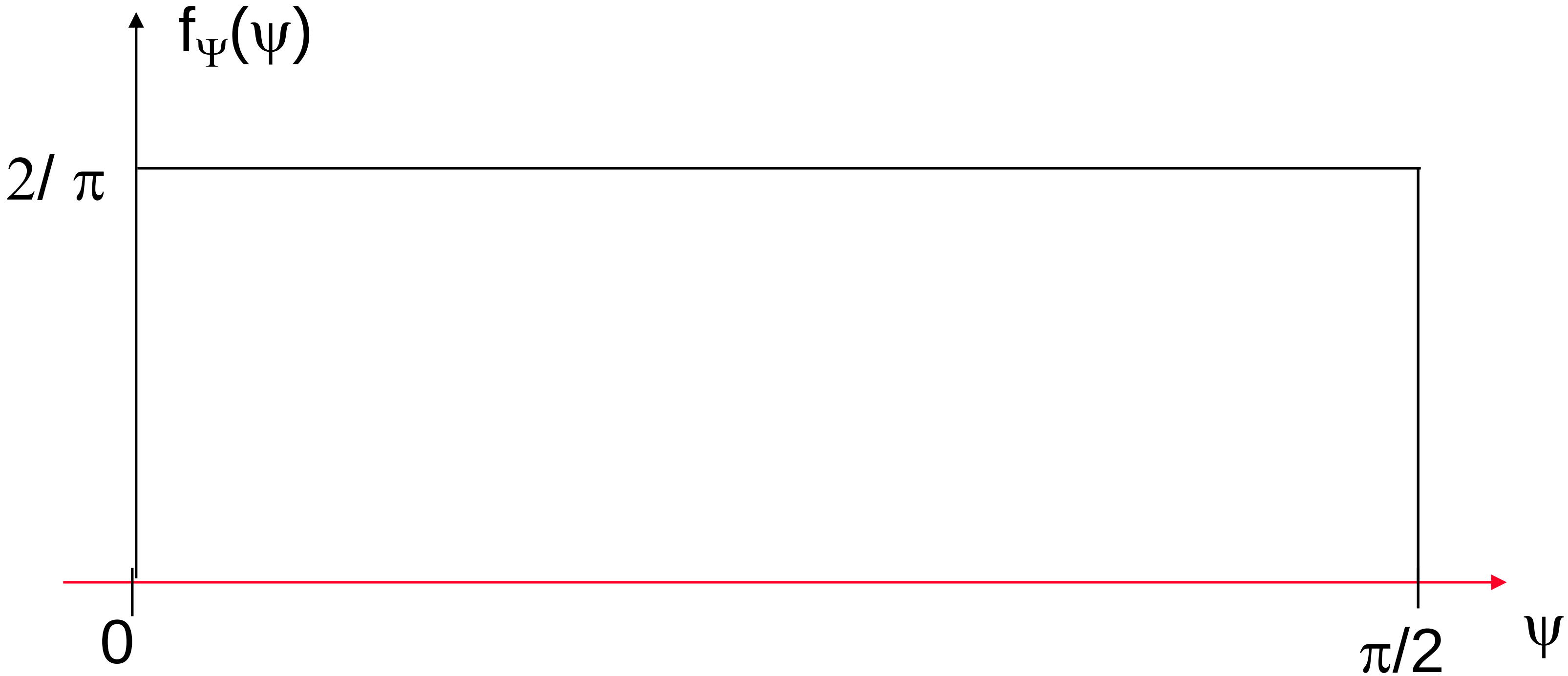
$(R|\Psi) = \cos\Psi + \sin\Psi = 2^{1/2}\cos(\Psi - \pi/4)$

2. Identify Sample Space

3. Probability Law over Sample Space:

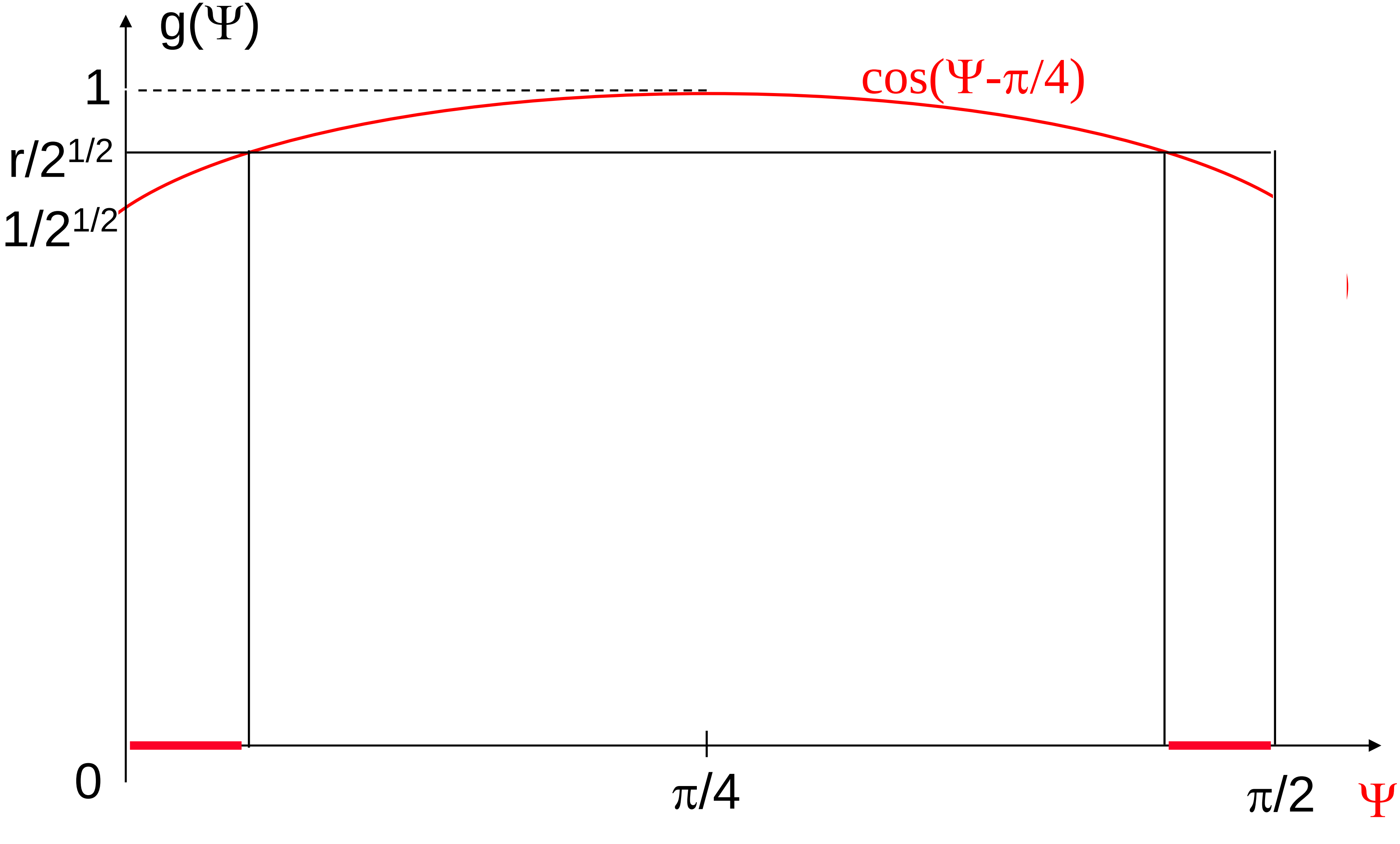
Invoke isotropy implying uniformity of angle

