Introduction to Transportation Systems

PART III:

TRAVELER TRANSPORTATION

Chapter 27:

Deterministic Queuing

Deterministic Queuing Applied to Traffic Lights

- Here we introduce the concept of deterministic queuing at an introductory level and then apply this concept to setting of traffic lights.

Deterministic Queuing

Deterministic Queuing

In the first situation, we consider $\lambda(t)$, the arrival rate, and $\mu(t)$, the departure rate, as deterministic.

Deterministic Arrival and Departure Rates

Figure 27.1

Deterministic Queuing

Deterministic Arrival and Departure Rates

(continued)

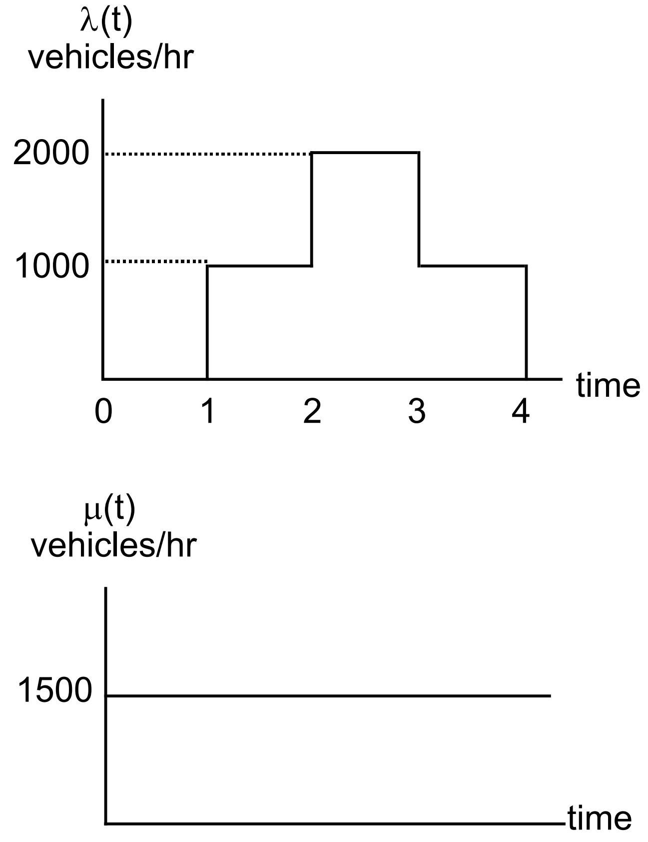


Figure 27.1

Queuing Diagram

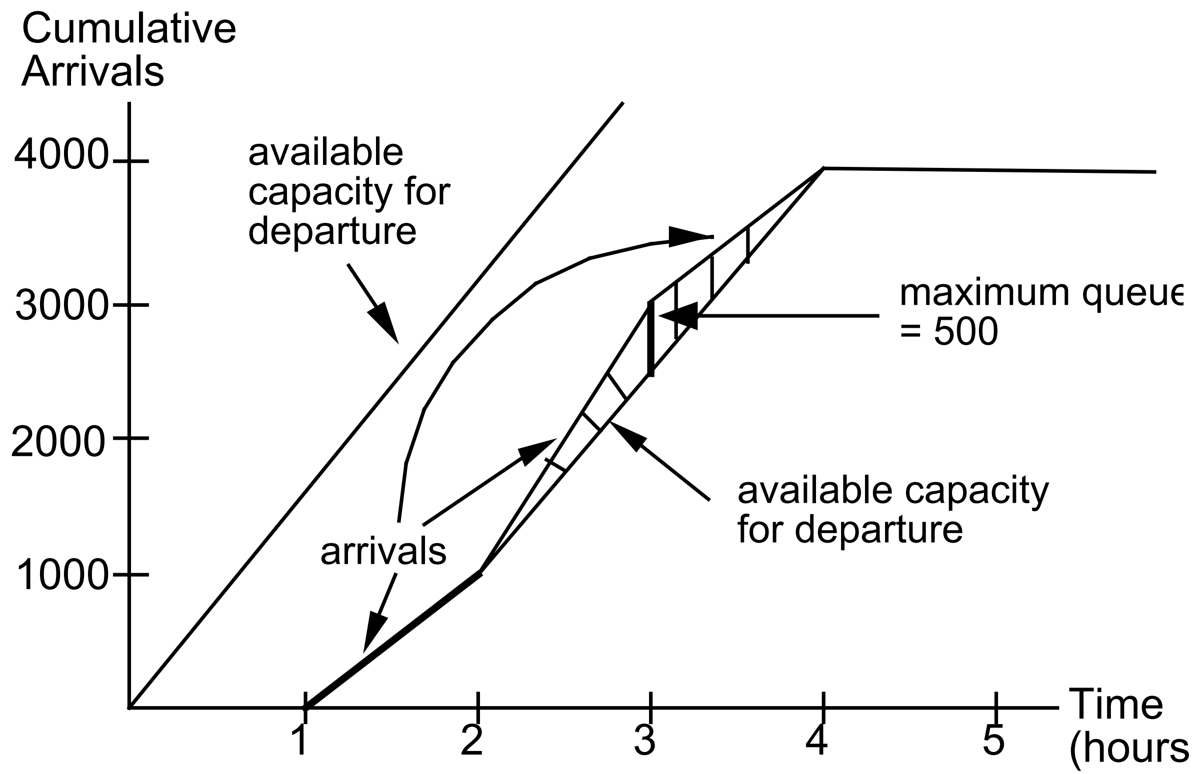
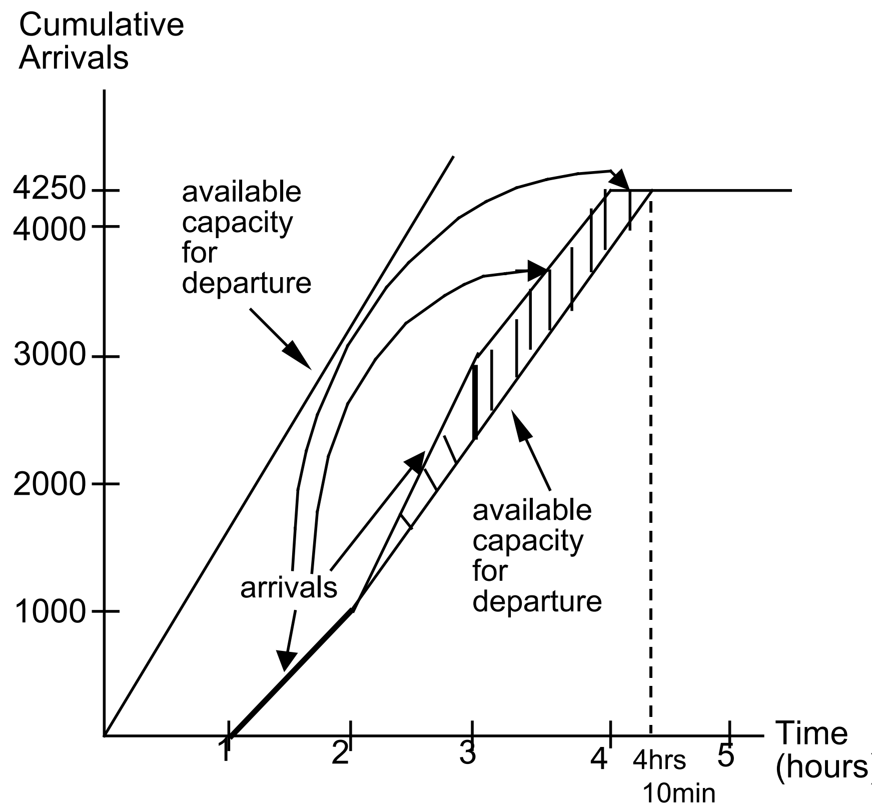


Figure 27.2

Another Case

- Now, the numbers were selected to make this simple; at the end of four hours the system is empty. The queue dissipated exactly at the end of four hours. But for example, suppose vehicles arrive at the rate of 1,250/hour from t=3 to t=4.

