1.203J/6.281J/15.073J/16.76J/ESD.216J **Fall 2004**

Quiz 2: Solutions

Problem 1:

(a) There are several, essentially equivalent ways to define the state of the system. One possibility is: (k, j, i) where

k = the type of customer (0, 1, or 2) currently in service (note that you cannot have one server occupied by a Type 1 customer and the other by a Type 2 customer)

j = the number of Type 2 customers (0, 1 or 2) in the system

i = the number of Type 1 customers (0, 1, 2, 3 or 4) in the system

The state-transition diagram for the system is then as shown on Figure 1.

(b) From Figure 1 it can be seen that the event of interest can happen only by having a set of three consecutive transitions from state (1, 0, 4) to (1, 0, 3) to (1, 0, 2) to (1, 1, 2). (Note that the probability of the first of these transitions is 1.) It can be seen that:

$$\text{Answer} = \left(\frac{2\mu_1}{2\mu_1 + \lambda_1}\right) \cdot \left(\frac{\lambda_2}{\lambda_2 + \lambda_1 + 2\mu_1}\right)$$

Problem 2:

(a) Note that T_0 is a traveling salesman tour through the 2n+1 points using the Christofides heuristic. We know that

$L(T_0) < (3/2)L(TSP)$

where TSP is the optimal traveling salesman tour through the same 2n+1 points.

Note that TSP does not necessarily observe the precedence constraints and, thus,

$L(TSP) \leq L(DARP)$

Where DARP is the optimal solution to the dial-a-ride problem.

Finally and obviously: $L(T_1) < 2L(T_0)$.

Putting these together, we have:

$L(T_1) < 2L(T_0) < 2[(3/2)L(TSP)] = 3L(TSP) \leq 3L(DARP)$

(b) The application of the 2-exhange heuristic to the DARP problem is not straightforward. First, it is obvious that a 2-exchange may lead to a violation of the precedence constraints, by placing the visit of a delivery point before the visit to the corresponding origin. Moreover, the process of checking whether such violations have occurred requires checking the entire tour that results from the exchange. In addition, it is possible that the resulting tour will be illegitimate (not respecting the precedence constraints) when traversed in one direction (say, clockwise), but legitimate in the opposite direction (say, counter-clockwise).

Problem 3 Solution:

1. The key to this problem was to realize that it was a one median problem.

   The distance matrix for the problem is:

	A	B	C	D	E
A	0	4	4	1	2
B	4	0	2	4	5
C	4	2	0	3	4
D	1	4	3	0	1
E	2	5	4	1	0

   The weighted distance matrix is:

	A	B	C	D	E
A	0	12	12	3	6
B	20	0	10	20	25
C	4	2	0	3	4
D	2	8	6	0	2
E	10	25	20	5	0
Total:	36	47	48	**31**	37

   Thus we should locate the mail facility at node D.

2. To solve this problem we let the number of units at node E be 5+x putting this into our weighted distance matrix we get:

	A	B	C	D	E
A	0	12	12	3	6
B	20	0	10	20	25
C	4	2	0	3	4
D	2	8	6	0	2
E	10+2x	25+5x	20+4x	5+x	0
Total:	36+2x	47+5x	48+4x	31+x	37

Clearly the median will move from D to E at some point thus we solve:

31+x>=37

x>=6

Thus if 6 families move into E then E becomes an optimal location for the mail center.

3.

In order to solve this part we add 4 units of weight to a node already in the network and see if the optimal facility will be located at the node. If we do this for all 4 nodes but D (which cannot be used) we find that only when we locate the families at node B do we have a tie between B and D and thus B is an optimal location, all other nodes are inferior. We then realize that we only need to perform a local analysis on the arcs that connect two optimal node locations. For the example above this turns out to be nodes B and D, thus only points along arc (B,D) need to be considered. Considering moving a very small number e units off B shows that D becomes the optimal facility as does moving a small number of units off D towards B shows that D will remain the optimal complex. Thus, the only place to locate the four new families is at complex B. It is important to note that for other weights (other than 4 families) it is indeed possible to have optimal locations be on arcs. For example, if the additional families were 10,000, any point on any arc in the network where these 10,000 families are located is optimal. It is not correct to say that the optimal location will always be at an existing node when the facility has demand units attached to it.

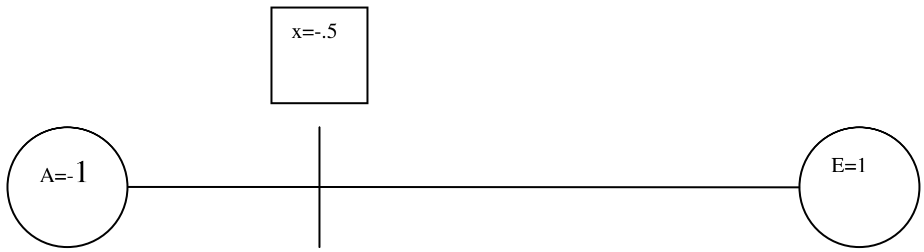
4. The idea here was to simply balance the weights so the facility doesn't move. The easiest way to do this is: "for every unit we locate at node A, we locate another unit at Node E" thus balancing the changes in demand weights. Thus an optimal way to locate the families without changing the location of the optimal facility is to locate 25 families at A and 25 families at E.

The next 3 parts were about the stochastic queue median. The first two can be answered without any math.

5. The answer is true. The reason is since the rate of demand is close to zero there are virtually no calls the facility must be located at the median. But because the weights on the two sides are equal, the facility can be located anywhere along the link. However if the rate of calls was greater then zero then the answer would only have been node D, this is due to the variance in service times.

6. By the symmetry of demand at nodes A and E the optimal location of the facility is clearly the midpoint node D.

7. Here we need to use the equations given to us in the handout on the stochastic queue median. We will follow the handout. But first we redraw the diagram so that our computations are easier.



$$E[R] = E[W] + E[S]$$

$$E[R] = E[S] + \frac{\lambda[(E[S])^2 + \sigma_s^2]}{2(1 - \lambda E[S])}$$

$$E[d] = f_a(1 + .5) + f_e(1 - .5)$$

$$f_a = \frac{3}{11}$$

$$f_e = \frac{8}{11}$$

$$E[d] = \frac{3}{11}(\frac{1}{2}) + \frac{8}{11}(\frac{3}{2}) = \frac{27}{22}$$

$$E[d^2] = f_a(1+.5)^2 + f_e(1-.5)^2$$

$$E[d^2] = \frac{3}{11}(\frac{1}{2})^2 + \frac{8}{11}(\frac{3}{2})^2 = \frac{75}{44}$$

$$\sigma_d^2 = E[d^2] - (E[d])^2$$

$$\sigma_d^2 = \frac{75}{44} - (\frac{27}{22})^2 = \frac{24}{121}$$

$$E[S] = E[Service] + \frac{2E[d]}{v}$$

$$E[S] = 1 + 2 * \frac{\frac{27}{22}}{1} =$$

$$E[S] = \frac{38}{11}$$

$$\sigma_s^2 = \sigma_{ServiceTime}^2 + 4\sigma_d^2$$

$$\sigma_s^2 = 1 + 4(\frac{24}{121}) = \frac{217}{121}$$

$$E[R] = \frac{38}{11} + \frac{.04((\frac{38}{11})^2 + \frac{217}{121})}{2(1 - .04 * \frac{38}{11})}$$

$$E[R] = 3.77311$$