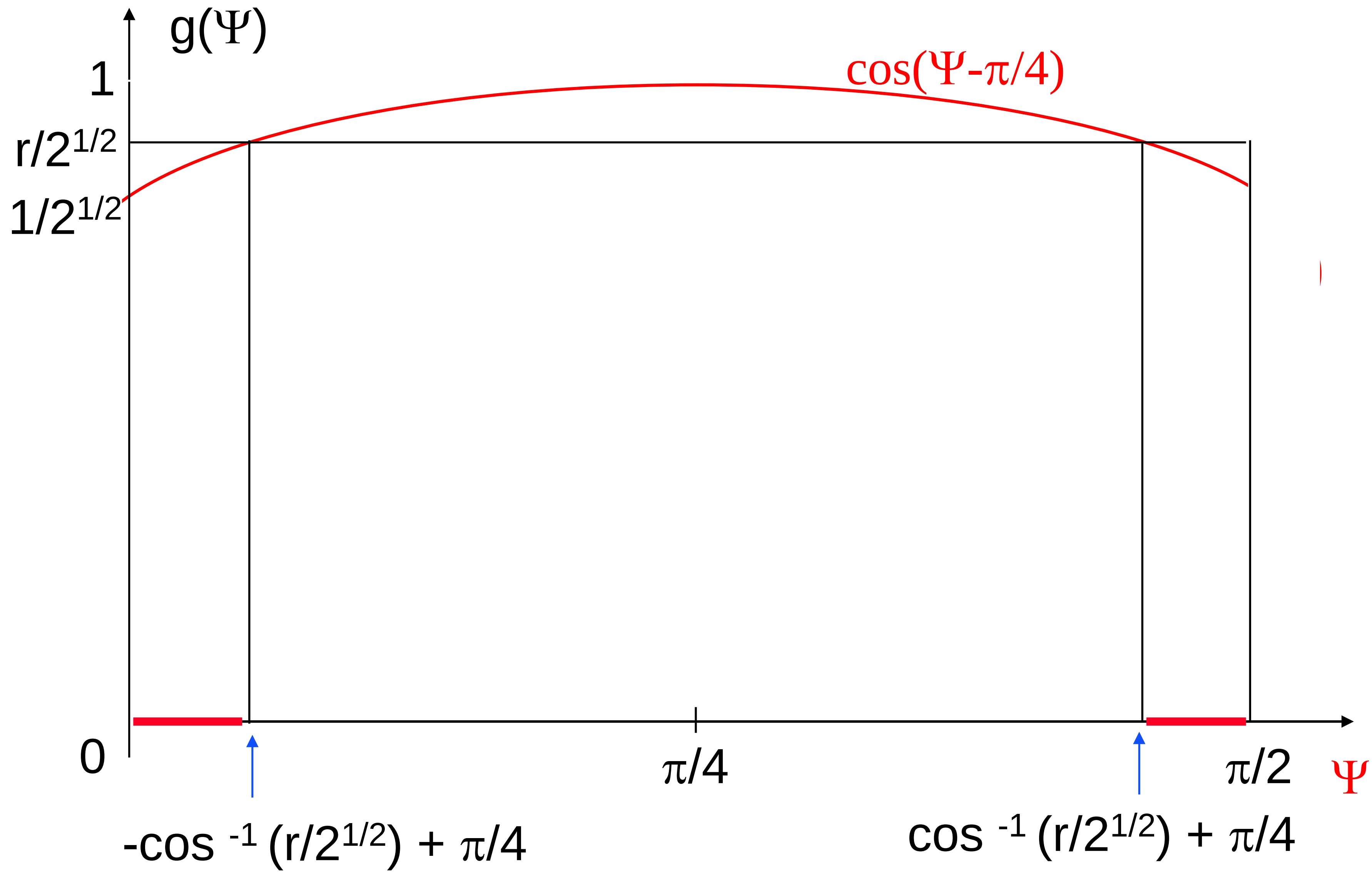


$f_{\Psi}(\psi)$
2/ π
0
π/2
ψ

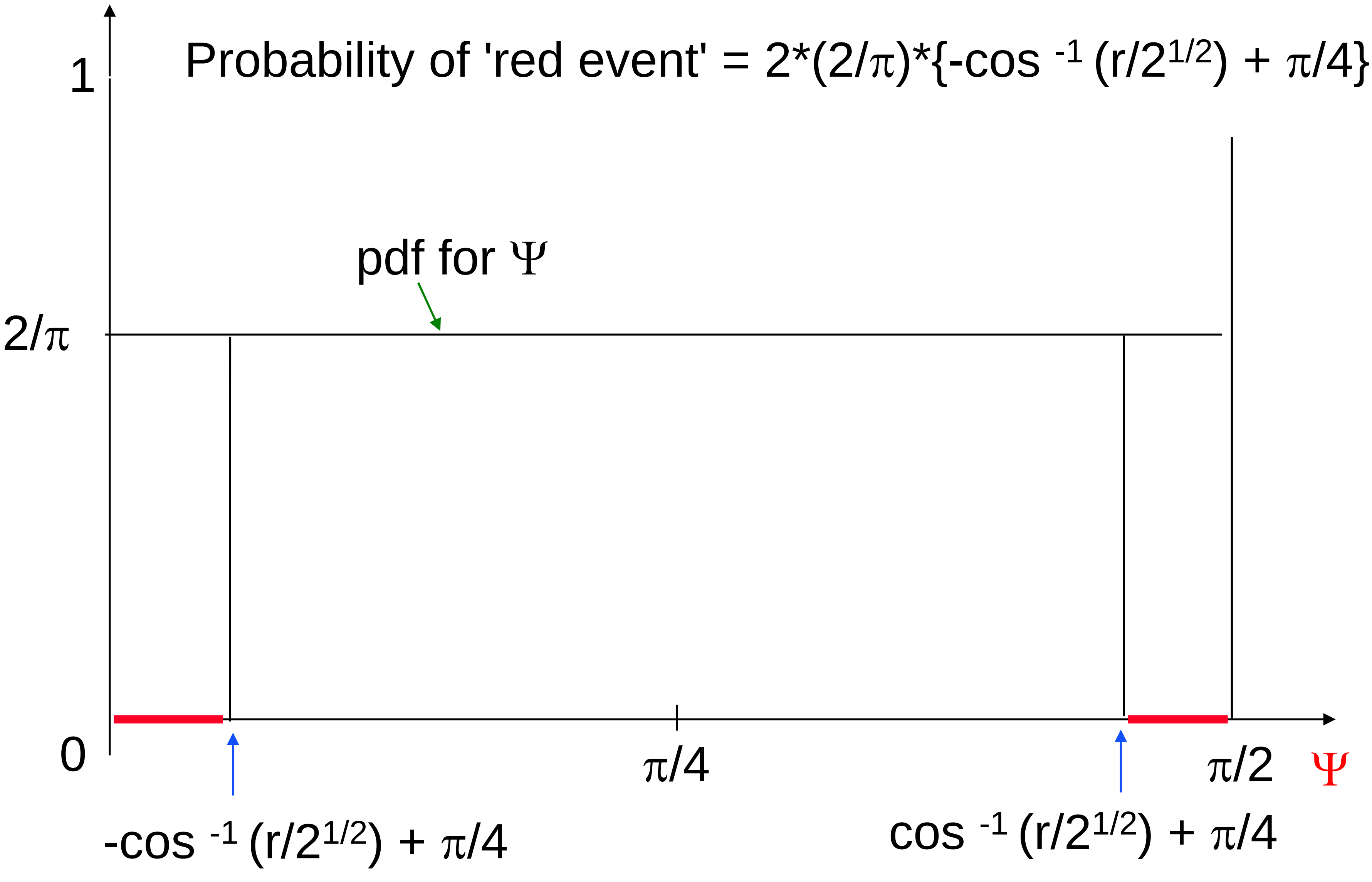
4. Find CDF

- $F_R(r) = P\{R < r\} = P\{2^{1/2} \cos(\Psi - \pi/4) < r\}$
- $F_R(r) = P\{R < r\} = P\{\cos(\Psi - \pi/4) < r/ 2^{1/2}\}$


