



MIT International Center for Air Transportation

Airline Revenue Management: Flight Leg and Network Optimization

***1.201 Transportation Systems Analysis: Demand
& Economics***

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Lecture Outline

1. Overview of Airline Pricing

- Differential Pricing Theory
- Fare Restrictions and Disutility

2. Revenue Management Systems

3. Overbooking Models

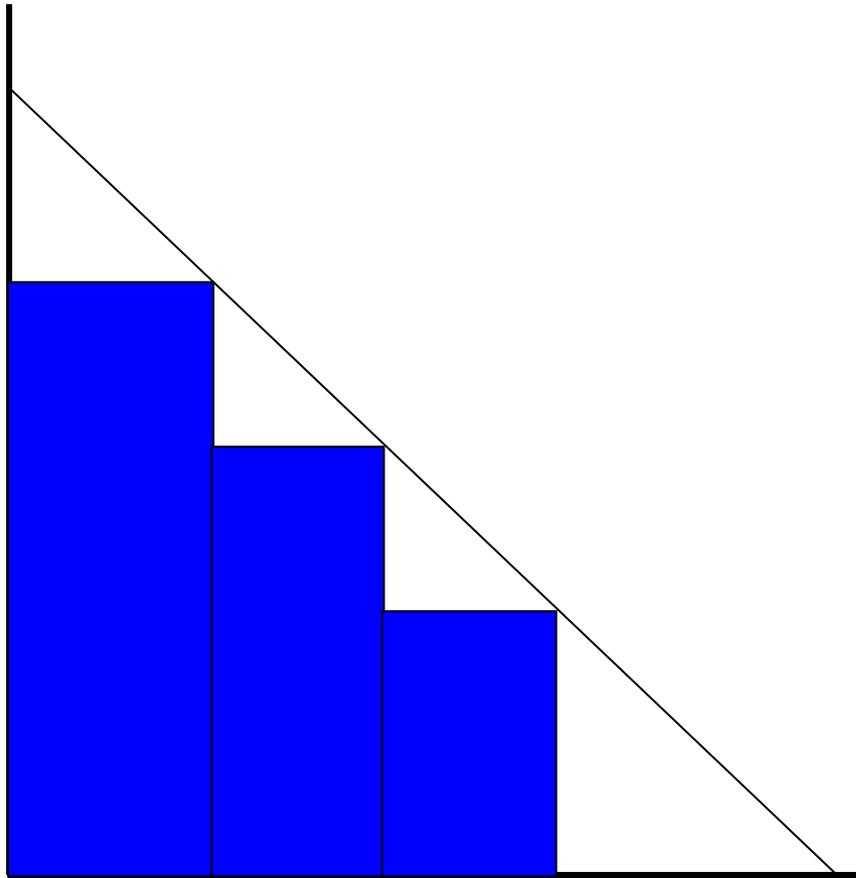
4. Single-leg Fare Class Seat Allocation Problem

- EMSRb Model for Seat Protection

5. Network Revenue Management

- Origin-Destination Control Mechanisms
- Network Optimization Methods

Differential Pricing Theory



- Market segments with different “willingness to pay” for air travel
- Different “fare products” offered to business versus leisure travelers
- Prevent diversion by setting restrictions on lower fare products and limiting seats available
- Increased revenues and higher load factors than any single fare strategy



Traditional Approach: Restrictions on Lower Fares

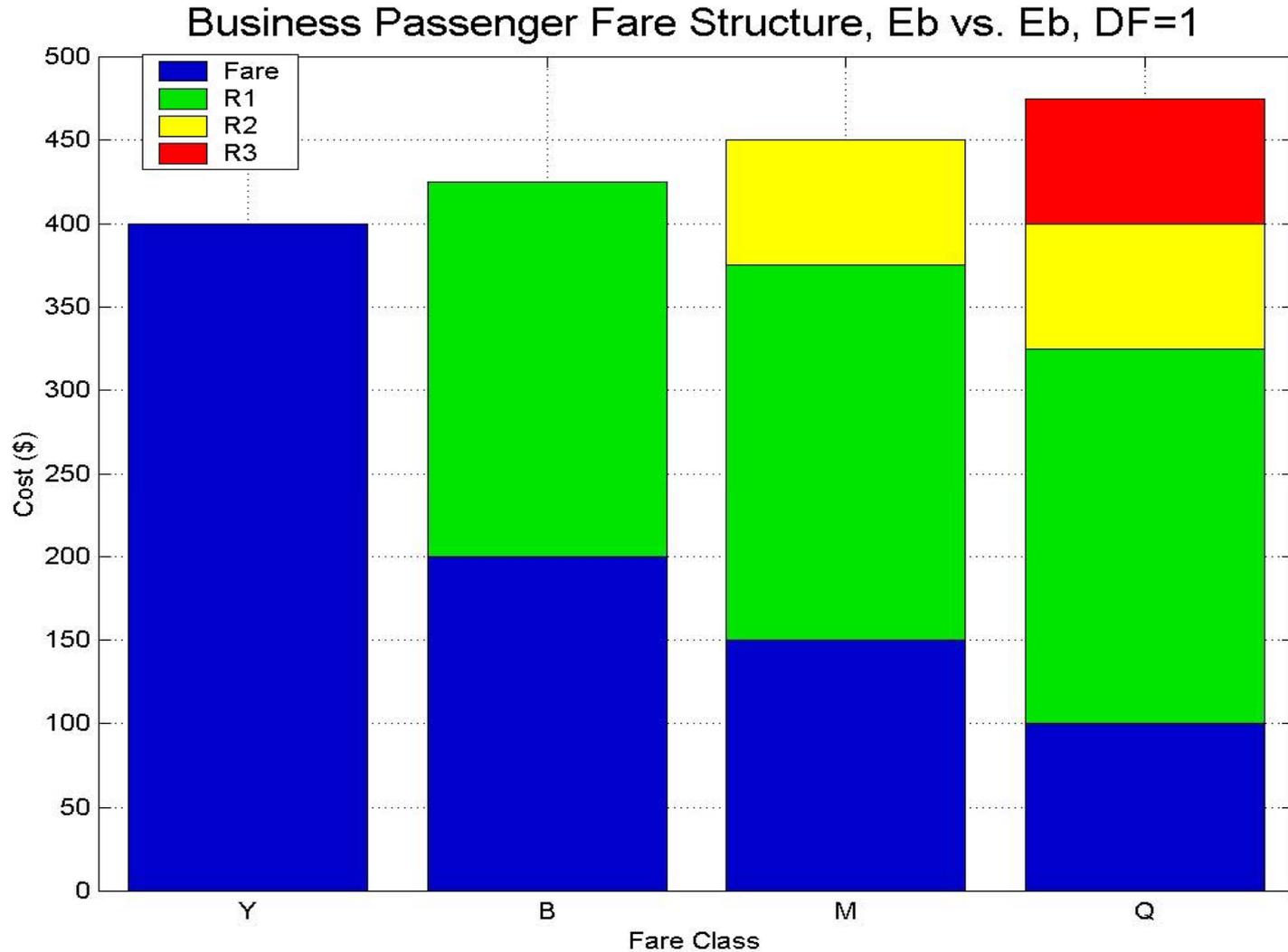
- **Progressively more severe restrictions on low fare products designed to prevent diversion:**
 - Lowest fares have advance purchase and minimum stay requirements , as well as cancellation and change fees
 - Restrictions increase the inconvenience or “disutility cost” of low fares to travelers with high WTP, forcing them to pay more
 - Studies show “Saturday night minimum stay” condition to be most effective in keeping business travelers from purchasing low fares
- **Still, it is impossible to achieve perfect segmentation:**
 - Some travelers with high WTP can meet restrictions
 - Many business travelers often purchase restricted fares