Another Queuing Diagram



CLASS DISCUSSION

- What is the longest queue in this system?
- What is the longest individual waiting time?

Figure 27.3

Computing Total Delay

Area Between Input and Output Curves

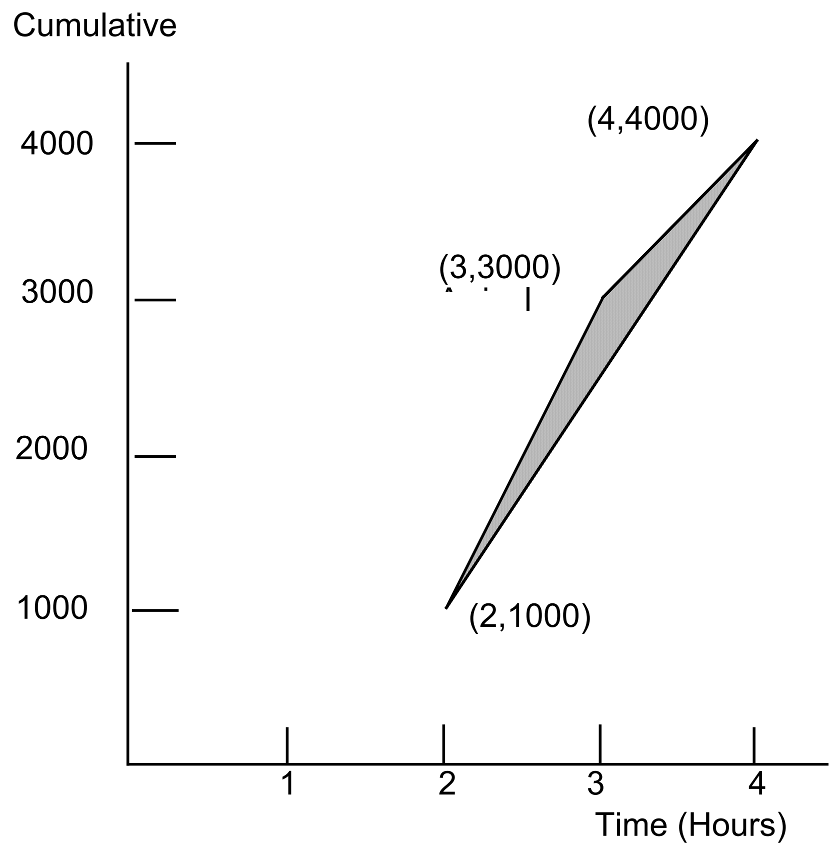


Figure 27.4

Choosing Capacity

$\mu(t) = 2000$

$\mu(t) = 1500$

$\mu(t) = 500$