g(Ψ)
1
r/2^{1/2}
1/2^{1/2}
cos(Ψ-π/4)
0
π/4
π/2
Ψ



g(Ψ)
cos(Ψ-π/4)
1
r/2^1/2
1/2^1/2
0
π/4
π/2
Ψ
-cos -1 (r/2^1/2) + π/4
cos -1 (r/2^1/2) + π/4

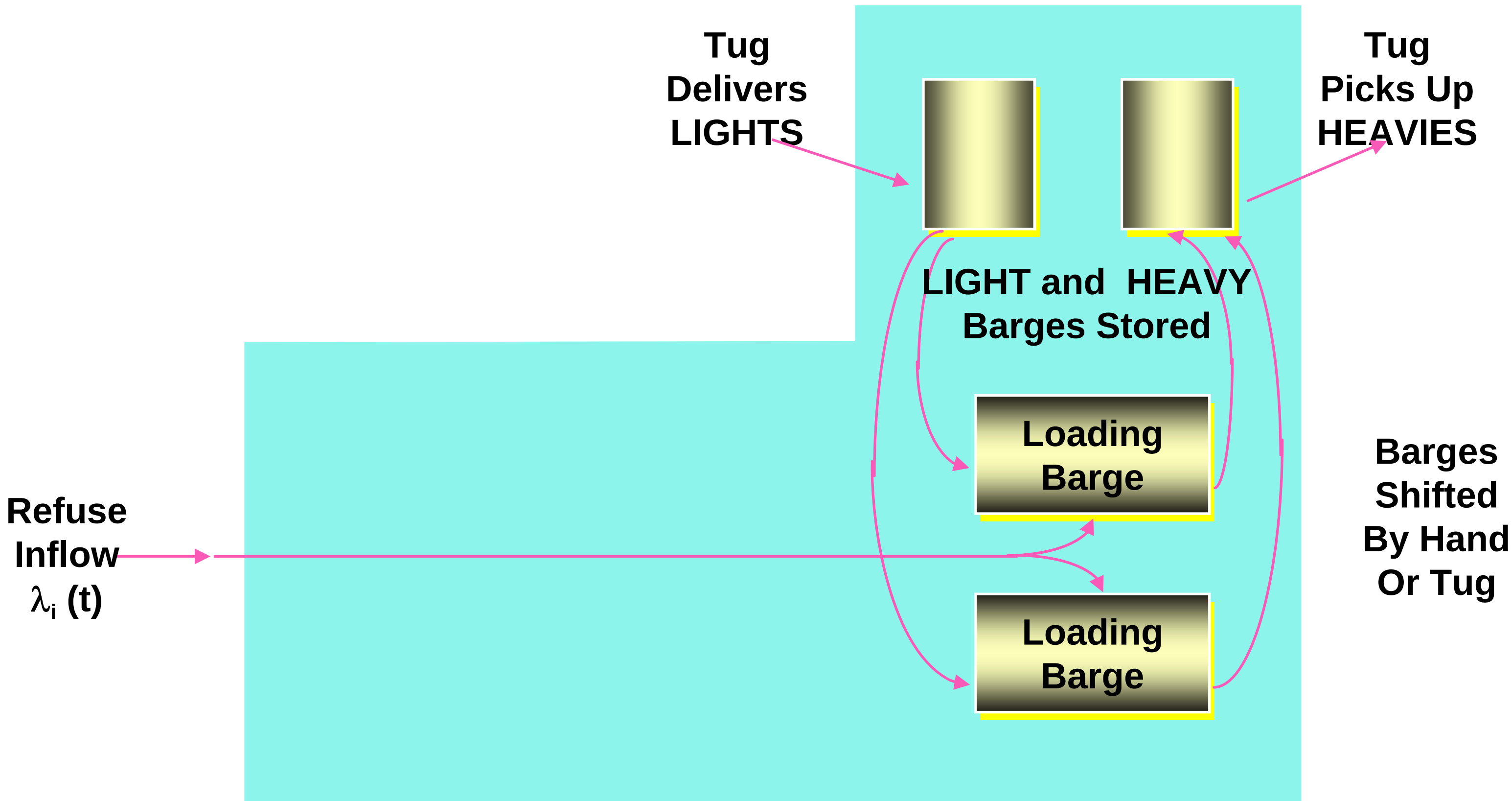

Probability of 'red event' = 2*(2/π)*{-cos -1 (r/21/2) + π/4}
1
pdf for Ψ
2/π
0
π/4
π/2
Ψ
-cos -1 (r/21/2) + π/4
cos -1 (r/21/2) + π/4

And finally...