Restrictions Help to Segment Demand

Fare Code	Dollar Price	Advance Purchase	Round Trip?	Sat. Night Min. Stay	Percent Non-Refundable
Y	\$400	--	--	--	--
B	\$200	7 day	Yes	--	50 %
M	\$150	14 day	Yes	Yes	100 %
Q	\$100	21 day	Yes	Yes	100 %

- **Business passengers unwilling to stay over Saturday night will not buy M or Q.**
- **RM system protects for Y, B demand but keeps M,Q classes open without losing revenue.**

Example: Restriction Disutility Costs





BOS-SEA Fare Structure

American Airlines, October 1, 2001

Roundtrip Fare (\$)	Cls	Advance Purchase	Minimum Stay	Change Fee?	Comment
458	N	21 days	Sat. Night	Yes	Tue/Wed/Sat
707	M	21 days	Sat. Night	Yes	Tue/Wed
760	M	21 days	Sat. Night	Yes	Thu-Mon
927	H	14 days	Sat. Night	Yes	Tue/Wed
1001	H	14 days	Sat. Night	Yes	Thu-Mon
2083	B	3 days	none	No	2 X OW Fare
2262	Y	none	none	No	2 X OW Fare
2783	F	none	none	No	First Class



Yield Management = Revenue Management

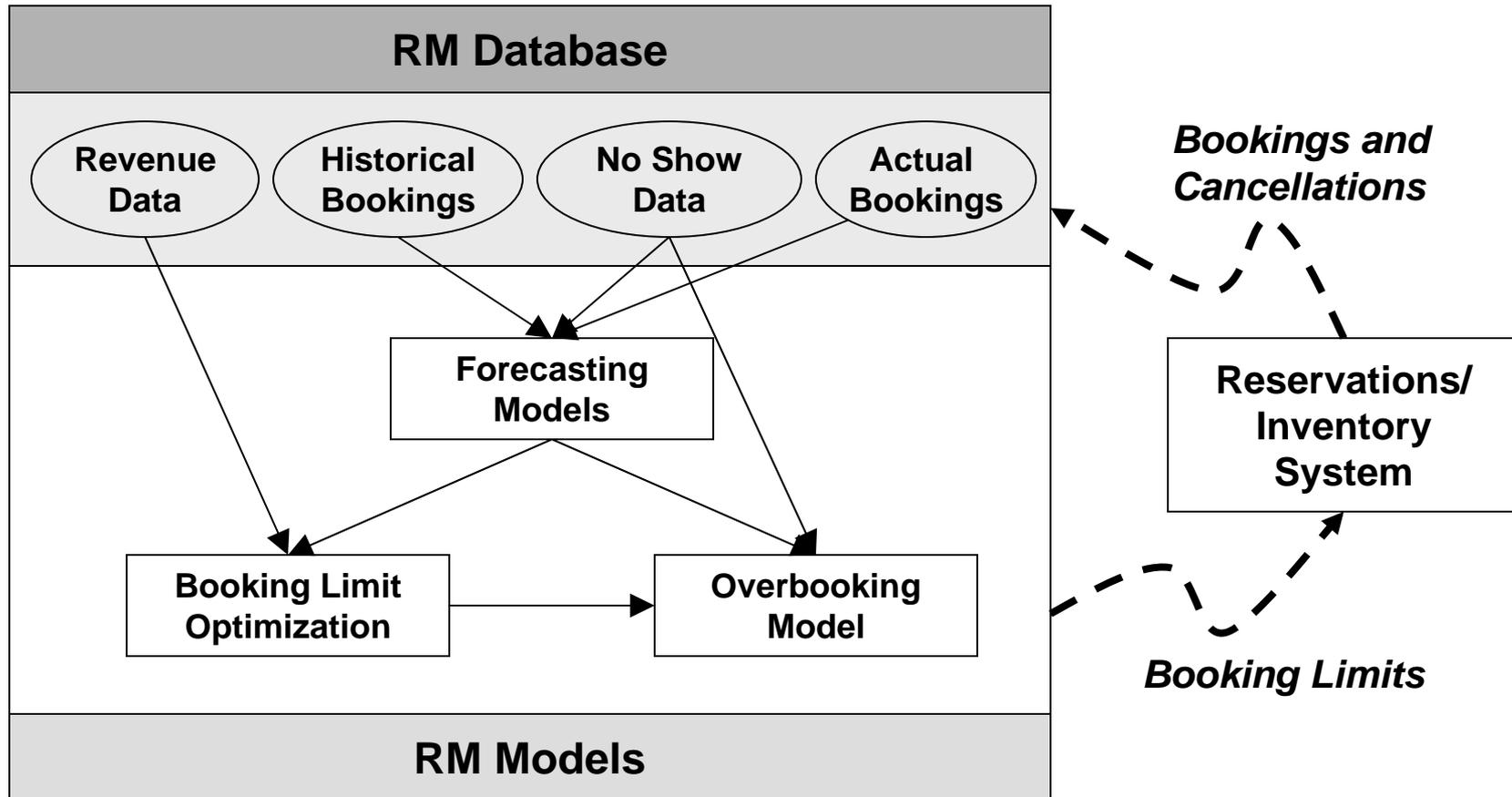
- **YM assumes a set of differentiated fare classes and available flight capacity as given:**
 - Forecast future booking demand for each fare product
 - Optimize number of seats to be made available to each fare class
- **Optimal control of available seat inventory:**
 - On high demand flights, limit discount fare and group bookings to increase overall yield (average fare) and revenue.
 - On low demand flights, sell empty seats at any low fare to increase load factors and revenue.
 - Revenue maximization requires a balance of yield and load factor
- **Most airlines now refer to “Revenue Management” (RM) instead.**



