Performance of a Single Route

Outline

1. Wait time models
2. Service variation along route
3. Running time models

Wait Time Models

Simple deterministic model:

$E(w) = E(h)/2$

where

$E(w)$ = expected waiting time

$E(h)$ = expected headway

Model assumptions:

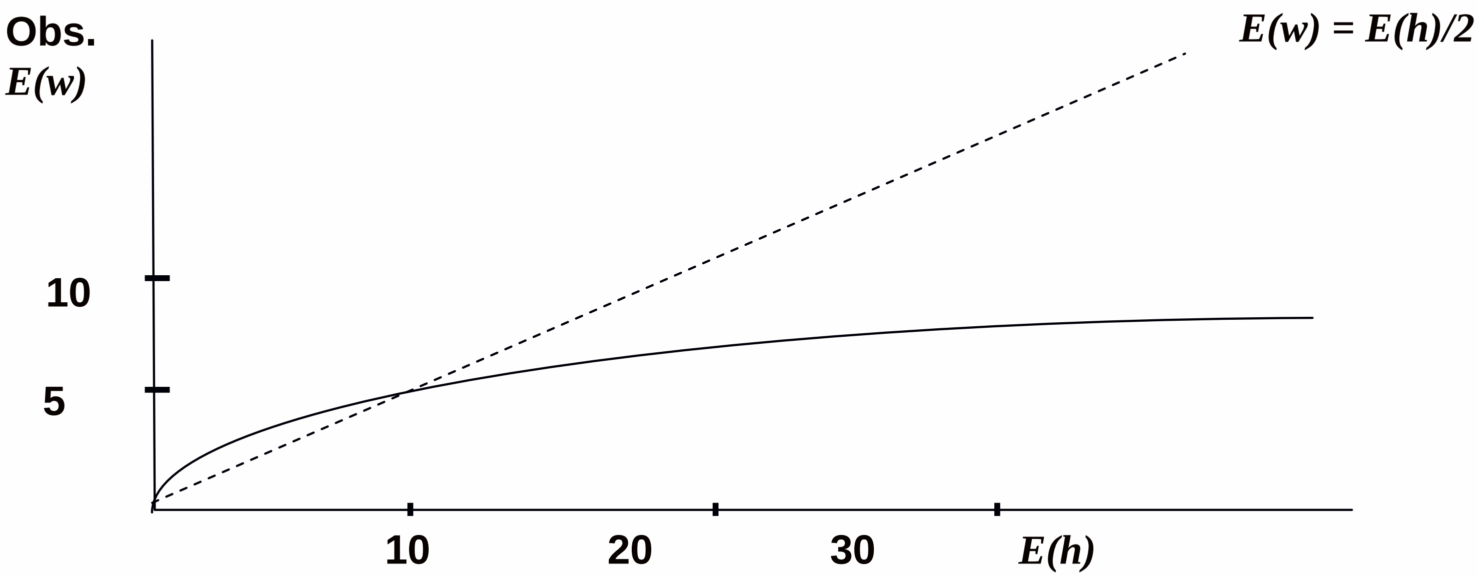
- passenger arrival times are independent of vehicle departure times
- vehicles depart deterministically at equal intervals
- every passenger can board the first vehicle to arrive

Passenger Arrival Process

- **Individual, group, and bulk passenger arrivals**
- **Passengers can be classified in terms of arrival process:**
  - **random arrivals**
  - **time arrival to minimize $E(w)$**
  - **arrive with the vehicle, i.e. have $w = 0$**

Passenger Arrival Process (cont'd)

- For long headway service have "schedule delay" as well as wait time



Vehicle Departure Process

Vehicle departures typically not regular and deterministic

Wait Time Model refinement:

If: $n(h)$ = **# of passengers arriving in a headway** h

$\overline{w}(h)$ = **mean waiting time for passengers arriving in headway** h

$g(h)$ = **probability density function of headway**

Then:

$E(w)$ = $\dfrac{\text{Expected Total Passenger Waiting Time per vehicle departure}}{\text{Expected Passengers per vehicle departure}}$

$$\frac{\int_0^{\infty} n(h)\overline{w}(h)g(h)dh}{\int_0^{\infty} n(h)g(h)dh}$$

Vehicle Departure Process

Now if: $n(h) = \lambda \bullet h$ where λ is passenger arrival rate

$$\bar{w}(h) = \frac{h}{2}$$

Then:

$$E(w) = \frac{E(h^2)}{2E(h)} = \frac{E(h)}{2}\left[1 + \frac{\text{var}(h)}{(E(h))^2}\right] = \frac{E(h)}{2}\left[1 + (\text{cov}(h))^2\right]$$

Vehicle Departure Process Examples

A. If $var(h) = 0$:

$$E(w) = E(h)/2$$

B. If vehicle departures are as in a Poisson process:

$var(h) = (E(h))^2$ and $E(w) = E(h)$

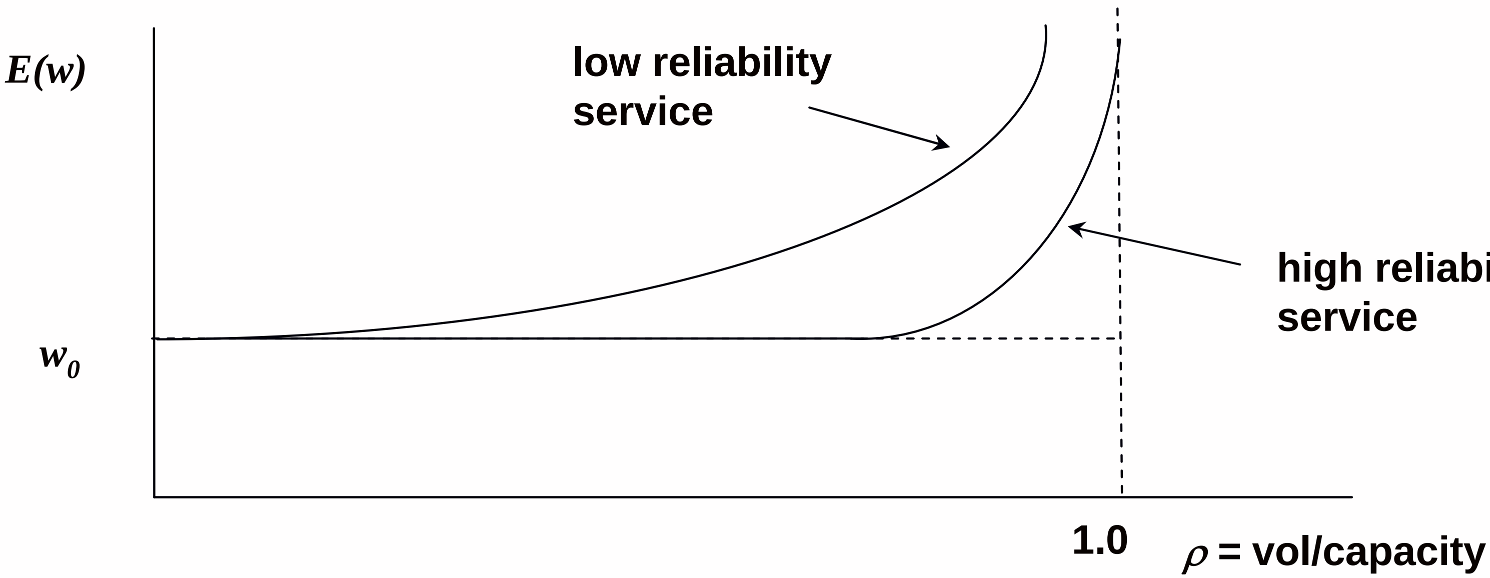
C. The headway sequence is 5, 15, 5, 15, ...
then:

$E(h) =$ 10

$E(w) =$ 2.5 * 0.25 + 7.5 * 0.75 = 6.25 mins

Passenger Loads Approach Vehicle Capacity

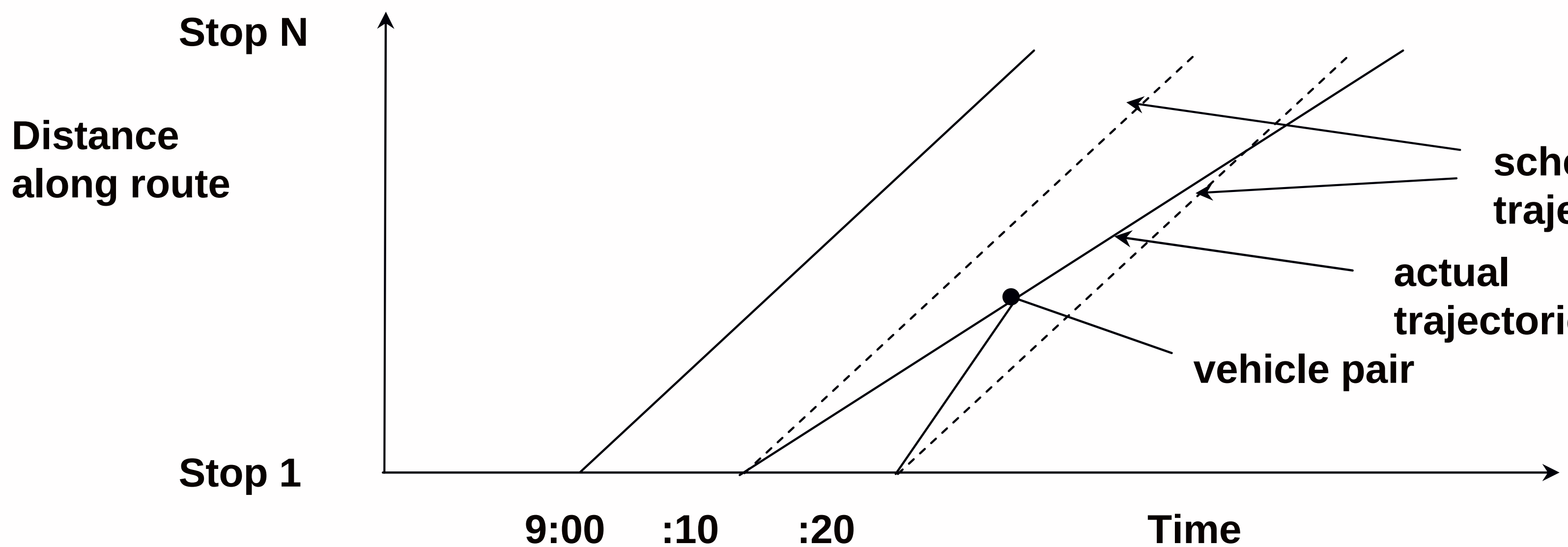
- **Not all passengers can board the first vehicle to depart:**



