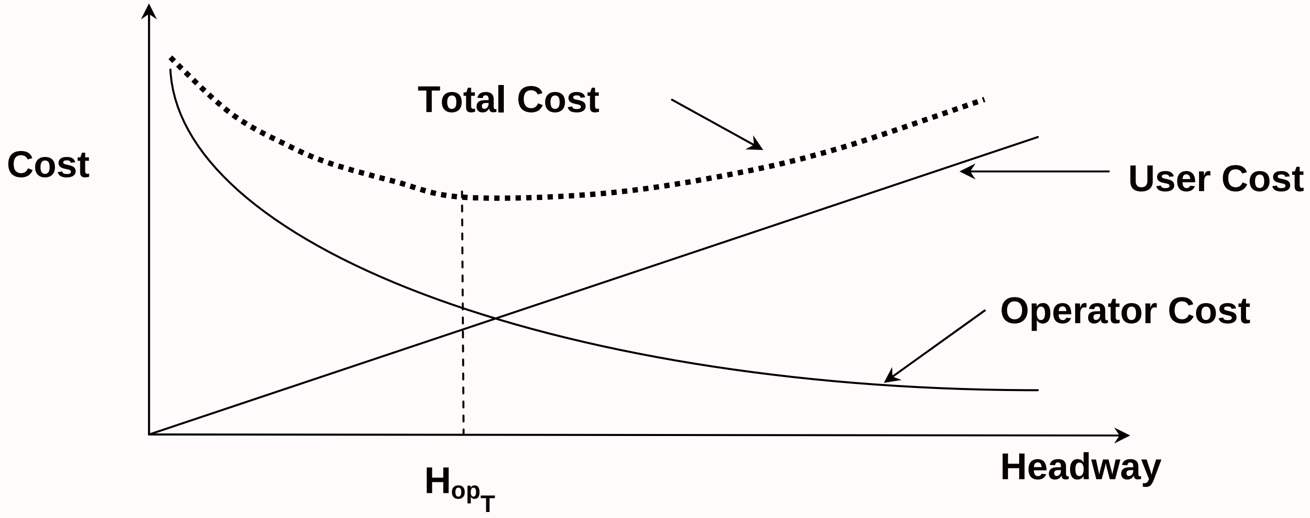
1. Introduction to analysis approach
2. Bus frequency model
3. Bus size model
4. Stop/station spacing model

Introduction to Analysis Approach

- Basic approach is to establish an aggregate total cost function including:
  - operator cost as *f* (design parameters)
  - user cost as *g* (design parameters)
- Minimize total cost function to determine optimal design parameter (s.t. constraints)

Variants include:

- Maximize service quality s.t. budget constraint
- Maximize consumer surplus s.t. budget constraint



Bus Frequency Model: the Square Root Model

Problem: define bus service frequency on a route as a function of ridership

Total Cost = operator cost + user cost

$$Z = c \bullet \frac{t}{h} + b \bullet r \bullet \frac{h}{2}$$

where
Z = total (operator + user) cost per unit time
c = operating cost per unit time
t = round trip time
h = headway – the decision variable to be determined
b = value of unit passenger waiting time
r = ridership per unit time

Minimizing Z w.r.t. h yields:

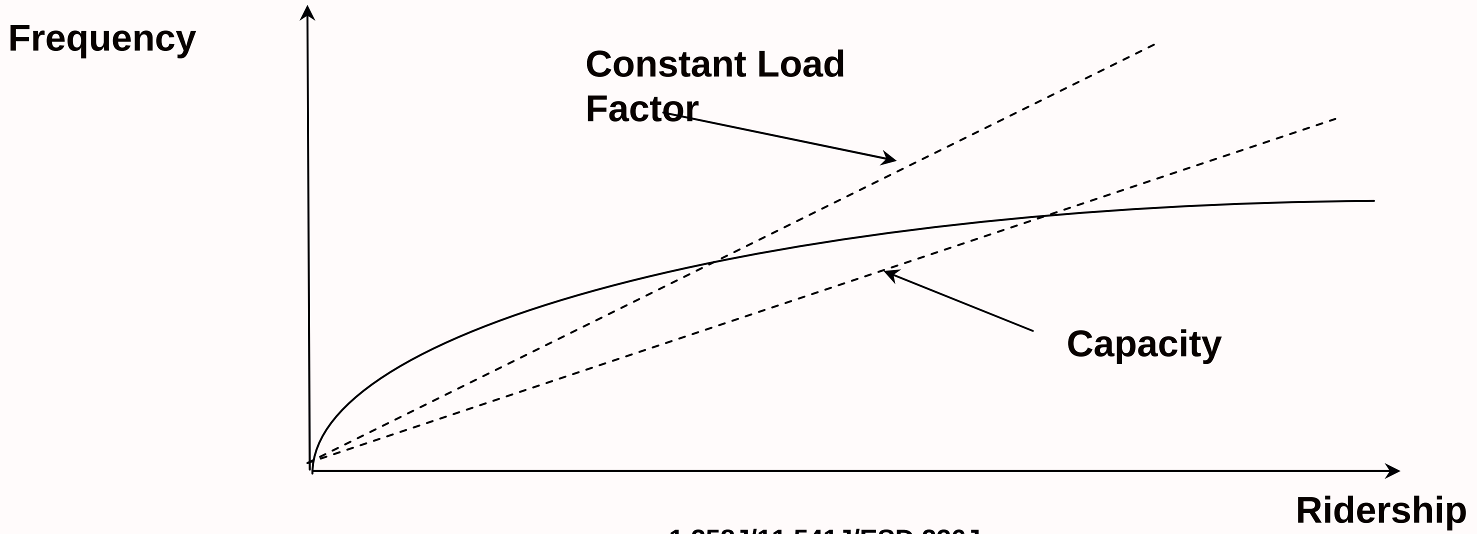
$$h = \sqrt{\frac{2ct}{br}} \; or \; \sqrt{2\left(\frac{c}{b}\right)\left(\frac{t}{r}\right)}$$

Square Root Model (cont'd)

This is the Square Rule with the following implications:

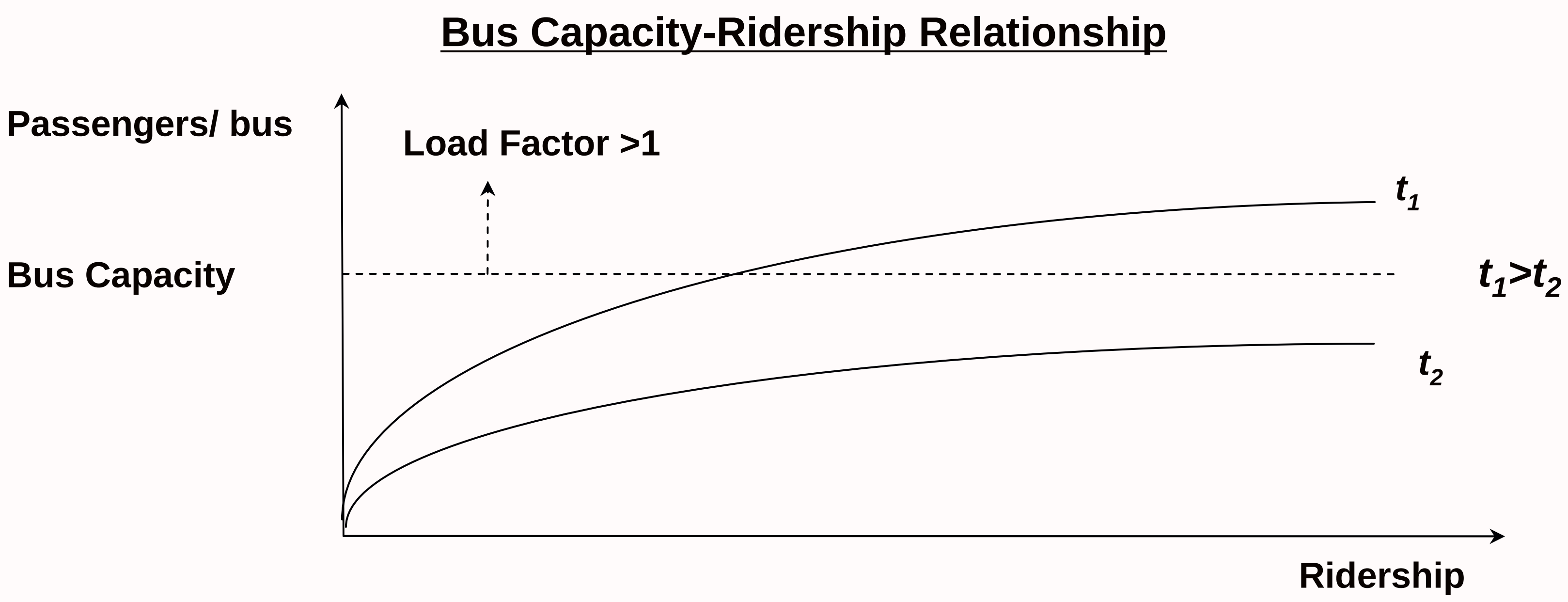
- **high frequency is appropriate where (cost of wait time/cost of operations time) is high**
- **frequency is proportional to the square root of ridership per unit time for routes of similar length**