Typical 3rd Generation RM System

- **Collects and maintains historical booking data by flight and fare class, for each past departure date.**
- **Forecasts future booking demand and no-show rates by flight departure date and fare class.**
- **Calculates limits to maximize total flight revenues:**
 - Overbooking levels to minimize costs of spoilage/denied boardings
 - Booking class limits on low-value classes to protect high-fare seats
- **Interactive decision support for RM analysts:**
 - Can review, accept or reject recommendations

Third Generation RM System





Revenue Management Techniques

- **Overbooking**
 - Accept reservations in excess of aircraft capacity to overcome loss of revenues due to passenger “no-show” effects
- **Fare Class Mix (Flight Leg Optimization)**
 - Determine revenue-maximizing mix of seats available to each booking (fare) class on each flight departure
- **Traffic Flow (O-D) Control (Network Optimization)**
 - Further distinguish between seats available to short-haul (one-leg) vs. long-haul (connecting) passengers, to maximize total network revenues



Flight Overbooking

- **Determine maximum number of bookings to accept for a given physical capacity.**
- **Minimize total costs of denied boardings and spoilage (lost revenue).**
- **U.S. domestic no-show rates can reach 15-20 percent of final pre-departure bookings:**
 - On peak holiday days, when high no-shows are least desirable
 - Average no-show rates have dropped, to 10-15% with more fare penalties and better efforts by airlines to firm up bookings
- **Effective overbooking can generate as much revenue gain as fare class seat allocation.**



Cost-Based Overbooking Model

- **Find AU that minimizes :**

[Cost of DB + Cost of SP]

- **For any given AU:**

$$\underline{\text{Total Cost}} = \$DB * E[DB] + \$SP * E[SP]$$

\$DB and \$SP= cost per DB and SP, respectively

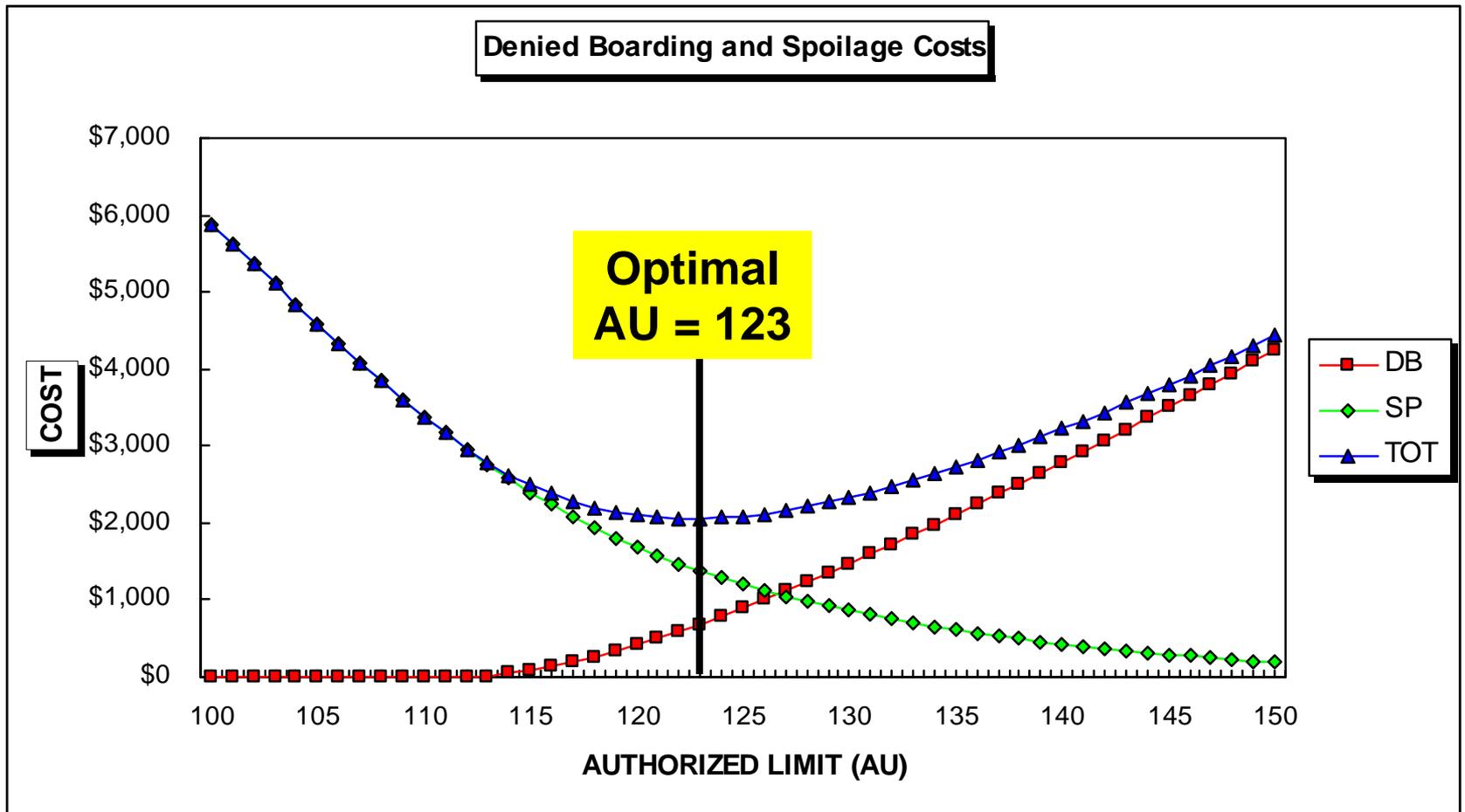
E[DB] = expected number of DBs, given AU