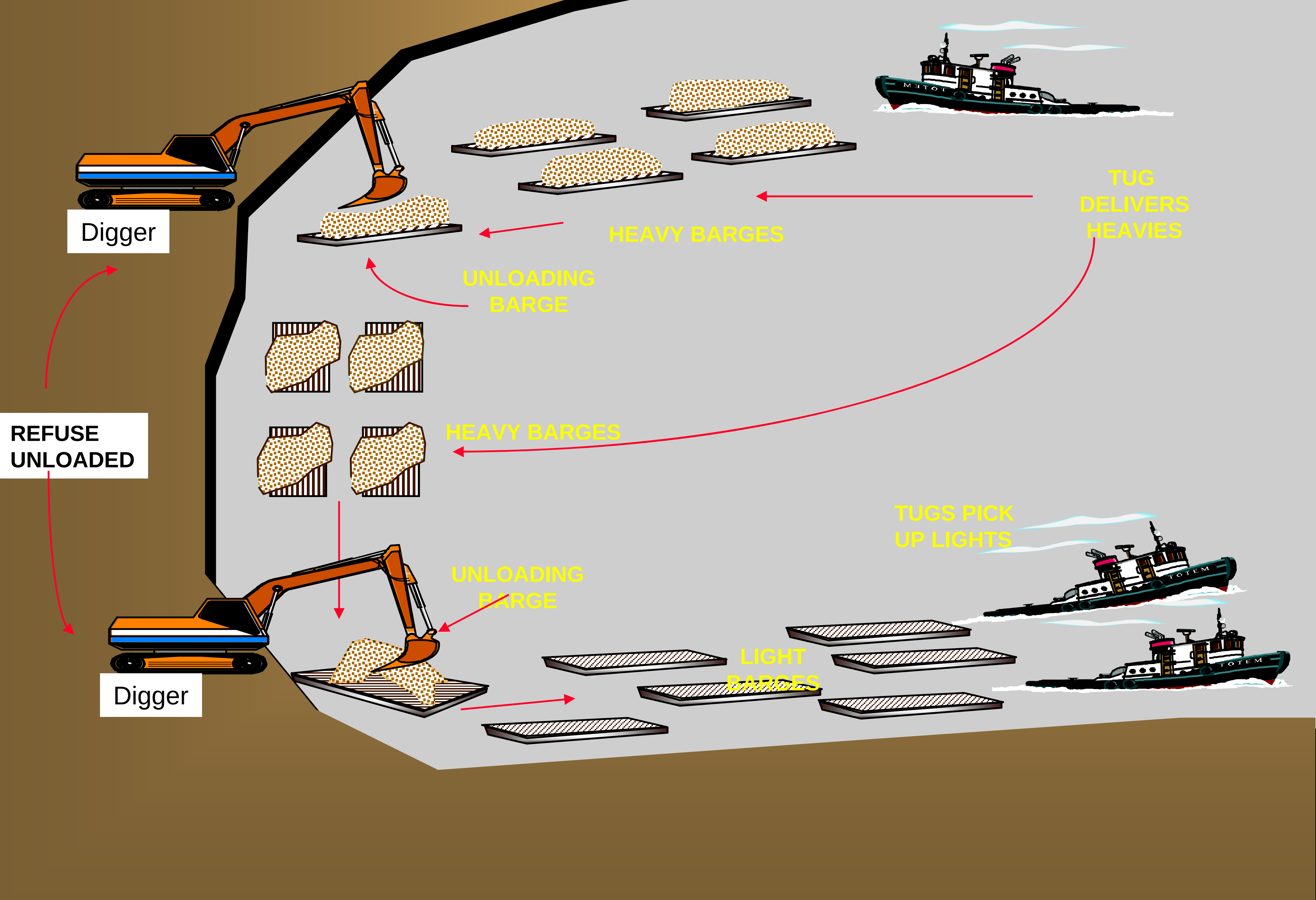
- After all the computing is done, we find:
- $F_R(r) = 1 - (4/\pi)\cos^{-1}(r/2^{1/2}),\ 1 < r < 2^{1/2}$
- $f_R(r) = d[F_R(r)]/dr = (4/\pi)\{1/(2 - r^2)^{1/2}\}$
- Median R = 1.306
- $E[R] = 4/\pi = 1.273$
- $\sigma_R/E[R] = 0.098$, implies very robust

