

# Transportation Revenue Forecasting: Theory and Models

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## Overview

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- Increasing reliance on private sector financing of transportation projects (particularly toll roads) has emphasized the importance of accurate revenue forecasting
  
- Main factors of revenue forecasting:
  - Pricing/tolling strategy
  - Travel demand forecasting
  - Traffic assignment

## Outline

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- **Review Basic Pricing Concepts**
  - Public Sector Pricing
  - Private Sector Pricing
- Revenue Forecasting
  - Forecasting Accuracy
  - Sources of Uncertainty
- Enhanced Methods
- Conclusion

## Review of Public Sector Pricing

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- Maximize welfare
- Marginal cost pricing
- Under constraints
  - Cost recovery: Ramsey pricing
  - Distortions: second-best pricing

## Review of Private Sector Pricing

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- Maximize profit or revenue
- Perfectly competitive market
  - Price close to marginal cost
- Less competitive market
  - Price discrimination

## Outline

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- Review Basic Pricing Concepts
- **Revenue Forecasting**
  - Forecasting Accuracy
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- Enhanced Methods
- Conclusion

## Revenue Forecasting

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- Revenue forecasting is essential for assessing financial feasibility and project approval
- Here we focus on toll roads and toll bridges

## Forecasting Accuracy

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- Few examples where actual revenue exceeded the forecast
- Many examples where actual traffic demand and revenue have significantly lagged the forecasts
  - Dulles Greenway in Virginia went into default in 1996, when toll revenues were less than the forecast (20% of the forecast in its first year of operation; and only 35% in its fifth year)

## Forecasting Accuracy (cont.)

- 68 case studies during 2003 of traffic forecasting for international toll roads
- Ratio of actual/forecast traffic volumes follows a normal distribution with average 0.74 and standard deviation 0.26

Image removed due to copyright issues.

Source: Bain, R. and Plantagie, T.W. (2003), *Traffic Forecasting Risk: Study Update 2003*, Standard & Poor's RatingsDirect, the McGraw-Hill Companies, New York, [Online], May 2008, Available at: [http://www.people.hbs.edu/besty/projfinportal/S&P\\_Traffic\\_Update.pdf](http://www.people.hbs.edu/besty/projfinportal/S&P_Traffic_Update.pdf).



## Forecasting Accuracy (cont.)

Examples: Actual revenue as percentage of forecast

Facility	Year of opening	Year 1	Year 2	Year 3	Year 4	Year 5
Florida's Turnpike Enterprise/Sawgrass Expressway	1986	17.8%	23.4%	32.0%	37.1%	38.4%
Orlando-Orange Expressway Authority/Central Florida Greenway North Segment	1989	96.8%	85.7%	81.4%	69.6%	77.1%
State Road and Tollway Authority (Georgia)/GA 400	1993	117.0%	133.1%	139.8%	145.8%	141.8%
Transportation Corridor Agencies (California)/San Joaquin Hills	1996	31.6%	47.5%	51.5%	52.9%	54.1%
Santa Rosa Bay Bridge Authority (Florida)/Garcon Point Bridge	1999	32.6%	54.8%	50.5%	47.1%	48.7%
Pocahontas Parkway Association (Virginia)/Pocahontas Parkway	2002	41.6%	40.4%	50.8%	NA	NA

Figure by MIT OpenCourseWare.

Source: Kriger, D., Shiu, S. and Naylor, S. (2006), *Estimating Toll Road Demand and Revenue*, National Cooperative Highway Research Program Synthesis 364, Transportation Research Board, [Online], May 2008, Available at: [http://onlinepubs.trb.org/onlinepubs/nchrp/nchrp\\_syn\\_364.pdf](http://onlinepubs.trb.org/onlinepubs/nchrp/nchrp_syn_364.pdf).