E[SP] = expected number of SP seats, given AU

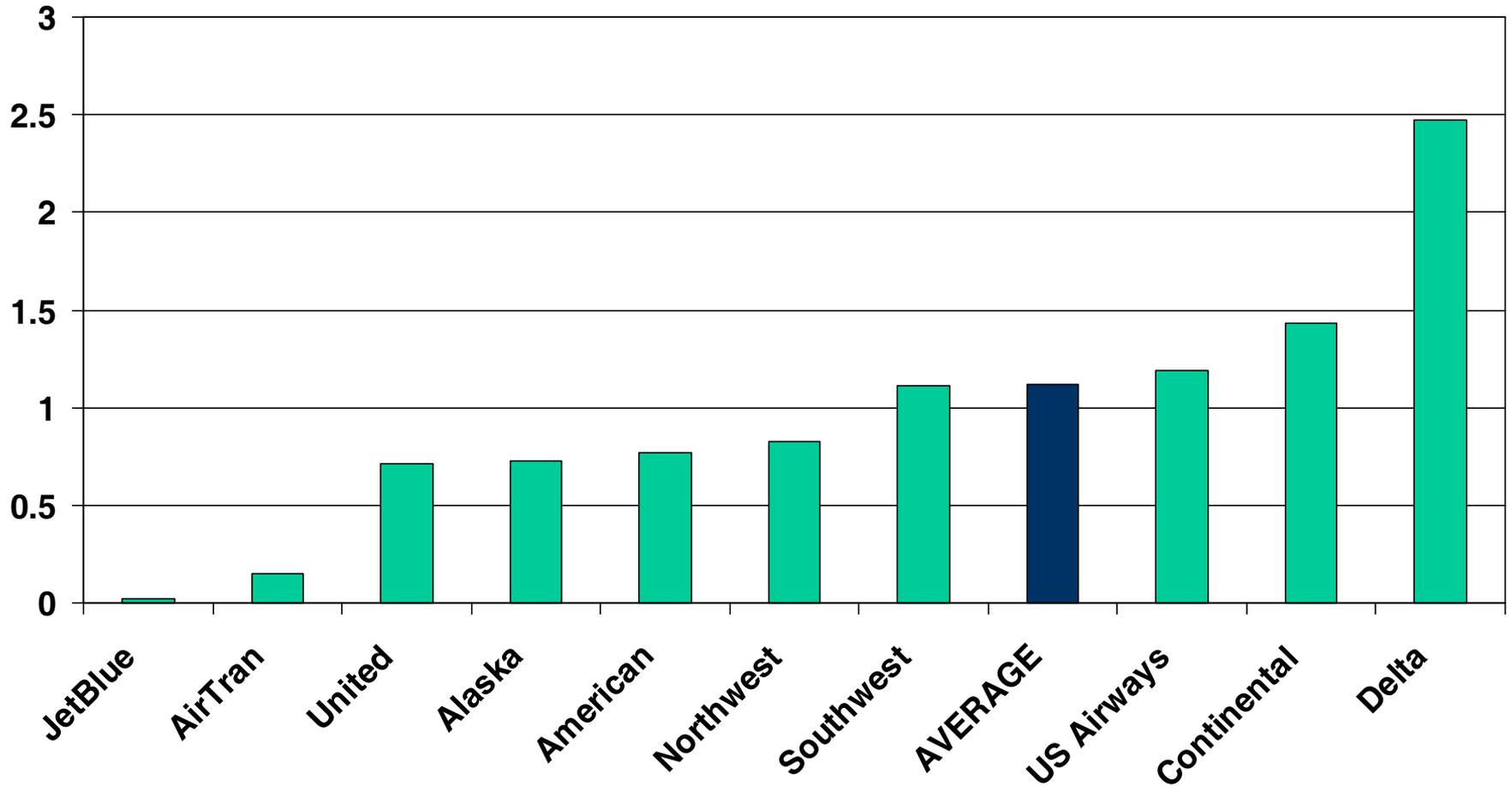
- **Mathematical search over range of AU values to find minimum total cost.**