A Quantization Problem

NYC Marine Transfer Station



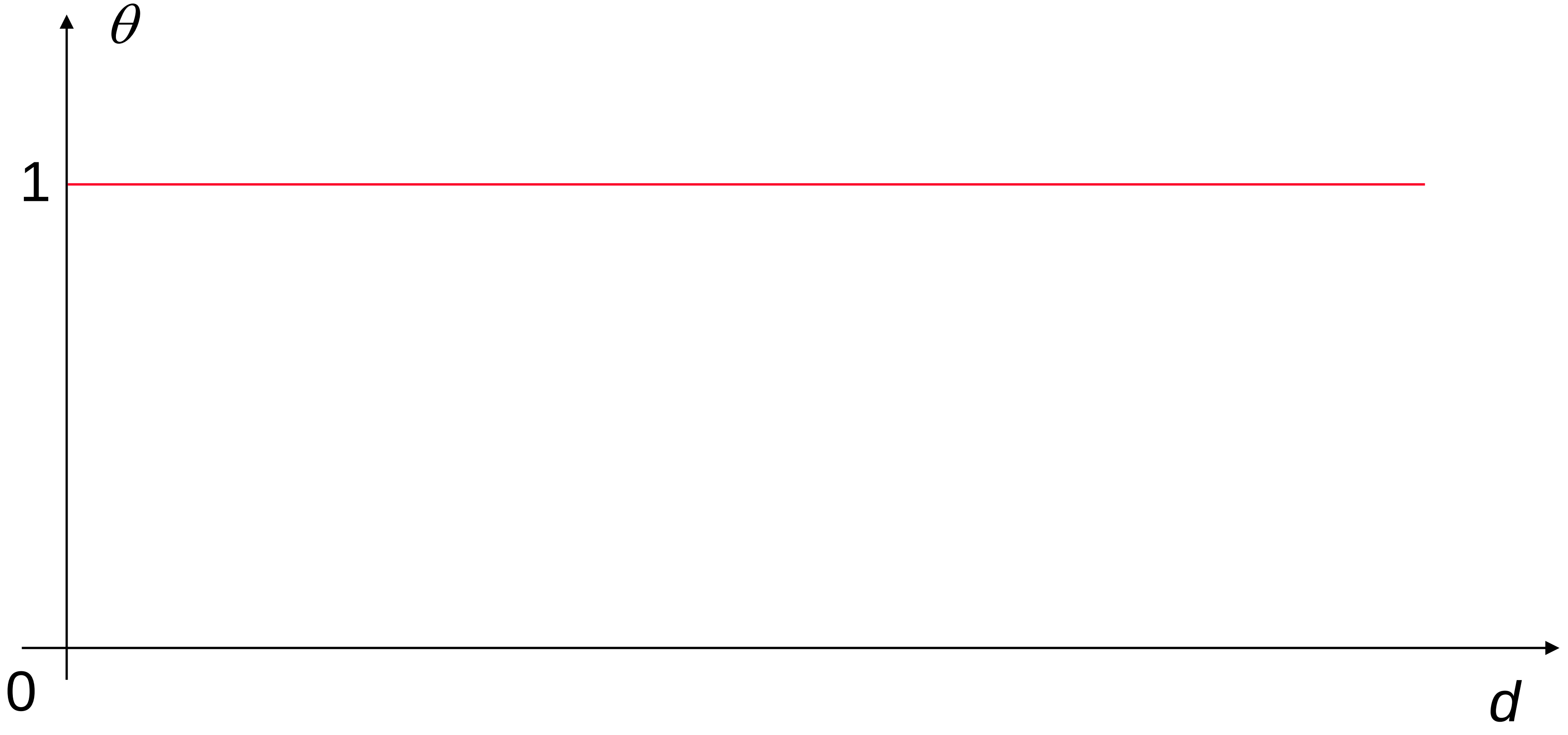
Fresh Kills Landfill

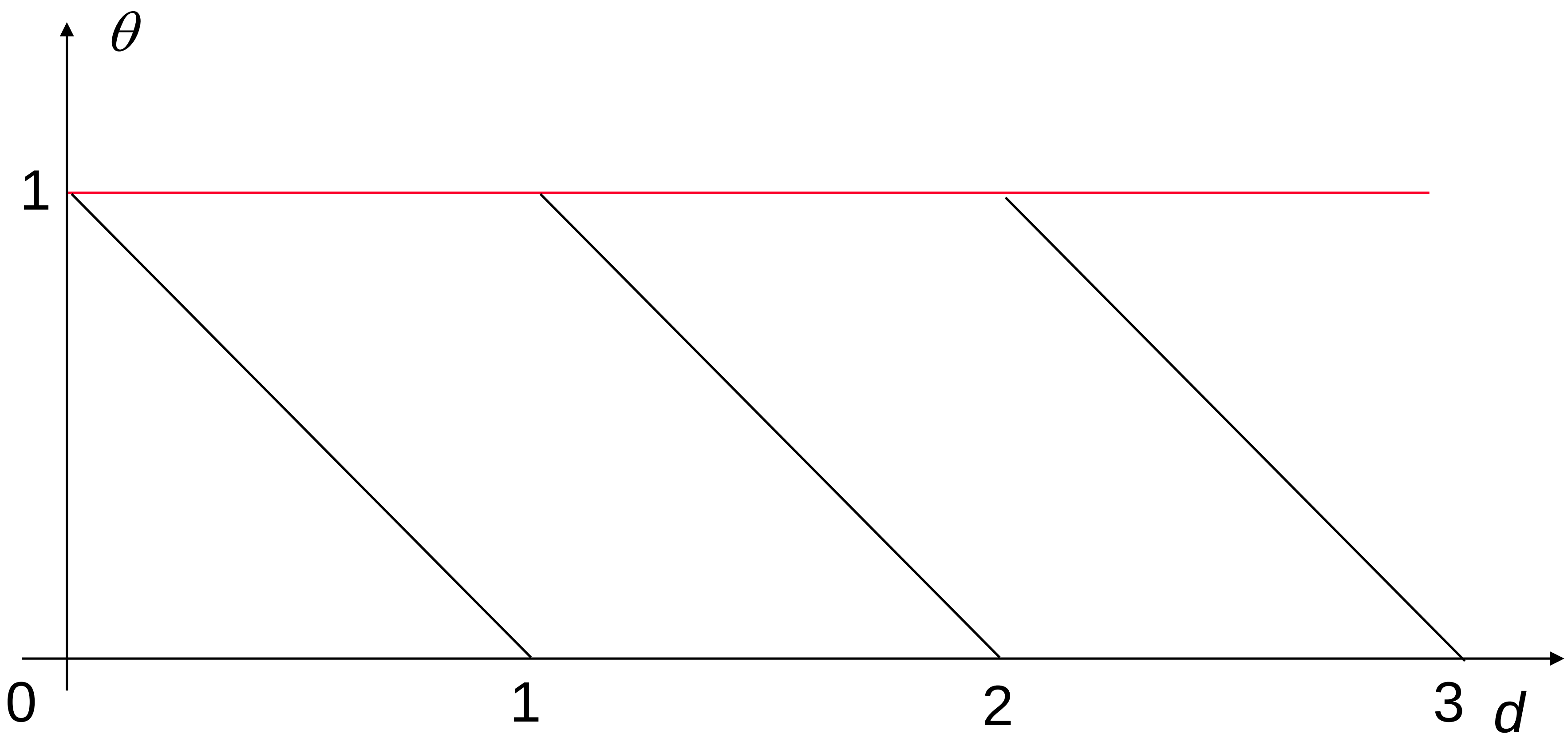


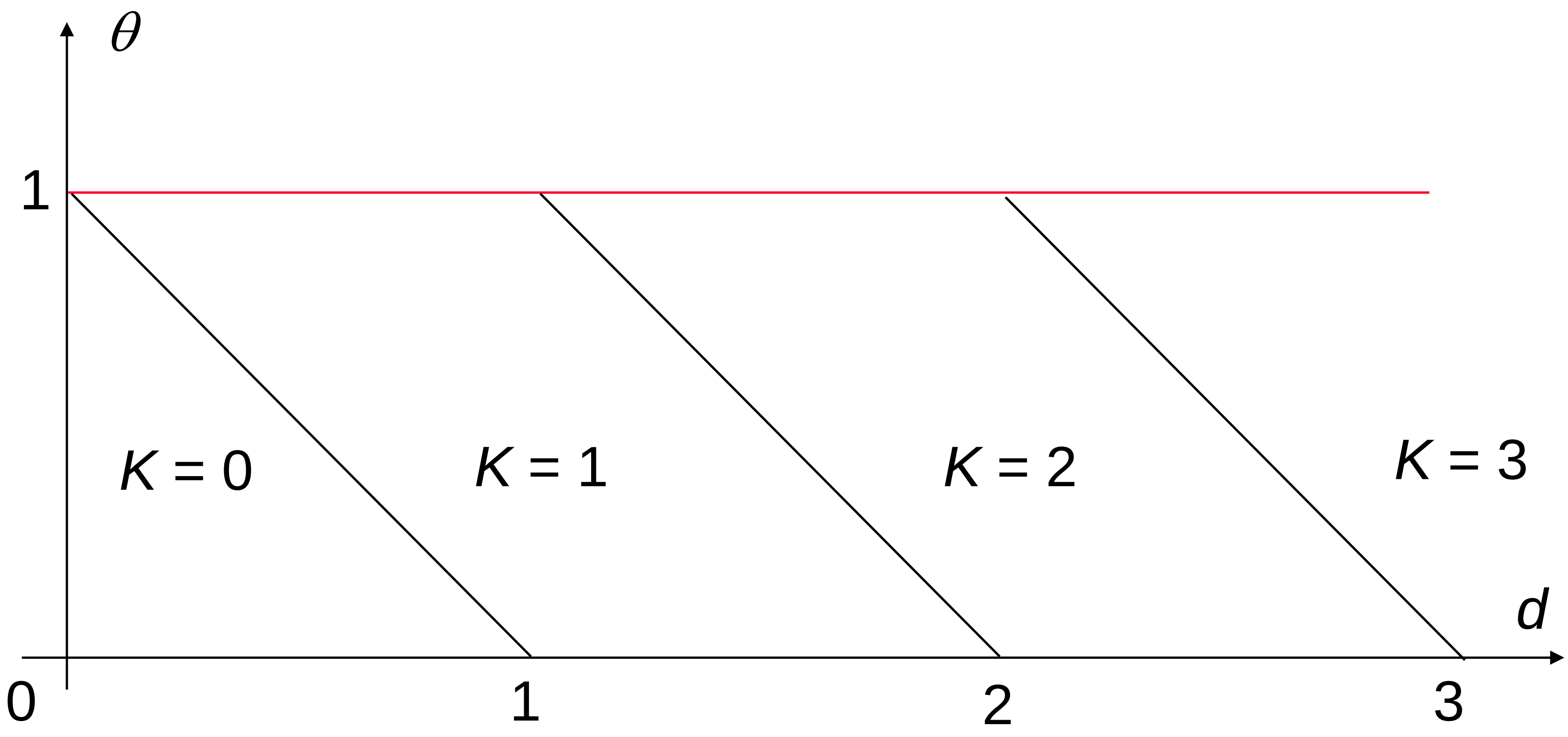
1. The R.V.'s

- D = barge loads of garbage produced on a random day (continuous r.v.)
- Θ = fraction of barge that is filled at beginning of day ($0 < \Theta < 1$)
- K = total number of completely filled barges produced by a facility on a random day (K integer)
- $K = \lfloor D + \Theta \rfloor$ = integer part of $D + \Theta$