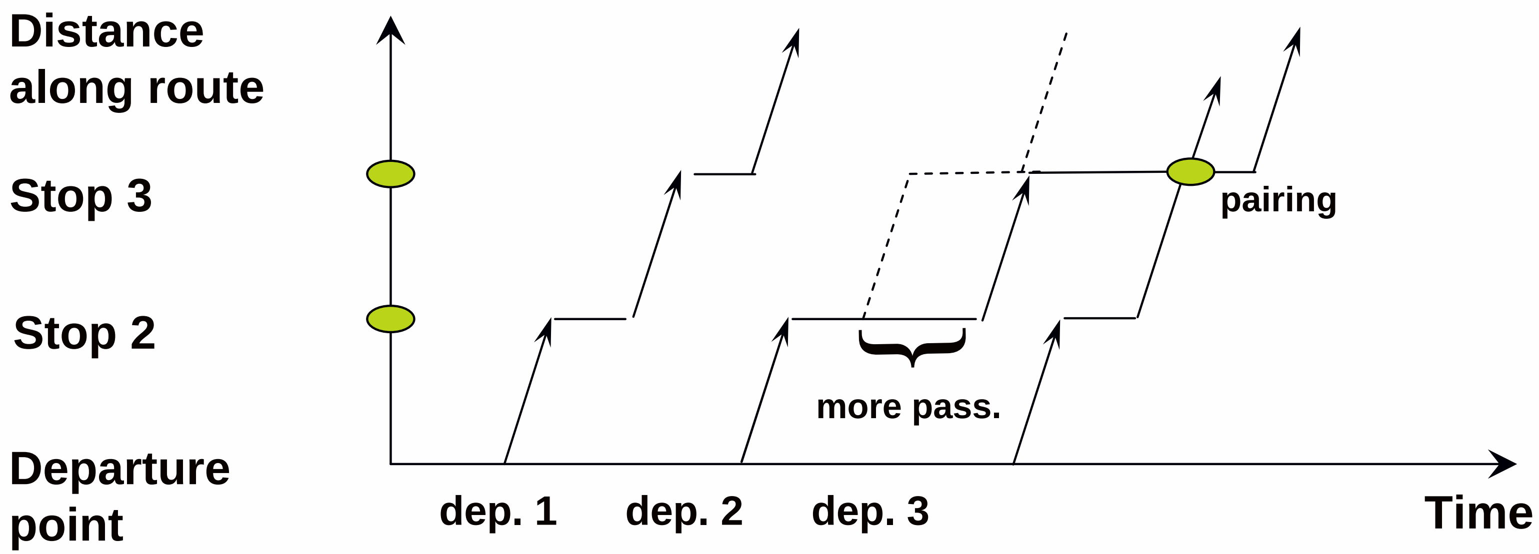
- **General queuing relationship**

Service Variation Along Route

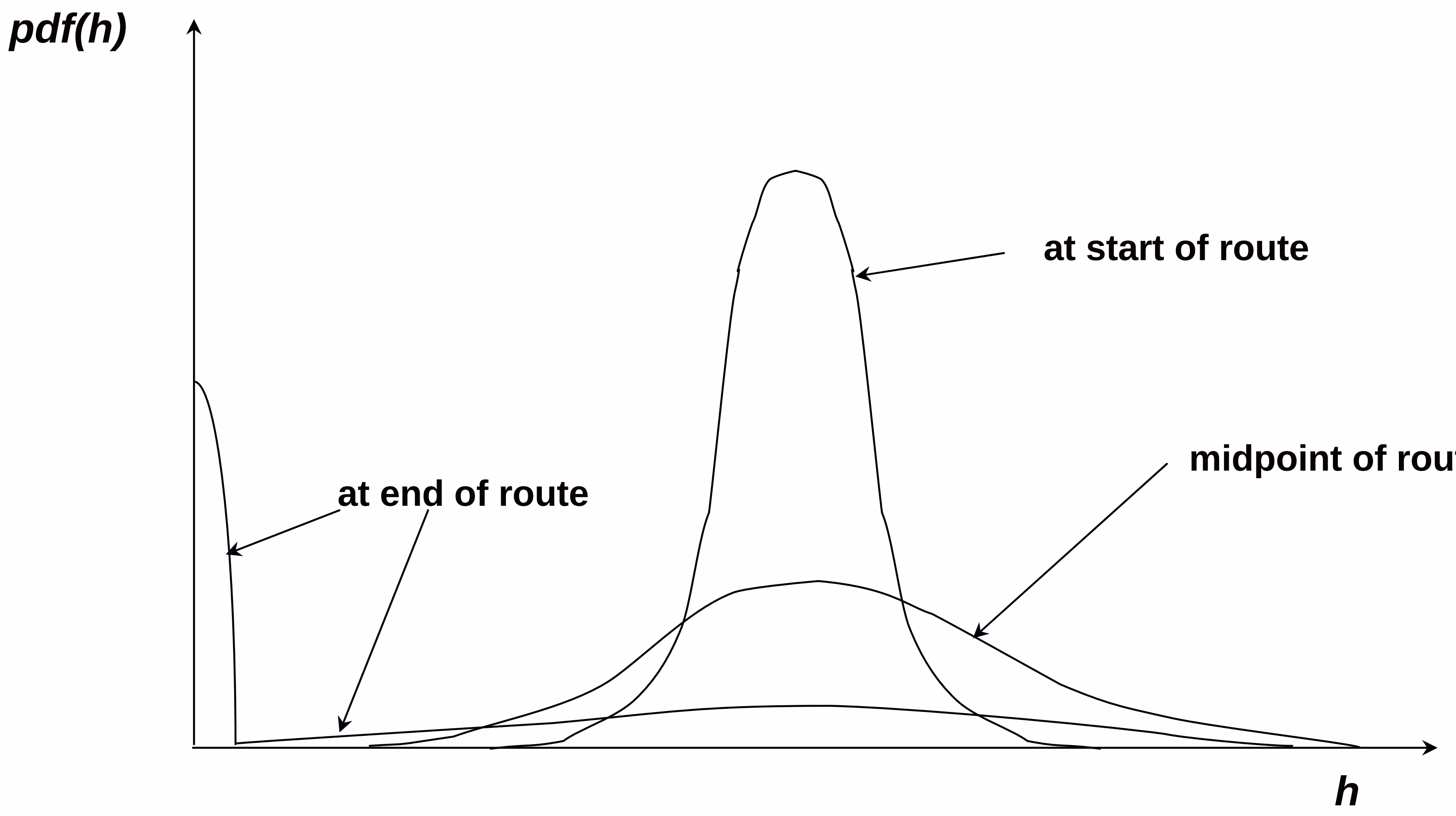
- **Service may vary along route even without capacity becoming binding:**
  - **the headway distribution can vary along the route, affecting $E(w)$**
  - **at the limit vehicles can be paired, or bunched**
  - **this can also result in passenger load variation between vehicles**