Cost-Based Overbooking Model

Minimize total cost of expected Denied Boardings plus Spoiled Seats



2007 US Involuntary DBs per 10,000

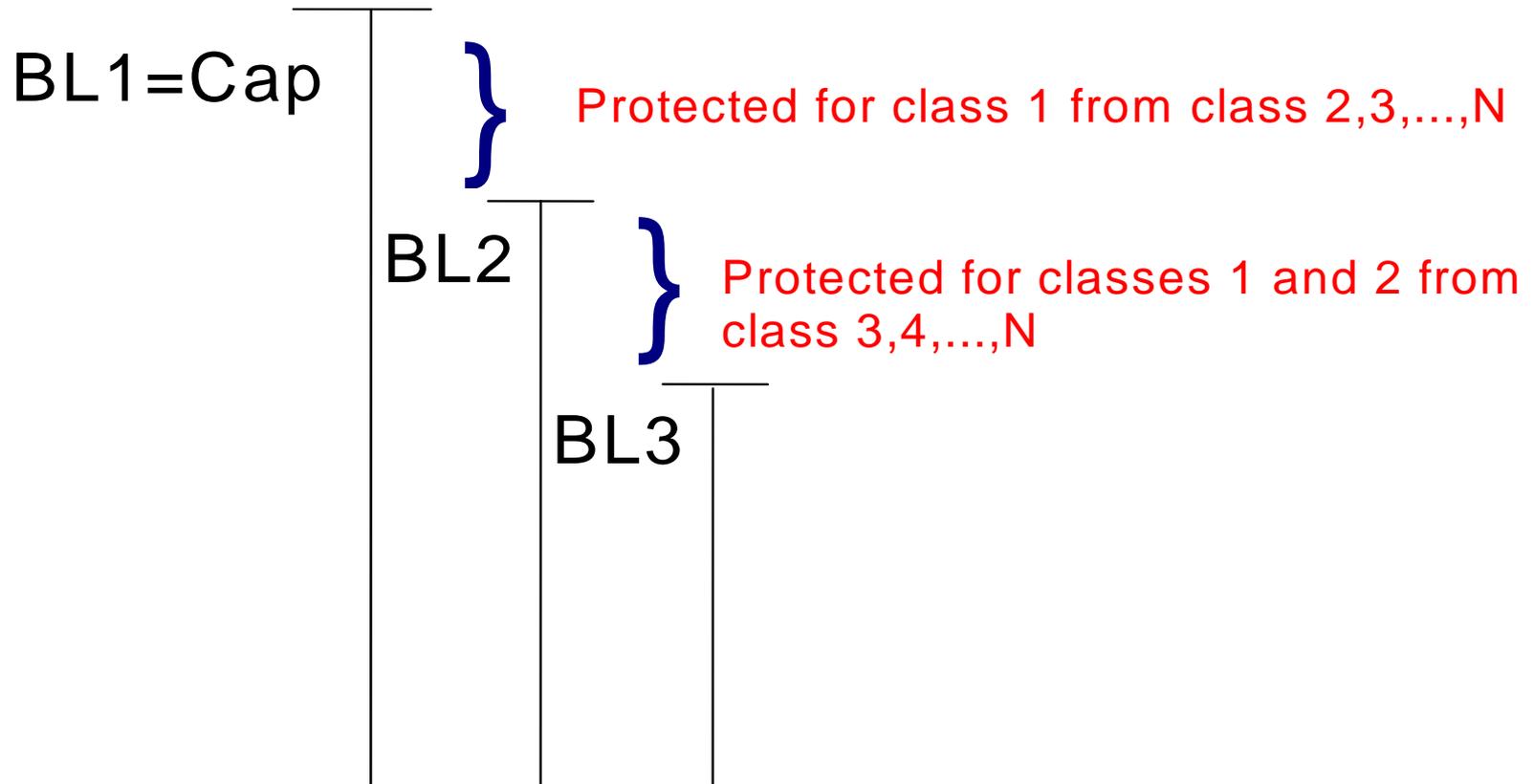




Flight Leg Revenue Maximization

- **Given for a future flight leg departure date:**
 - Total remaining booking capacity of (typically) the coach compartment
 - Several fare (booking) classes that share the same inventory of seats in the compartment
 - Forecasts of future booking demand by fare class between current DCP and departure
 - Revenue estimates for each fare (booking) class
- **Objective is to maximize total expected revenue:**
 - Protect seats for each fare class based on revenue value, taking into account forecast uncertainty and probability of realizing the forecasted demand

Serially Nested Buckets





EMSRb Model for Seat Protection: Assumptions

- **Modeling assumptions for serially nested classes:**
 - a) demand for each class is separate and independent of demand in other classes.
 - b) demand for each class is stochastic and can be represented by a probability distribution
 - c) lowest class books first, in its entirety, followed by the next lowest class, etc.
 - d) booking limits are only determined once (i.e., static optimization model)
- **Problem is to find protection levels for higher classes, and booking limits on lower classes**

- **To calculate the optimal protection levels:**

Define $P_i(S_i) = \text{probability that } X_i \geq S_i$,
where S_i is the number of seats made available to class i , X_i is
the random demand for class i

- **The expected marginal revenue of making the S th seat available to class i is:**

$\text{EMSR}_i(S_i) = R_i * P_i(S_i)$ where R_i is the average revenue (or fare)
from class i

- **The optimal protection level, π_1 for class 1 from class 2 satisfies:**

$$\text{EMSR}_1(\pi_1) = R_1 * P_1(\pi_1) = R_2$$

Consider the following flight leg example:

<u>Class</u>	<u>Mean Fcst.</u>	<u>Std. Dev.</u>	<u>Fare</u>
Y	10	3	1000
B	15	5	700
M	20	7	500
Q	30	10	350

- To find the protection for the Y fare class, we want to find the largest value of π_Y for which
$$\text{EMSR}_Y(\pi_Y) = R_Y * P_Y(\pi_Y) \geq R_B$$

Example (cont'd)

$$\text{EMSR}_Y(\pi_Y) = 1000 * P_Y(\pi_Y) \geq 700$$
$$P_Y(\pi_Y) \geq 0.70$$

where $P_Y(\pi_Y)$ = probability that $X_Y \geq \pi_Y$.

- **Assume demand in Y class is *normally* distributed, then we can create a standardized normal random variable as $(X_Y - 10)/3$:**
 - for $\pi_Y = 7$, $\text{Prob} \{ (X_Y - 10)/3 \geq (7 - 10)/3 \} = 0.841$
 - for $\pi_Y = 8$, $\text{Prob} \{ (X_Y - 10)/3 \geq (8 - 10)/3 \} = 0.747$
 - for $\pi_Y = 9$, $\text{Prob} \{ (X_Y - 10)/3 \geq (9 - 10)/3 \} = 0.63$
- **$\pi_Y = 8$ is the largest integer value of π_Y that gives a probability ≥ 0.7 and we will protect 8 seats for Y class.**

- Joint protection for classes 1 through n from class n+1

$$\overline{X}_{1,n} = \sum_{i=1}^n \overline{X}_i$$

$$\hat{\sigma}_{1,n} = \sqrt{\sum_{i=1}^n \hat{\sigma}_i^2}$$

$$R_{1,n} = \frac{\sum_{i=1}^n R_i * \overline{X}_i}{\overline{X}_{1,n}}$$

- We then find the value of π_n that makes

$$\text{EMSR}_{1,n}(\pi_n) = R_{1,n} * P_{1,n}(\pi_n) = R_{n+1}$$

- Once π_n is found, set $\text{BL}_{n+1} = \text{Capacity} - \pi_n$



EMSRb Seat Protection Model

CABIN CAPACITY =		135				
AVAILABLE SEATS =		135				
BOOKING CLASS	AVERAGE FARE	SEATS BOOKED	<u>FORECAST DEMAND</u>		JOINT PROTECT	BOOKING LIMIT
			MEAN	SIGMA		
Y	\$ 670	0	12	7	6	135
M	\$ 550	0	17	8	23	129
B	\$ 420	0	10	6	37	112
V	\$ 310	0	22	9	62	98
Q	\$ 220	0	27	10	95	73
L	\$ 140	0	47	14		40
	SUM	0	135			