## Sources of Uncertainty

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- Revenue forecasting is based on:
  - Travel forecasts: models and assumptions on which the travel forecasts were based, such as economic growth, land use, and changes in traffic patterns
  - The tolling schedule and structure, e.g., by vehicle type
  - The enforcement of toll collection
  
- Revenue is estimated by multiplying the forecasted traffic volumes by the toll amount, taking into account different toll rates by vehicle type, potential toll evasion, and discounts

## Sources of Uncertainty: Modeling

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The four-step modeling process is widely used for traffic demand forecasting

- Travel demand models intended for regional planning purpose may not be appropriate for toll roads
- Steady-state forecast does not incorporate the likelihood of traffic fluctuations during economic cycles
- Peak-hour travel characteristics do not represent annual traffic demand, typically required for toll road revenue forecasts
- Limited data on weekend/truck traffic patterns
- Value of time is heterogeneous and difficult to estimate

## Sources of Uncertainty: Input Data

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- Demographic, socioeconomic and land-use variables, e.g., population, economic growth, employment, and zone definition
- Transportation network
- Travel characteristics, e.g. origin-destination demand, travel cost, traffic counts and speeds, stated preferences data
- Individual value of time or willingness to pay
- Tolling amount and structure

## Other Sources of Uncertainty

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- Traffic ramp-up period
  - High financial risk during the initial years of operation
  - Traffic volumes may be lower than forecasted as drivers slowly become aware of the toll facility and time saving, or if population or economic growth is less than forecasted
- Event and political effects, e.g. competing roads

## Outline

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- Review Basic Pricing Concepts
- Revenue Forecasting
- **Enhanced Methods**
  - Activity-based models
  - Travel choice models (eg time, mode, path)
  - Traffic Assignment (simulation & dynamic models)
  - **Models with heterogeneous value of time**
  - **Travel time reliability**
  - **Uncertainty of forecast results ...**
- Conclusion

## Preference Heterogeneity: An Example

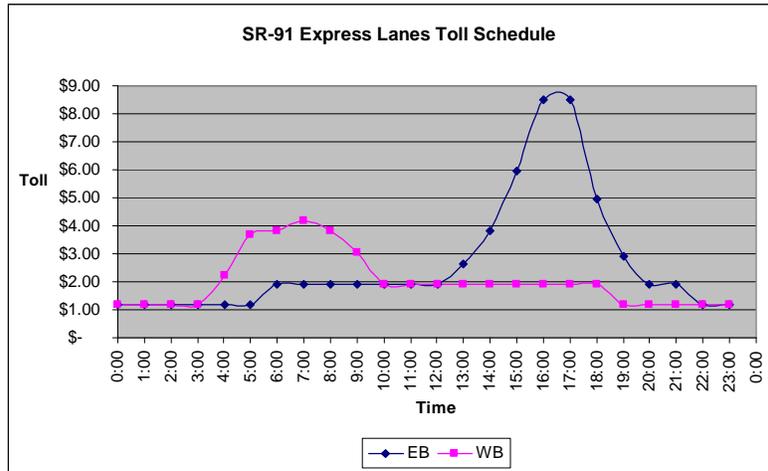
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### SR-91 in California, USA

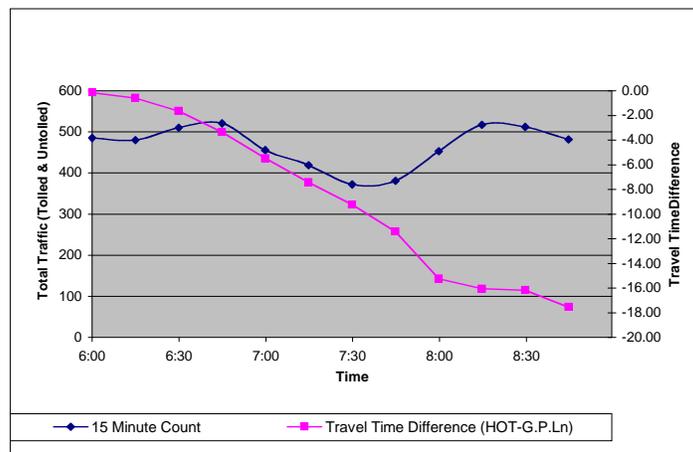
Source: Tian, X., Kulkarni, A. and Jha, M. (2008), Modeling Congestion Pricing, presented at Workshop on Traffic Modeling: Traffic Behavior and Simulation, University of Technology, Graz, Austria

Google map image of California highway SR-91 removed due to copyright restrictions.