Frequency-Ridership Relationship



Square Root Model (cont'd)

- Load factor is proportional to the square root of the product of ridership and route length.



Square Root Model (cont'd)

Critical Assumptions:

- bus capacity is never binding
- only frequency benefits are wait time savings
- ridership $\neq f$ (frequency)
- simple wait time model
- budget constraint is not binding

Possible Remedies:

- introduce bus capacity constraint
- modify objective function
- introduce $r=f(h)$ and re-define objective function
- modify objective function
- introduce budget constraint

Bus Frequency Example

If: $c = \$90$/bus hour

$b = \$10$/passenger hour

$t = 90$ mins

$r = 1{,}000$ passengers/hour

Then: $h_{OPT} = 11$ mins

Bus Size Model

Problem: define optimal bus size on a route

Assumptions:

- Desired load factor is constant
- Labor cost/bus hour is independent of bus size
- Non-labor costs are proportional to bus size
- Bus dwell time costs per passenger are independent of bus size

Using same notation as before plus:

w = labor cost per bus hour
p = passenger flow past peak load point
k = desired bus load - the decision variable to be determined

Then $Z = w \bullet \frac{t}{h} + b \bullet r \bullet \frac{h}{2}$

Now $h = \frac{k}{p}$ by assumption above

$\therefore \quad Z = \frac{wtp}{k} + \frac{brk}{2p}$

Minimizing Z w.r.t. k gives: $k_{OPT} = \sqrt{\frac{2p^2wt}{rb}}$

Bus Size Model (cont'd)

Result is another square root model, implying that optimal bus size increases with:

- round trip time
- ratio of labor cost to passenger wait time cost
- peak passenger flow
- concentration of passenger flows

Previous example extended with:

p = 500 pass/hour

w = $40/bus hour

all other parameters as before:

Then: h_{OPT} = 55

Stop/Station Spacing Model

Problem: determine optimal stop or station spacing

Trade-off is between walk access time (which increases with station spacing), and in-vehicle time (which decreases as station spacing increases) for the user, and operating cost (which decreases as station spacing increases)

Define Z = total cost per unit distance along route and per headway

and T_{st} = time lost by vehicle making a stop

c = vehicle operating cost per unit time

s = station/stop spacing - the decision variable to be determined

N = number of passengers on board vehicle

v = value of passenger in-vehicle time

D = demand density in passenger per unit route length per headway

v_{acc} = value of passenger access time

w = walk speed

c_s = station/stop cost per headway

Stop/Station Spacing Model (cont'd)

$$Z = \frac{T_{st}}{s}(c + N \bullet v) + \frac{c_s}{s} + \frac{s}{4} \bullet D \bullet \frac{v_{acc}}{w}$$

Minimizing Z w.r.t. s gives:

$$s_{OPT} = \left[\frac{4w}{Dv_{acc}} \left[c_s + T_{st}(c_v + Nv) \right] \right]^{1/2}$$

Yet another square root relationship, implying that station/stop spacing increases with:

- **walk speed**
- **station/stop cost**
- **time lost per stop**
- **vehicle operating cost**
- **number of passengers on board vehicle**
- **value of in-vehicle time**

and decreases with:

- **demand density**
- **value of access time**

Bus Stop Spacing

U.S. Practice

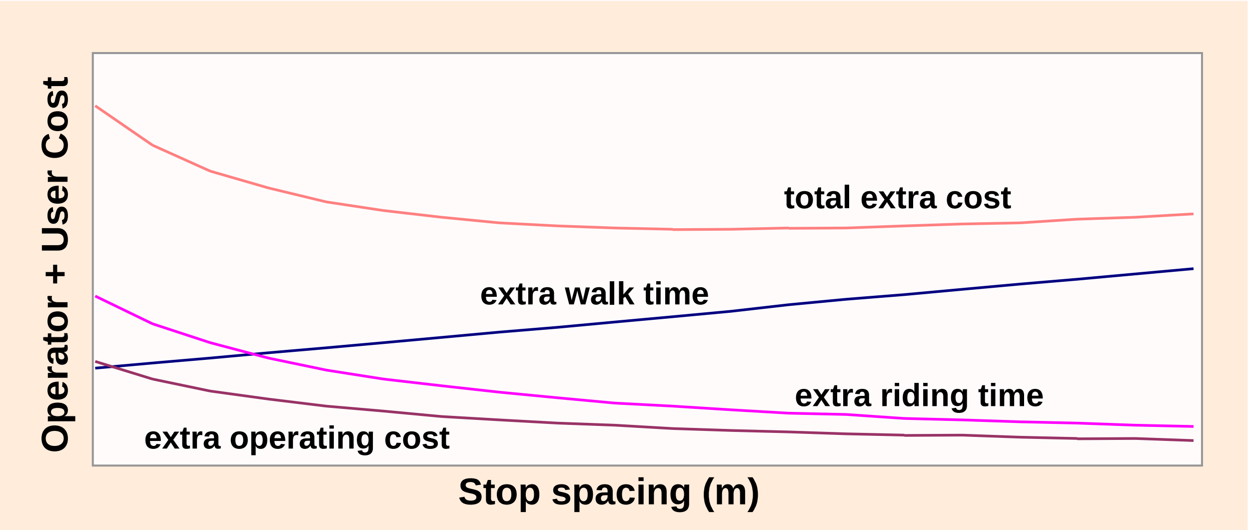
- 200 m between stops (8 per mile)
- shelters are rare
- little or no schedule information

European Practice

- 320 m between stops (5 per mile)
- named & sheltered
- up to date schedule information
- scheduled time for every stop