Service Variation Along Route (cont'd)



Service Variation Along Route (cont'd)



Factors Affecting Headway Deterioration

- **Length of route**
- **Marginal dwell time per passenger**
- **Stopping probability**
- **Scheduled headway**
- **Driver behavior**

Simple model:

$$e_i = (e_{i-1} + t_i)(1 + p_{i-1} \cdot b)$$

where e_i = headway deviation (actual-scheduled) at stop i

t_i = travel time deviation (actual-scheduled) from stop $i-1$ to i

p_i = passenger arrival rate at stop i

b = boarding time per passenger

Mathematical Model for Headway Variance*

$$\text{var}(h_i) = \text{var}(h_{i-1}) + \text{var}(\Delta t_{i-1}) + 2p_{i-1}(1-p_{i-1})(c \cdot \bar{q}_{i-1} + \ell)^2$$
$$+ 2c^2 \text{var}(q_{i-1})\left[1-\rho_q + p_{i-1}\rho_q\right](1-p_{i+1})$$
$$+ c(1-p_{i-1})^2 \cdot \text{cov}(\Delta q_{i-1}, h_{i-1})$$

where: $\text{var}(h_i)$ = headway variance at stop i

$\text{var}(\Delta t_i)$ = variance of the difference in running time between successive buses between stops $i-1$ and i

p_i = probability bus will skip stop i

c = marginal dwell time per passenger served at a stop

$\bar{q}_i$ = mean number of passengers per bus served at stop i

ℓ = the constant term of the dwell time function

$\text{var}(q_i)$ = variance of the number of passengers served per bus at stop i

ρ_q = correlation coefficient between the passengers served by successive buses at a stop

$\text{cov}(\Delta q_i, h_i)$ = covariance of the difference in number of passengers served by successive buses and the headway at stop i

*** Adebisi, O., "A Mathematical Model for Headway Variance of Fixed Bus Routes." Transportation Research B, Vol. 20B, No. 1, pp 59-70 (1986).**

Vehicle Running Time Models

Different levels of detail:

A. Very detailed, microscopic simulation:

- **represents vehicle motion and interaction with other vehicles, e.g. buses operating in mixed traffic, or train interaction through control system**

B. Macroscopic:

- **identify factors which might affect running times**
- **collect data and estimate model**

Vehicle Running Time Models

Running Time includes dwell time, movement time, and delay time:

- **dwell time is generally a function of number of passengers boarding and alighting as well as technology characteristics**
- **movement time and delay depend on other traffic and control system attributes**

Typical bus running time breakdown in mixed traffic:

- **50-75% movement time**
- **10-25% stop dwell time**
- **10-25% traffic delays including traffic signals**