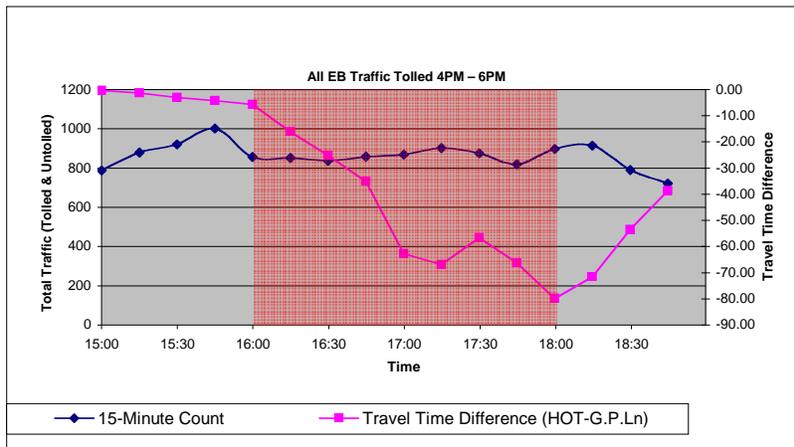
## Toll Rates



## Travel Time Difference between HOT and General Purpose Lanes: AM



## Travel Time Difference between HOT and General Purpose Lanes: PM



## Heterogeneous Value of Time

Value of time (VOT) or willingness to pay (WTP) varies by

- Trip purpose
- Time-of-travel
- Travel mode
- Vehicle occupancy
- Congestion levels
- Trip length
- Socio-economic variables, e.g. income, gender, occupation and education level

## Heterogeneous Value of Time: Explicit Segmentation

- An explicit segmentation of the corresponding assignment, mode choice and time-of-travel choice models, while assuming a single average VOT within each segment
- Limitations:
  - Requires multi-class assignment with a large number of segments
  - Problematic when different choice dimensions use different segmentation
  - Does not account for within-segment variability

## Heterogeneous Value of Time: Explicit Segmentation (cont.)

Example: VOT estimates for toll road users in Montreal model

Gender	Income	Time-of-travel	VOT by purpose		
			Work	Maintenance	Discretionary
Male	Low	Off-peak	\$7.30	\$4.00	\$3.00
		Peak	\$10.30	\$4.00	\$3.00
	High	Off-peak	\$10.20	\$4.00	\$3.00
		Peak	\$10.20	\$4.00	\$3.00
Female	Low	Off-peak	\$7.30	\$6.40	\$6.00
		Peak	\$10.30	\$6.40	\$6.00
	High	Off-peak	\$10.60	\$7.30	\$7.60
		Peak	\$10.60	\$7.30	\$7.60

Source: Vovsha, P., Davidson, W. and Donnelly, R. (2005), 'Making the state of the art the state of the practice: advanced modeling techniques for road pricing', *Expert Forum on Road Pricing and Travel Demand Modeling*, Volpe National Transportation Systems Center, [Online], May 2008, Available at: <http://tmip.fhwa.dot.gov/clearinghouse/docs/DOT-OST-P-001-06>

## Distributed Value of Time

- Probabilistic distribution of VOT instead of deterministic values, with a corresponding adjustment of the structure of the assignment, mode choice, and time-of-travel choice models
  
- Latent Class Choice Model
  - Discrete distribution of VOT
  - Gopinath, D.A. and Ben-Akiva, M. (1997), 'Estimation of randomly distributed value of time', Working paper, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology.
  
- Random Coefficient Logit Model
  - Continuous distribution of VOT
  - Coefficients for travel time and travel cost, or their ratio (VOT), is assumed to be randomly distributed



## Distributed Value of Time (cont.)

Ben-Akiva, M., Bolduc, D. and Bradley, M. (1993), 'Estimation of travel choice models with randomly distributed value of time', *Transportation Research Record*, vol. 1413, pp.88-97.

$$U_i = \mu \left[ c_i + \beta' Y_i + \nu \sqrt{\text{Value of time}} (t_i + \gamma' Z_i) \right] + \varepsilon_i$$

- $c_i$  = travel cost of alternative  $i$
- $t_i$  = travel time of alternative  $i$
- $Y_i$  = vector of additional variables for alternative  $i$  (may include interactions with  $c_i$ )
- $Z_i$  = vector of additional variables for alternative  $i$ , whose coefficients vary proportionally to the time coefficient (may include interactions with  $t_i$ )
- $\varepsilon_i$  = additive error terms
- $\mu$  = scale parameter
- $\beta, \gamma$  = unknown parameters



## Distributed Value of Time (cont.)

Choice probability for a lognormal distribution of VOT

$$\ln v : N(\omega, \sigma^2)$$

$$P(i) = \frac{1}{\sigma\sqrt{2\pi}} \times \int_0^{\infty} \frac{\exp\{\mu[c_i + \beta'Y_i + v(t_i + \gamma'Z_i)]\}}{\sum_{j=1}^J \exp\{\mu[c_j + \beta'Y_j + v(t_j + \gamma'Z_j)]\}} \times \frac{1}{v} \exp\left\{-\frac{1}{2}\left(\frac{\ln v - \omega}{\sigma}\right)^2\right\} dv$$

