An Example of M/M/1 Queue

- An airport runway for arrivals only
- Arriving aircraft join a single queue for the runway
- Exponentially distributed service time with a rate
  $\mu = 27 \; arrivals/hour$ (As you computed in PS1.)
- Poisson arrivals with a rate $\lambda = 20 \; arrivals/hour$
- $$W = \frac{1}{\mu - \lambda} = \frac{1}{27-20} = \frac{1}{7} \, hour \approx 8.6 \min$$
- $$L = \lambda W = \frac{\lambda}{\mu - \lambda} = \frac{20}{27-20} \approx 2.9 \, aircrafts$$
- $$W_q = W - \frac{1}{\mu} = \frac{1}{\mu - \lambda} - \frac{1}{\mu} = \frac{1}{27-20} - \frac{1}{27} \approx 6.4 \min$$
- $$L_q = \lambda W_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{20^2}{27(27-20)} \approx 2.1 \, aircrafts$$

An Example of M/M/1 Queue (cont.)

- Now suppose we are in holidays and the arrival rate increases

  $\lambda = 25\, arrivals / hour$

- How will the quantities of the queueing system change?

- $W = \dfrac{1}{\mu - \lambda} = \dfrac{1}{27 - 25} = \dfrac{1}{2}\, hour = 30 \min$

- $L = \lambda W = \dfrac{\lambda}{\mu - \lambda} = \dfrac{25}{27 - 25} = 12.5\, aircrafts$

- $W_q = W - \dfrac{1}{\mu} = \dfrac{1}{\mu - \lambda} - \dfrac{1}{\mu} = \dfrac{1}{27 - 25} - \dfrac{1}{27} = 27.8 \min$

- $L_q = \lambda W_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{25^2}{27(27 - 25)} \approx 11.6\, aircrafts$

An Example of M/M/1 Queue (cont.)

- Now suppose we have a bad weather and the service rate decreases
  $\mu = 22\, arrivals / hour$
- How will the quantities of the queueing system change?
- $W = \frac{1}{\mu - \lambda} = \frac{1}{22 - 20} = \frac{1}{2}\, hour = 30 \min$
- $L = \lambda W = \frac{\lambda}{\mu - \lambda} = \frac{20}{22 - 20} = 10\, aircrafts$
- $W_q = W - \frac{1}{\mu} = \frac{1}{\mu - \lambda} - \frac{1}{\mu} = \frac{1}{22 - 20} - \frac{1}{22} \approx 27.3 \min$
- $L_q = \lambda W_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{20^2}{22(22 - 20)} \approx 9.1\, aircrafts$