Dynamic Revision and Intervention

- **RM systems revise forecasts and re-optimize booking limits at numerous “checkpoints”:**
 - Monitor actual bookings vs. previously forecasted demand
 - Re-forecast demand and re-optimize at fixed checkpoints or when unexpected booking activity occurs
 - Can mean substantial changes in fare class availability from one day to the next, even for the same flight departure
- **Substantial proportion of fare mix revenue gain comes from dynamic revision of booking limits:**
 - Human intervention is important in unusual circumstances, such as “unexplained” surges in demand due to special events



Revision of Forecasts and Limits as Bookings Accepted

CABIN CAPACITY =		135				
AVAILABLE SEATS =		63				
BOOKING CLASS	AVERAGE FARE	SEATS BOOKED	<u>FORECAST DEMAND</u>		JOINT PROTECT	BOOKING LIMIT
			MEAN	SIGMA		
Y	\$ 670	2	10	5	5	63
M	\$ 550	4	13	7	19	58
B	\$ 420	5	5	2	27	44
V	\$ 310	12	10	5	40	36
Q	\$ 220	17	20	6	63	23
L	\$ 140	32	15	4		0
	SUM	72	73			

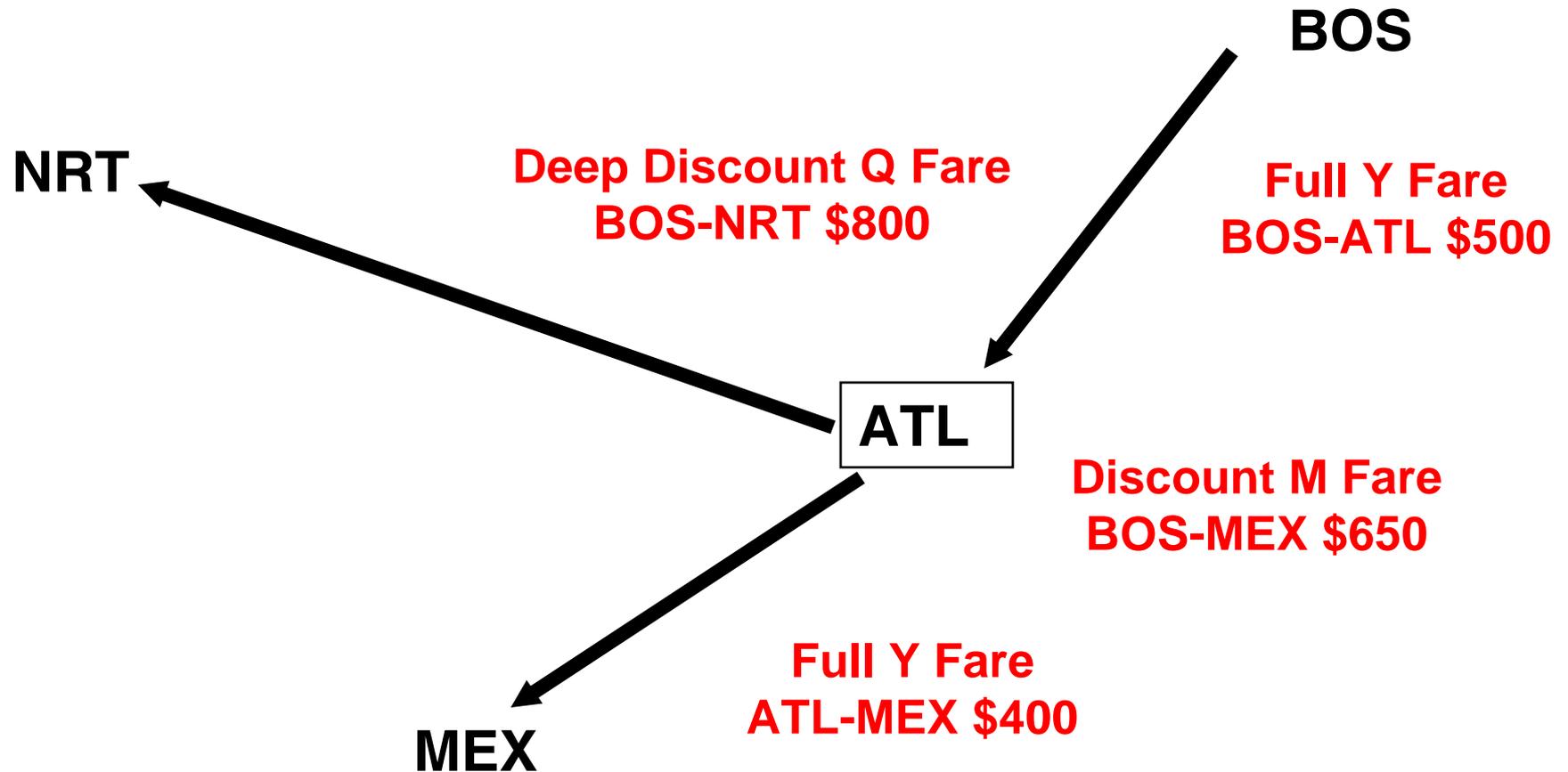
Higher than expected Q bookings close L class



Network RM: O-D Control

- **Advanced airlines are developing O-D control after having mastered basic leg/class RM controls**
 - Effective leg-based fare class control and overbooking alone can increase total system revenues by 4 to 6%
- **“The capability to respond to different O-D requests with different seat availability.”**
- **Effective O-D control can further increase total network revenues by 1 to 2%**
 - Depends on network structure and connecting flows
 - O-D control gains increase with average load factor
 - But implementation is more difficult than leg-based RM systems