Stop Spacing Tradeoffs

- Walking time
- Riding time
- Operating cost
- Ride quality



Walk Access: Block-Level Modeling

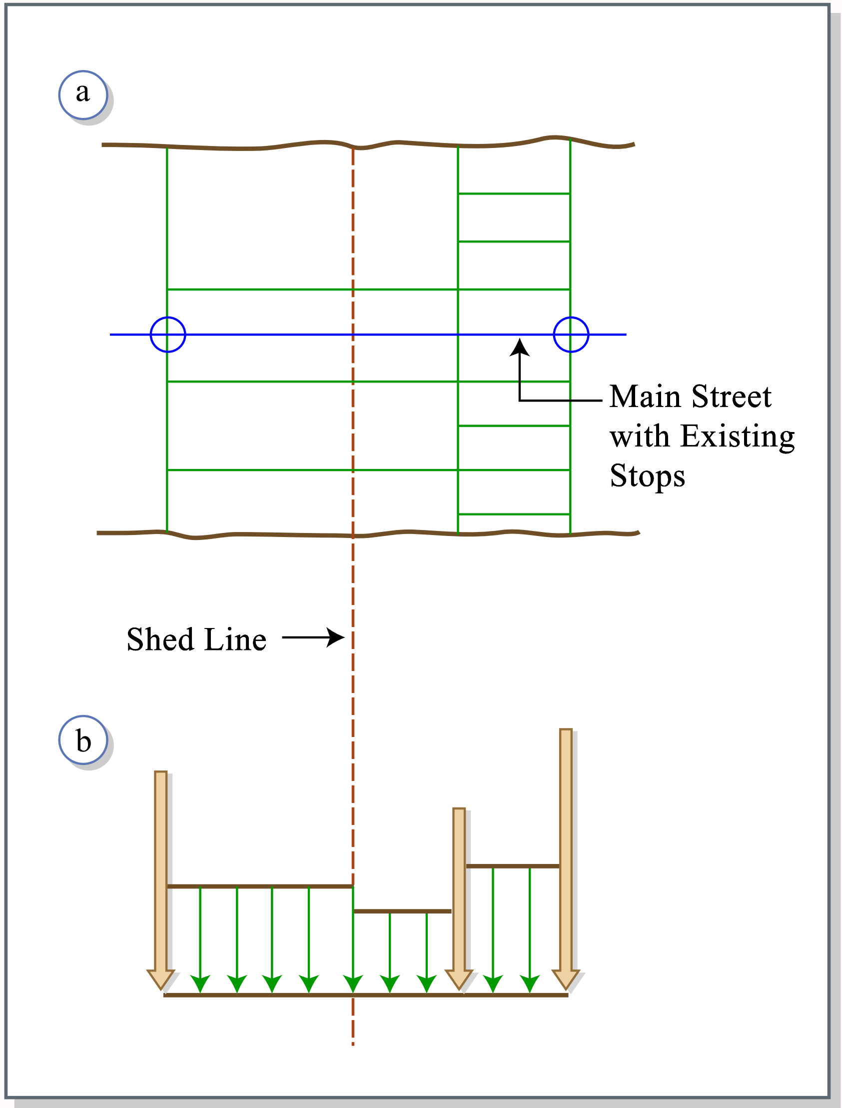


Figure by MIT OpenCourseWare.

Results: MBTA Route 39*

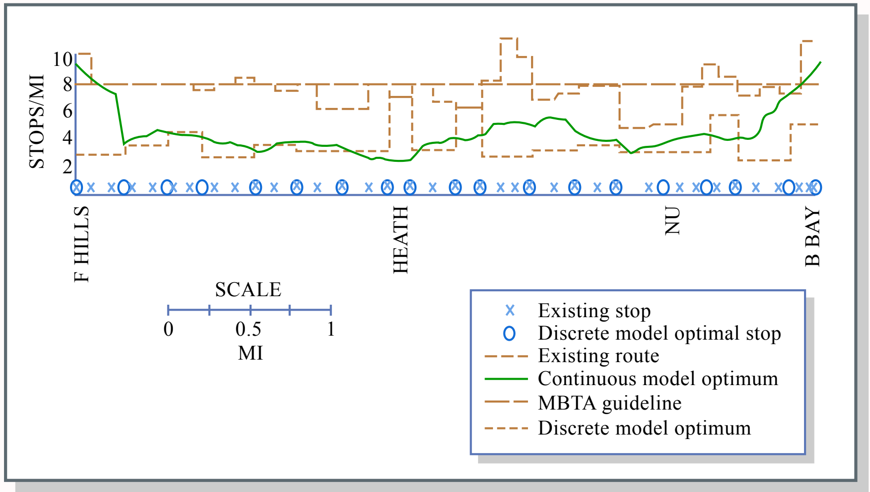


Figure by MIT OpenCourseWare.

AM Peak Inbound results

- Avg walking time up 40 s
- Avg riding time down 110 s
- Running time down 4.2 min
- Save 1, maybe 2 buses

Source: Furth, P.G. and A. B. Rahbee, "Optimal Bus Stop Spacing Using Dynamic Programming and Geographic Modeling." Transportation Research Record 1731, pp. 15-22, 2000.

Bus Stop Locations and Policies

- Far-side (vs. Near-side)
  - less queue interference
  - easier pull-in
  - fewer ped conflicts
  - snowbank problem demands priority in maintenance
- Curb extensions benefit transit, peds, and traffic (0.9 min/mi speed increase)
- Pull-out priority (it’s the law in some states)
- Reducing dwell time (vehicle design, fare collection, fare policy)

1.258J / 11.541J / ESD.226J Public Transportation Systems
Spring 2010