- where  $\mu, \beta, \gamma, \omega, \sigma$  are the parameters to be estimated simultaneously using maximum likelihood

## Distributed Value of Time (cont.)

1988 stated preferences survey in Netherlands

- Asymmetric distribution skewed to the left of the mean, with a minimum value 0 and a tail to the right

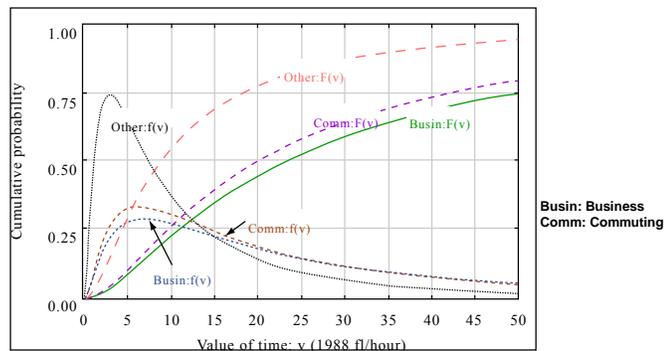


Figure by MIT OpenCourseWare.

## Travel Time Reliability

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### ➤ Motivation

- Unreliability is one of the primary costs of road congestion
- There is evidence that people choose toll roads for reliability, even when there are no obvious travel time savings
- Examples of reliability measures
  - The standard deviation
  - The difference between the 90<sup>th</sup> percentile and the median

### ➤ Value of reliability (VOR)

- Willingness to pay for reduction in the day-to-day variability of travel time

## Modeling Route Choice with Travel Time Reliability

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Lam, T.C. and Small, K. (2001), 'The value of time and reliability: measurement from a value pricing experiment', *Transportation Research Part E*, vol. 37, pp. 231-251.

- Revealed preferences survey (1998) of commuters on State Route 91 in Orange County, California
- Travel time measured with loop detectors
- Binary Logit choice model of two parallel routes (either free or with variable toll)

## Modeling Route Choice with Travel Time Reliability:

# Model Specification

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Variables included:

- Travel time median
  - + interaction with a distance function
- Travel time unreliability (difference between the 90<sup>th</sup> percentile and the median)
  - + interaction with gender
- Travel cost
- Socio-economic variables



## Modeling Route Choice with Travel Time Reliability:

# Estimating VOR

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$$U_{in} = V(t_{in}, v_{in}, c_{in}, x_{in}) + \varepsilon_{in}$$

$$VOR_n = (\partial V / \partial v_n) / (\partial V / \partial c_n)$$

$$VOT_n = (\partial V / \partial t_n) / (\partial V / \partial c_n)$$

where :

$U_{in}$  = utility of alternative  $i$   
for individual  $n$

$t_{in}$  = travel time

$v_{in}$  = travel time variability

$c_{in}$  = travel cost

$x_{in}$  = other attributes or characteristics

$\varepsilon_{in}$  = error term



## Modeling Route Choice with Travel Time Reliability: Estimation Results

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### VOT

Trip distance n miles (%)	13 (5%)	27 (25%)	37 (50%)	40 (mean)	50 (75%)	74 (95%)	92 (99%)
VOT (\$/h)	5.18	18.45	24.00	25.02	26.31	16.04	-4.05
VOT/mean wage (%)	16%	59%	76%	79%	84%	51%	-13%

### VOR

Gender	Male	Female
VOR (\$/h)	12.08	29.62
VOR/mean wage (%)	38%	94%

## Estimation of Uncertainty

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- Uncertainty of Revenue Forecasting is unavoidable
  - Inherent uncertainty in the assumptions about future events and errors in the forecasting procedure
  - Expected value may have large variance and bias
  
- Probability distribution of forecast results
  - Monte-Carlo simulation
  - Simplified methods

## Simplified Method of Estimating the Uncertainty

- Bowman, J.L., Gopinath, D. and Ben-Akiva, M. (2002), 'Estimating the probability distribution of a travel demand forecast', Working paper, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology.
- Requires fewer model runs to estimate elasticities of forecast w.r.t. critical factors affecting uncertainty