2. The Sample Space




θ
1
0
1
2
3
d

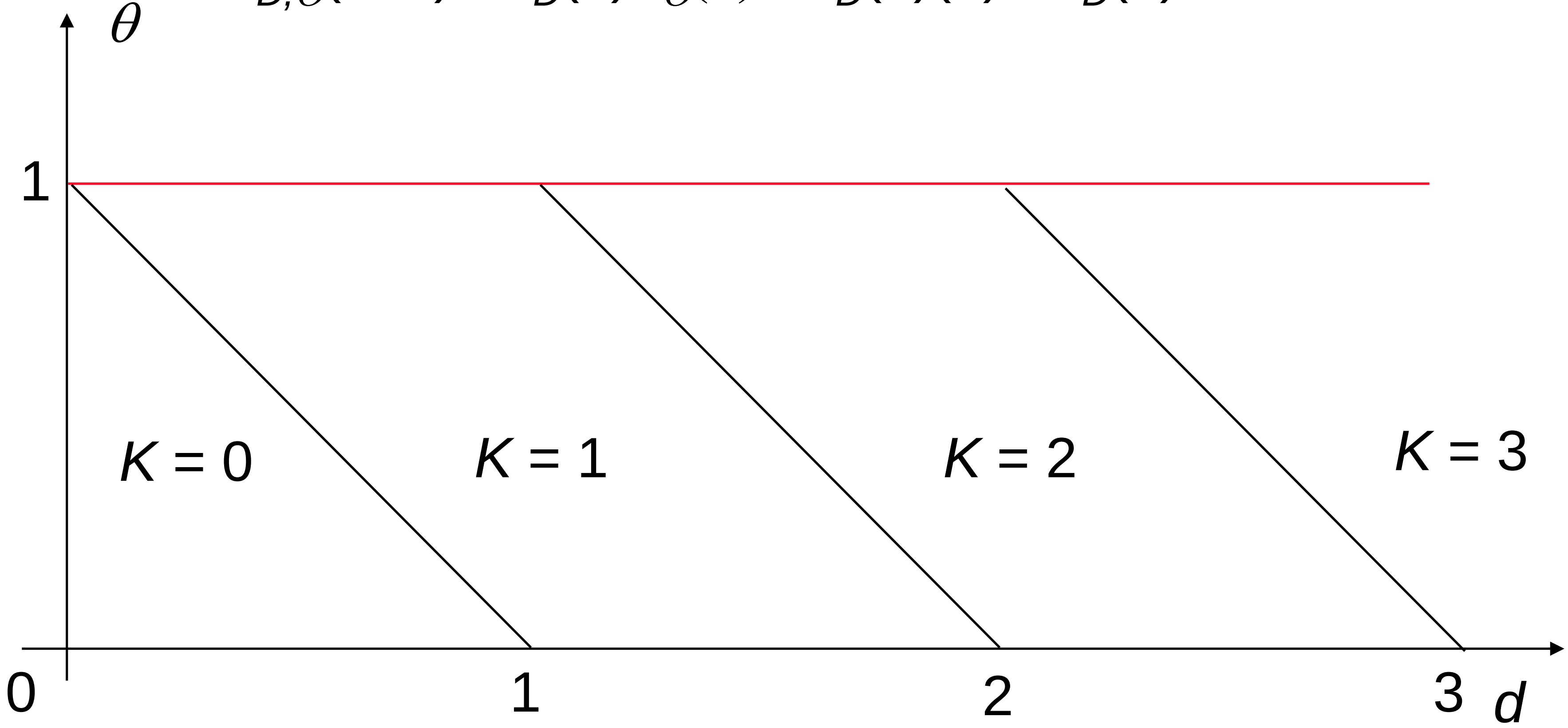

θ
1
K = 0
K = 1
K = 2
K = 3
d
0
1
2
3

3. Joint Probability Distribution

a) D and Θ are independent.

b) Θ is uniformly distributed over $[0, 1]$

$$f_{D,\Theta}(d, \theta) = f_D(d)\, f_\Theta(\theta) = f_D(d)(1) = f_D(d),\ d > 0,\ 0 < \theta < 1$$



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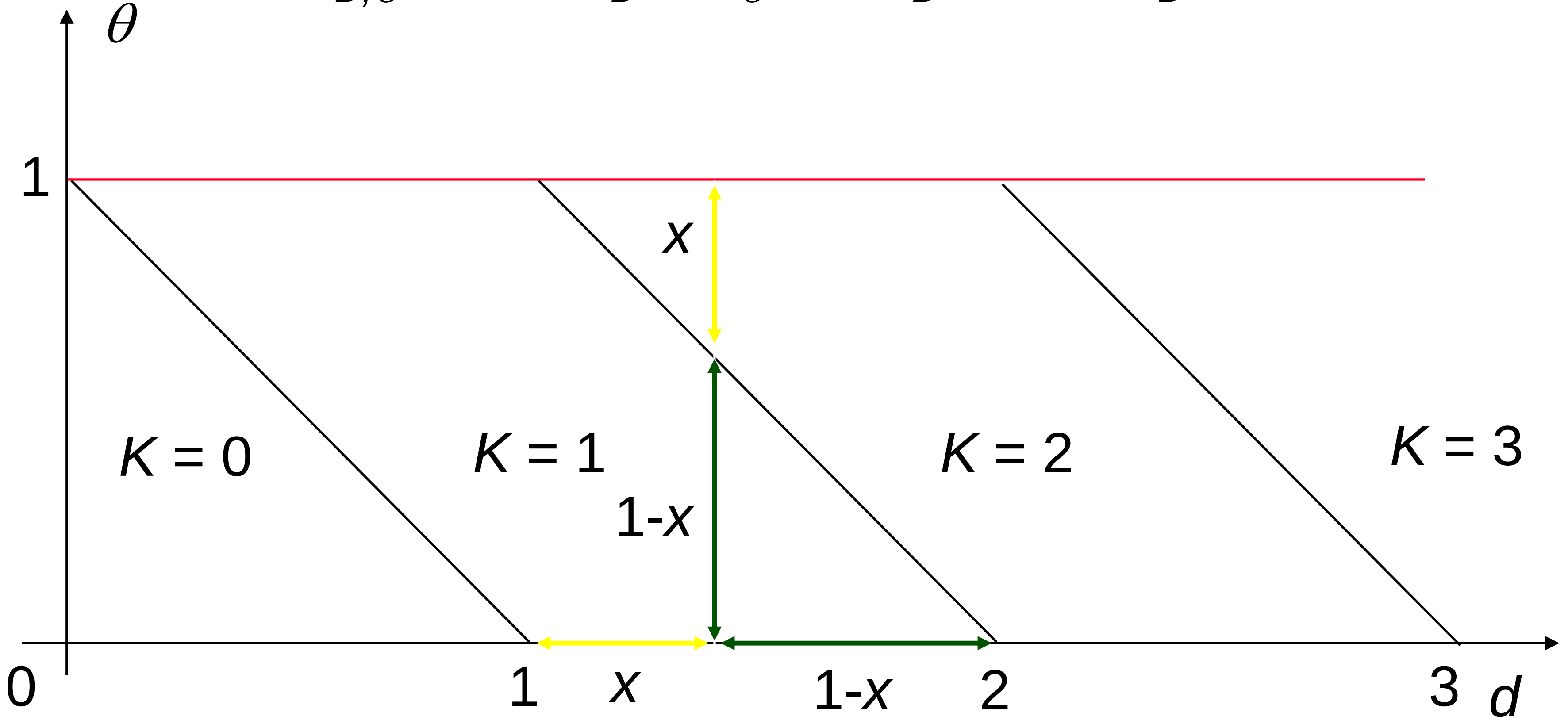