CLASS DISCUSSION

A Traffic Light as a Deterministic Queue

Service Rate and Arrival Rate at Traffic Light

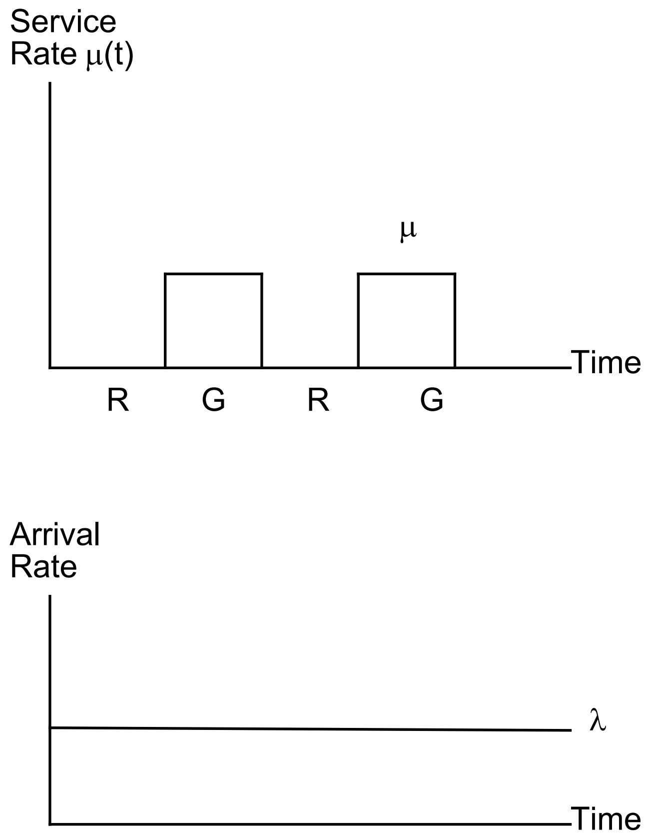


Figure 27.5

Queuing Diagram per Traffic Light

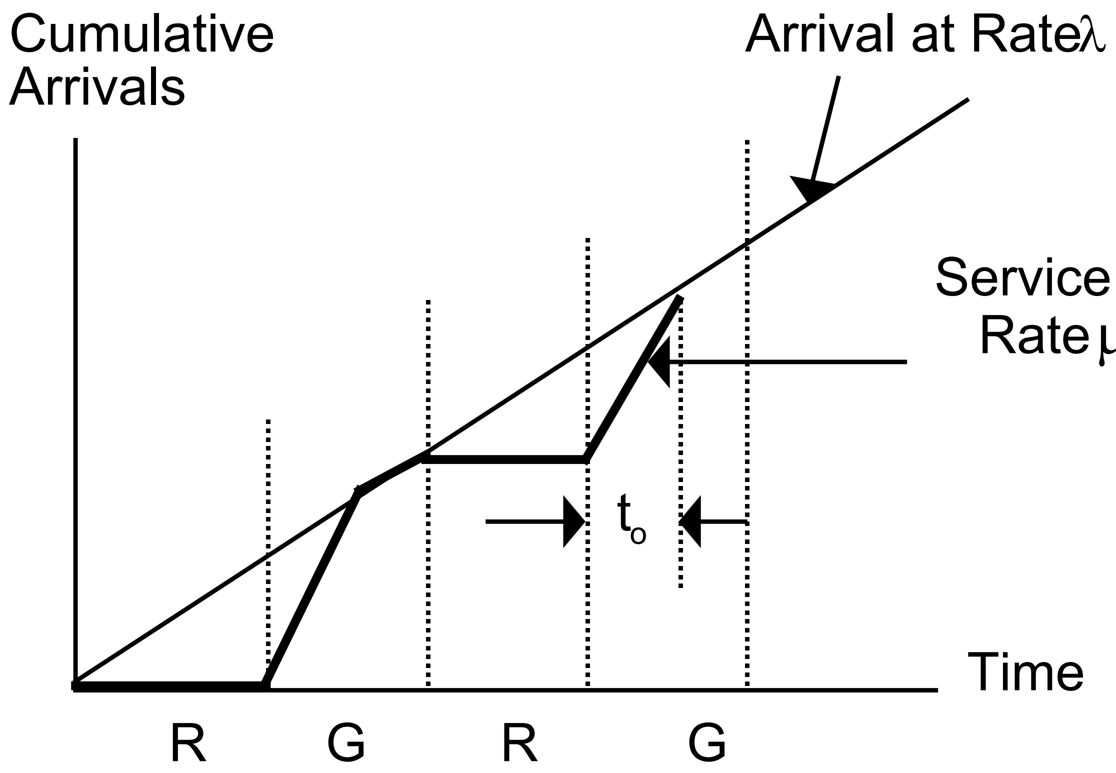


Figure 27.6

Queue Stability

All the traffic must be dissipated during the green cycle.