## Simplified Method of Estimating the Uncertainty (cont.)

- Step 1: Identify independent sources of uncertainty (e.g., economic growth, model error and value of time), and estimate a probability distribution of each source

$$\begin{aligned}x &= (x_1, \dots, x_k, \dots, x_K) \\x_k^{n_k}, n_k &= 1, \dots, N_k \\p(x_k^{n_k}), n_k &= 1, \dots, N_k \quad \text{s.t.} \quad \sum_{n_k=1}^{N_k} p(x_k^{n_k}) = 1\end{aligned}$$

where  $x$  is the vector of sources that induces errors in the forecast;  $x_k^{n_k}$  is a discrete outcome of  $x_k$ , and  $p(x_k^{n_k})$  is its corresponding probability

## Simplified Method of Estimating the Uncertainty (cont.)

- Step 2: Define a set of scenarios  $S$ , and compute the probability of each scenario,  $p(s)$ , under the assumption of independence of error sources

$$S = \{(x_1^{n_1}, \dots, x_k^{n_k}, \dots, x_K^{n_K}); n_k = 1, \dots, N_k; k = 1, \dots, K\}$$

$$p(s) = \prod_{k=1}^K p(x_k^{n_k(s)}); s \in S$$

- Step 3: Assume travel demand  $r$  depends on each source with a constant elasticity  $e_k$

$$r = a \prod_{k=1}^K (x_k)^{e_k}$$

and perform model runs to estimate these elasticities



## Simplified Method of Estimating the Uncertainty (cont.)

- Step 4: Calculate travel demand for each scenario  $r^{(s)}$  based on the predicted value  $r^{(p)}$

$$r^{(s)} = r^{(p)} \prod_{k=1}^K \left( \frac{x_k^{n_k(s)}}{x_k^{n_k(p)}} \right)^{e_k}, s \in S$$

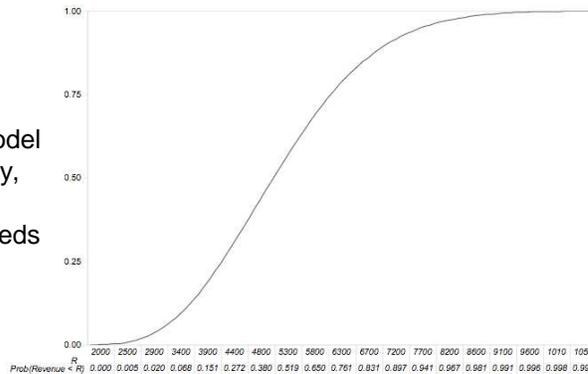
→ All pairs of  $r^{(s)}$  and  $p(s)$  provide an estimated discrete approximation of  $r$ 's probability distribution function



## Simplified Method of Estimating the Uncertainty (cont.)

Example: Estimated cumulative distribution function of 2001 revenue of a new transit system in a major Asian city

Total 16 sources of uncertainty, such as economic growth, model errors, transit captivity, VOT, measurement errors, operating speeds and headways



## Conclusion

- Knowledge of demand and price sensitivities is critical
- Traffic simulation, dynamic traffic assignment, activity-based models, time-of-travel choice models, distributed value of time, and travel time reliability are needed to improve forecasting accuracy
- Uncertainty in revenue forecasts can be estimated

## Additional Readings

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- Ben-Akiva, M., Bolduc, D. and Bradley, M. (1993), 'Estimation of travel choice models with randomly distributed value of time', *Transportation Research Record*, vol. 1413, pp.88-97.
- Bowman, J.L., Gopinath, D. and Ben-Akiva, M. (2002), 'Estimating the probability distribution of a travel demand forecast', Working paper, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology.
- Gopinath, D.A. and Ben-Akiva, M. (1997), 'Estimation of randomly distributed value of time', Working paper, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology.

## Additional Readings (cont.)

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- Kriger, D., Shiu, S. and Naylor, S. (2006), Estimating Toll Road Demand and Revenue, National Cooperative Highway Research Program Synthesis 364, Transportation Research Board, [Online], May 2008, Available at: [http://onlinepubs.trb.org/onlinepubs/nchrp/nchrp\\_syn\\_364.pdf](http://onlinepubs.trb.org/onlinepubs/nchrp/nchrp_syn_364.pdf)
- Lam, T.C. and Small, K. (2001), 'The value of time and reliability: measurement from a value pricing experiment', *Transportation Research Part E*, vol. 37, pp. 231-251.

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