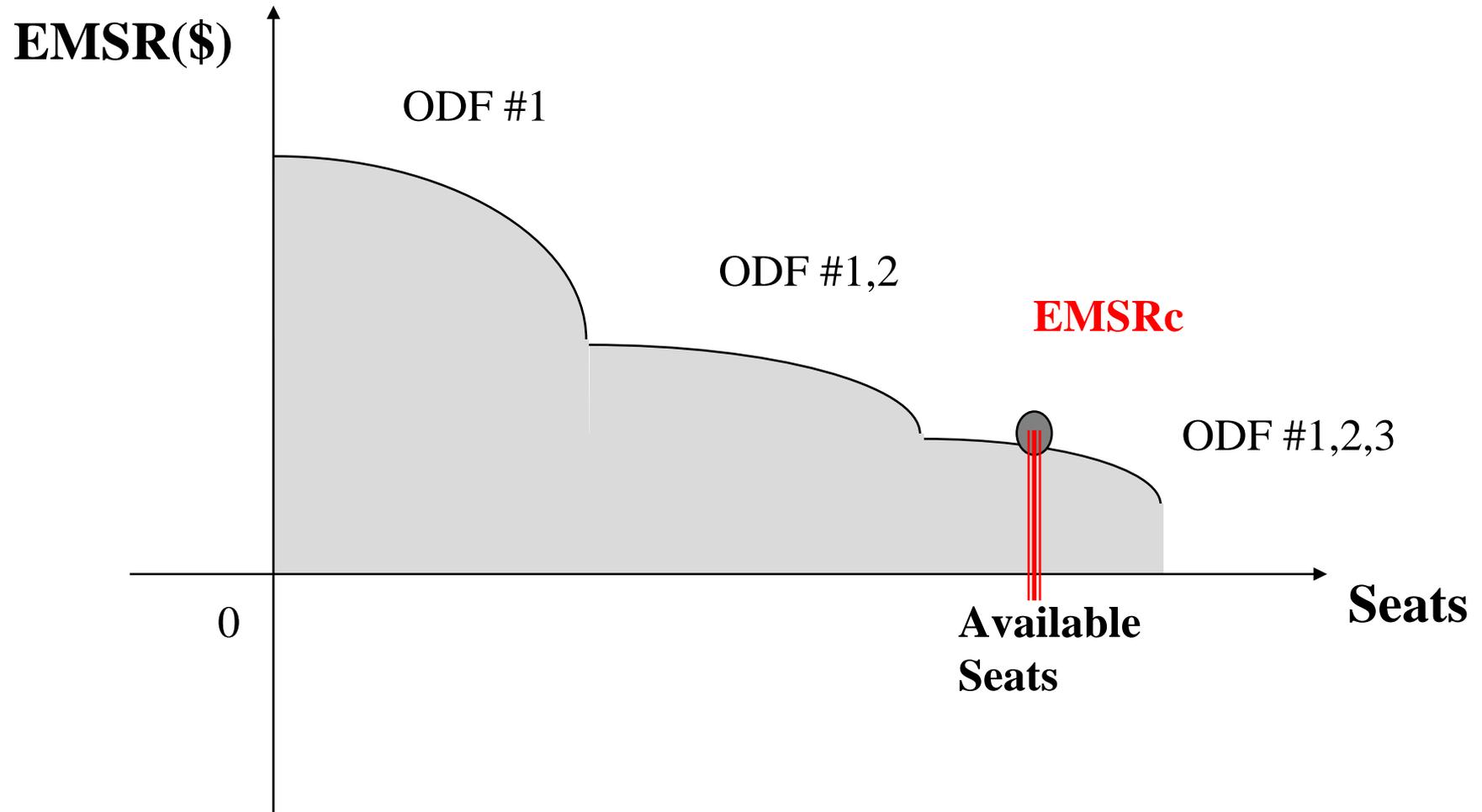
O-D Control Example: Hub Network





Marginal Value of Last Seat on a Leg

- **Marginal value concept is basis of leg RM:**
 - Accept booking in fare class if revenue value exceeds marginal value of last (lowest valued) remaining available seat on the flight leg
- **In network RM, need to estimate marginal network value of last seat on each leg:**
 - Can be used as “displacement cost” of a connecting vs. local passenger
 - Or, as a minimum acceptable “bid price” for the next booking on each leg





Displacement Cost Concept

- **Contribution of an ODF to network revenue on a leg is less than or equal to its total fare:**
 - Connecting passengers can displace revenue on down-line (or up-line) legs
- **Given estimated down-line displacement, ODFs are mapped based on network value:**
 - Network value on Leg 1 = Total fare minus sum of down-line leg displacement costs
 - Under high demand, availability for connecting passengers is reduced, locals get more seats

Virtual Class Mapping with Displacement

NCE/FRA	
CLASS	FARE (OW)
Y	\$450
B	\$380
M	\$225
Q	\$165
V	\$135

NCE/HKG (via FRA)	
CLASS	FARE (OW)
Y	\$1415
B	\$975
M	\$770
Q	\$590
V	\$499

NCE/JFK (via FRA)	
CLASS	FARE (OW)
Y	\$950
B	\$710
M	\$550
Q	\$425
V	\$325

MAPPING OF ODFs ON NCE/FRA LEG TO VIRTUAL VALUE CLASSES

VIRTUAL CLASS	REVENUE RANGE	MAPPING OF O-D MARKETS/CLASSES
1	1200 +	Y NCEHKG
2	900-1199	B NCEHKG Y NCEJFK
3	750-899	M NCEHKG
4	600-749	B NCEJFK
5	500-599	Q NCEHKG M NCEJFK
6	430-499	V NCEHKG Y NCEFRA
7	340-429	B NCEFRA Q NCEJFK
8	200-339	V NCEJFK M NCEFRA
9	150-199	Q NCEFRA
10	0 - 149	V NCEFRA



- **Marginal value of last seat can also represent the flight leg “Bid Price”:**
 - A minimum “cutoff” value required to accept a booking request
 - For a single-leg itinerary, a request is accepted if the corresponding fare is greater than the bid price for the leg.
 - For a multi-leg itinerary, the ODF fare must be greater than the sum of the bid prices of all flight legs used by the itinerary.
- **Much simpler inventory control mechanism than virtual buckets:**
 - Simply need to store bid price value for each leg
 - Must revise bid prices frequently to prevent too many bookings of ODFs at current bid price

Example: Bid Price Control



- Given leg bid prices

A-B: \$34 B-C: \$201 C-D: \$169

- Availability for O-D requests B-C:

	Bid Price = \$201	Available?
Y	\$440	Yes
M	\$315	Yes
B	\$223	Yes
Q	\$197	No

A-B: \$34 B-C: \$201 C-D: \$169

A-C Bid Price = \$235 Available?