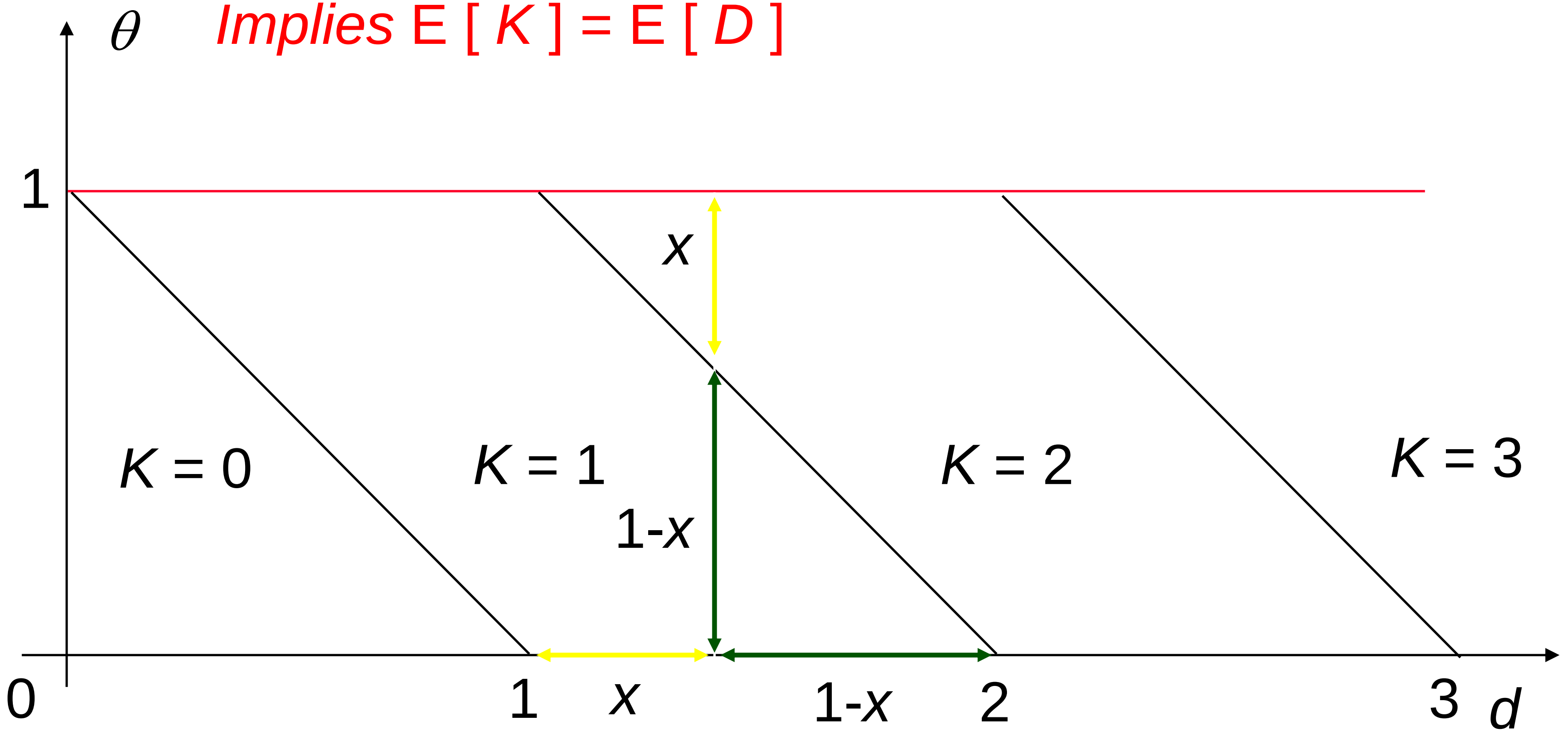
4. Working in the Joint Sample Space

Look at $E[K \mid D = d]$

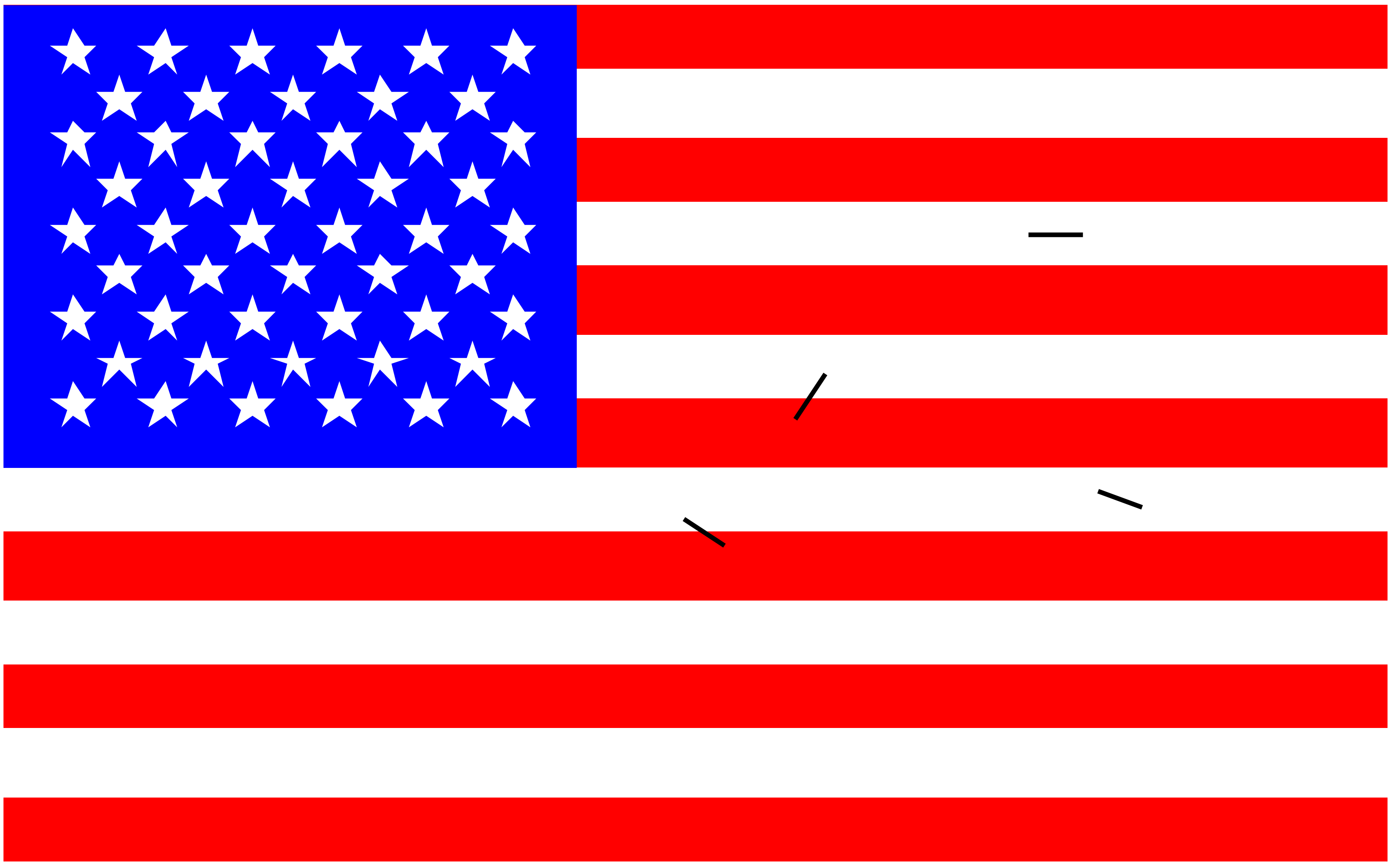
Let $d = i + x \quad 0 < x < 1$

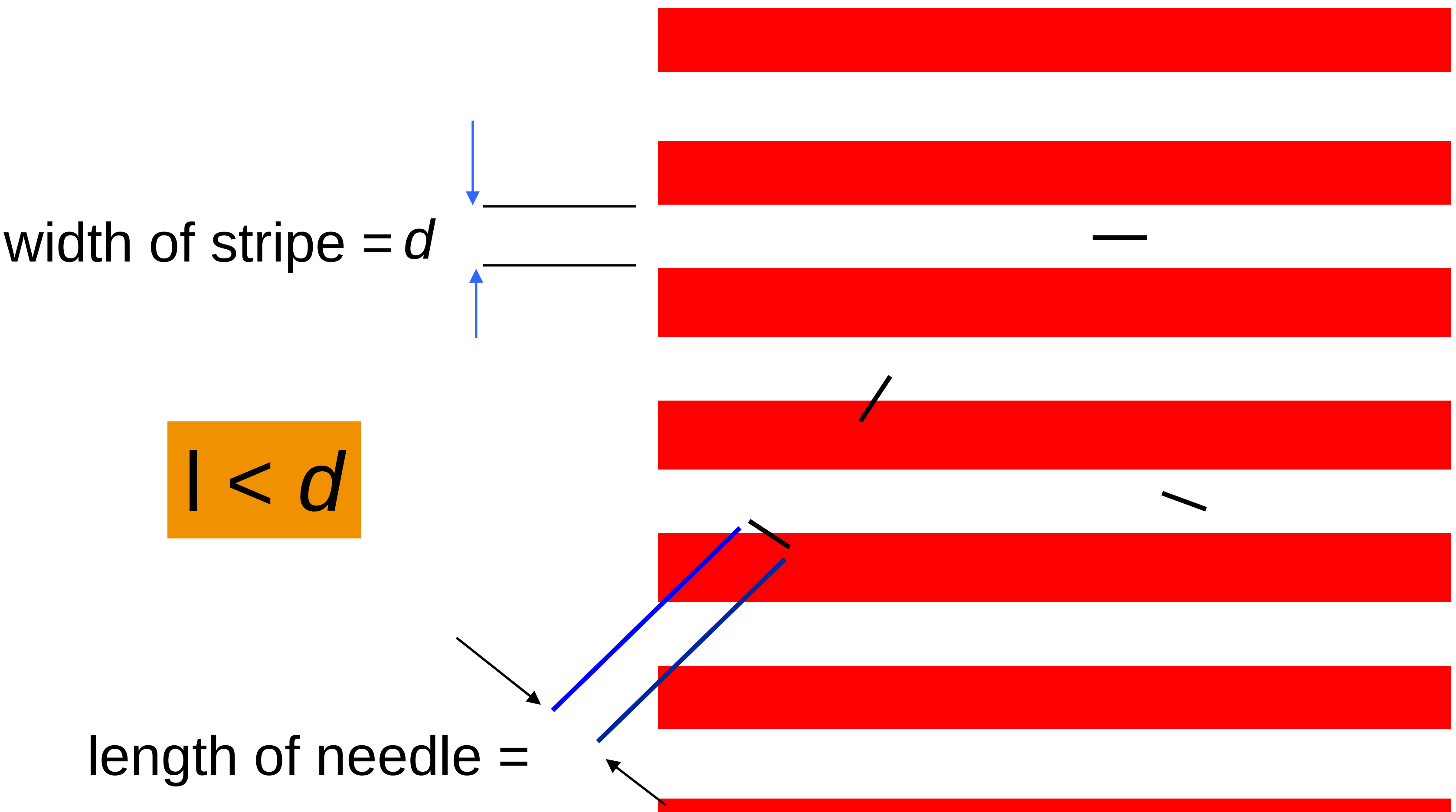
$E[K \mid D = i + x] = i(1 - x) + (i + 1)x = i + x = d$

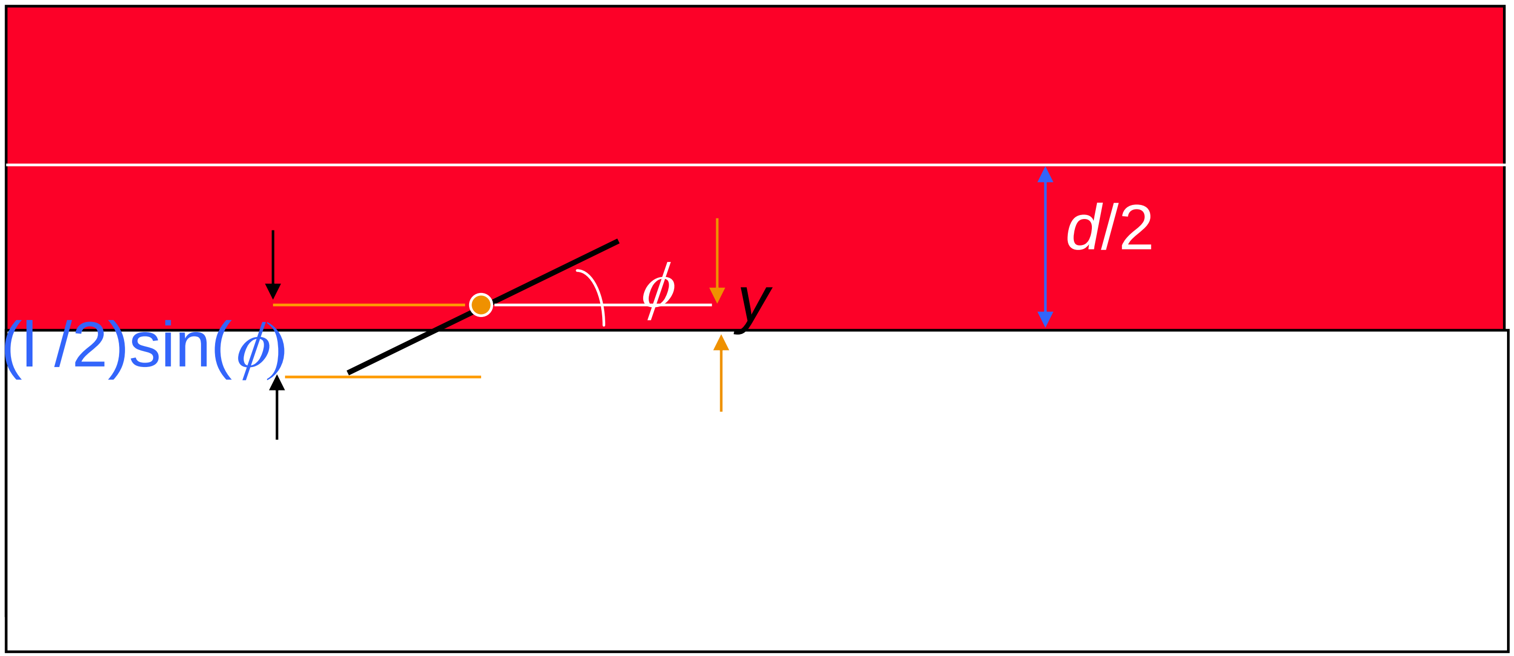
Implies $E[K] = E[D]$



Buffon’s Needle Experiment




width of stripe = d
l < d
length of needle =



d/2
ϕ
y
(l /2)sin(ϕ)

1. The R.V.'s

- Y = distance from the center of the needle to closest of equidistant parallel lines $0 < y < d/2$
- Φ = angle of needle wrt horizontal
$0 < \phi < \pi$