If $R + G = C$ (the cycle time),

then $\lambda(R + t_0) = \mu t_0$.

Rearranging $t_0 = \dfrac{\lambda R}{\mu - \lambda}$

If we define $\dfrac{\lambda}{\mu} = \rho$ (the "traffic intensity"),

Then $t_0 = \dfrac{\rho R}{1 - \rho}$

For stability $t_0 \leq G = C - R$.

Delay at a Traffic Signal -- Considering One Direction

$$D = \frac{\lambda R^2}{2(1 - \rho)}$$

The total delay *per cycle* is d

$$d = \frac{D}{\lambda C} = \frac{R^2}{2C(1 - \rho)}$$

Two Direction Analysis of Traffic Light

Flows in East-West and North-South Directions

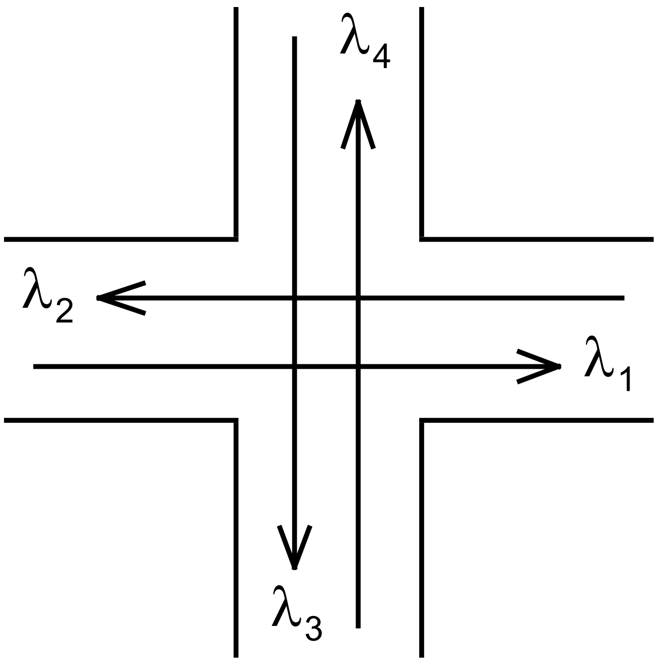


Figure 27.7

$$D_1 = \frac{\lambda_1 R_1^2}{2(1 - \rho_1)}$$

where $\rho_1 = \dfrac{\lambda_1}{\mu}$

We can write similar expressions for D_2, D_3 , D_4. We want to minimize D_T , the total delay, where

$$D_T = D_1 + D_2 + D_3 + D_4$$

Choosing an Optimum

Remembering that

$R_2 = R_1$

$R_4 = R_3 = (C - R_1)$

we want to minimize D_T where

$$D_T = \frac{\lambda_1 R_1^2}{2(1 - \rho_1)} + \frac{\lambda_2 R_1^2}{2(1 - \rho_2)} + \frac{\lambda_3 (C-R_1)^2}{2(1 - \rho_3)} + \frac{\lambda_4 (C-R_1)^2}{2(1 - \rho_4)}$$

To obtain the optimal R_1, we differentiate the expression for total delay with respect to R_1 (the only unknown) and set that equal to zero.

$$\frac{dD_T}{dR_1} = \frac{\lambda_1 R_1}{1 - \rho_1} + \frac{\lambda_2 R_1}{1 - \rho_2} - \frac{\lambda_3 (C-R_1)}{1 - \rho_3} - \frac{\lambda_4 (C-R_1)}{1 - \rho_4} = 0$$

Try a Special Case

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$$

Therefore, $\rho_1 = \rho_2 = \rho_3 = \rho_4$.

The result, then, is

$$R_1 = \frac{C}{2}, \quad R_3 = \frac{C}{2}$$

This makes sense. If the flows are equal, we would expect the optimal design choice is to split the cycle in half in the two directions.

- The text goes through some further mathematical derivations of other cases for the interested student.