Y \$519 Yes

M \$344 Yes

B \$262 Yes

Q \$231 No

A-D Bid Price = \$404 Available?

Y \$582 Yes

M \$379 No

B \$302 No

Q \$269 No



Network Optimization Methods

- **Network optimization mathematics needed for both bid price and value bucket controls.**
- **Several optimization methods to consider:**
 - Deterministic Linear Programming
 - Dynamic Programming
 - Nested Probabilistic Network Bid Price
- **Simulated revenue gains are quite similar:**
 - ODF database, forecast accuracy and robustness under realistic conditions make a bigger difference



Network Linear Program (LP)

Maximize Total Revenue = Sum [Fare * Seats]

- Summed over all ODFs on network

Subject to following constraints:

Seats for each ODF \leq Mean Forecast Demand
Sum[Seats on Each Leg] \leq Leg Capacity

Outputs of LP solution:

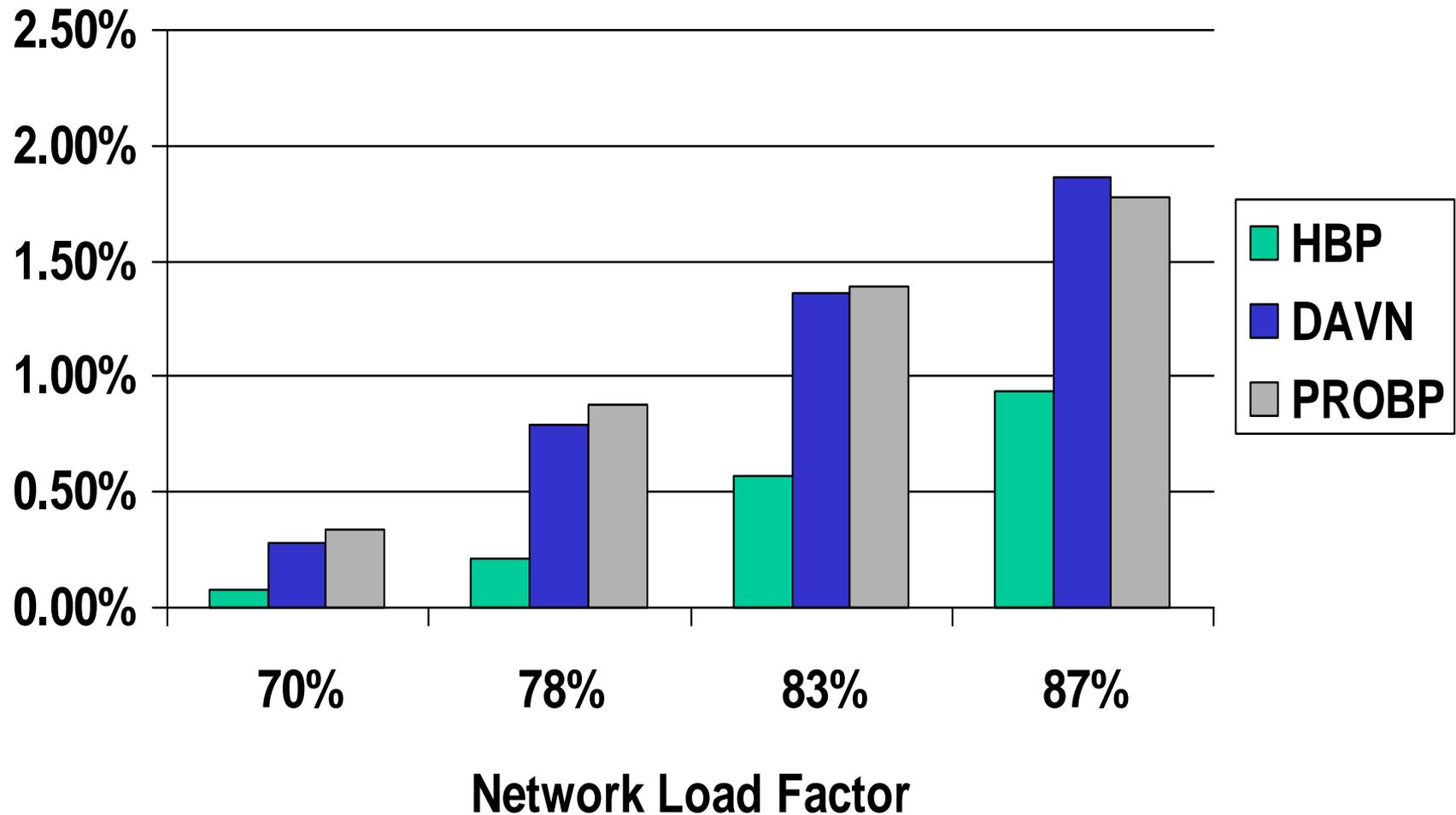
- Seats allocated to each ODF (not useful)
- “Shadow price” on each leg (reflects network revenue value of last seat on each flight leg)

O-D Control System Alternatives

O-D Control System	Data and Forecasts	Optimization Model	Control Mechanism
Rev. Value Buckets	Leg/bucket	Leg EMSR	Leg/bucket Limits
Heuristic Bid Price	Leg/bucket	Leg EMSR	Bid Price for Connex only
Disp. Adjust. Value Bkts.	ODF	Network + Leg EMSR	Leg/bucket Limits
Network Bid Price	ODF	Network	O-D Bid Prices

O-D Revenue Gain Comparison

Airline A, O-D Control vs. Leg/Class RM





Summary: Airline O-D RM Systems

- **O-D control is the 4th generation of RM:**
 - Data collection, forecasting, optimization and control by origin-destination-fare type as well as distribution channel
- **Provides control by itinerary and network value of requests, not simply by flight leg and class**
 - Incremental network revenue gains of 1-2% over basic RM
 - Essential to protect against revenue loss to competitors
 - Increased control of valuable inventory in the face of pricing pressures, new distribution channels, and strategic alliances

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