Theory of the Firm

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1.201 / 11.545 / ESD.210
Transportation Systems Analysis: Demand & Economics

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Outline

- **Basic Concepts**
- Production functions
- Profit maximization and cost minimization
- Average and marginal costs

Basic Concepts

- Describe behavior of a firm
- Objective: maximize profit

$$\max \pi = R(a) - C(a)$$

$$s.t. \quad a \geq 0$$

  – *R, C, a* – revenue, cost, and activities, respectively

- Decisions: amount & price of inputs to buy
  amount & price of outputs to produce
- Constraints: technology constraints
  market constraints

Production Function

- Technology: method for turning inputs (including raw materials, labor, capital, such as vehicles, drivers, terminals) into outputs (such as trips)
- Production function: description of the technology of the firm. Maximum output produced from given inputs.

$$q = q(X)$$

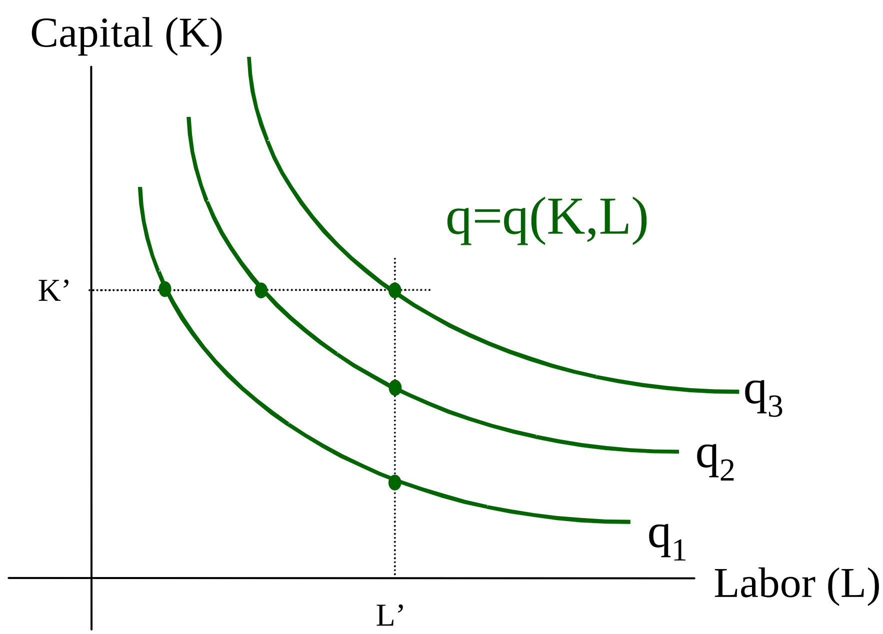
  - q – vector of outputs
  - X – vector of inputs (capital, labor, raw material)

Using a Production Function

- The production function predicts what resources are needed to provide different levels of output
- Given prices of the inputs, we can find the most efficient (i.e. minimum cost) way to produce a given level of output

Isoquant

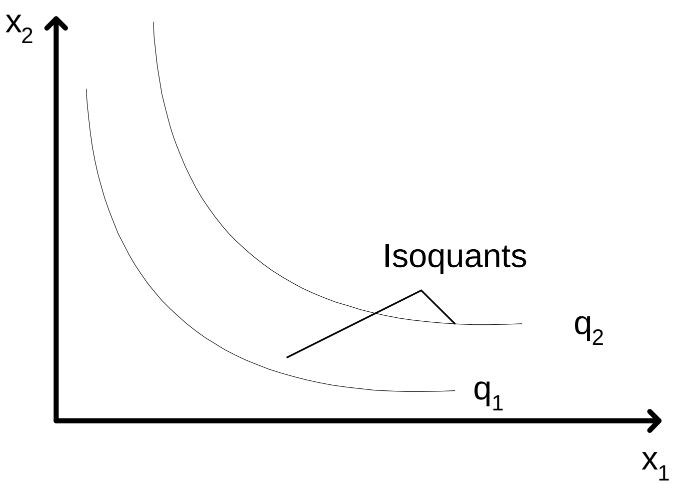
- For two-input production:



Production Function: Examples

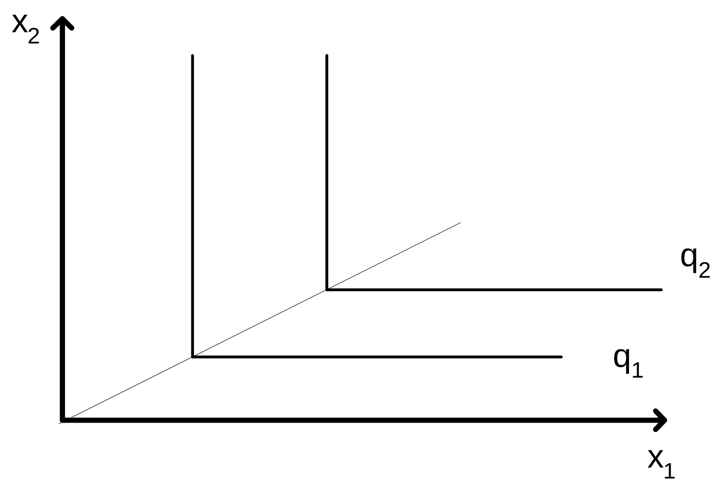
- Cobb-Douglas :

$$q = \alpha x_1^{\ a} x_2^{\ b}$$



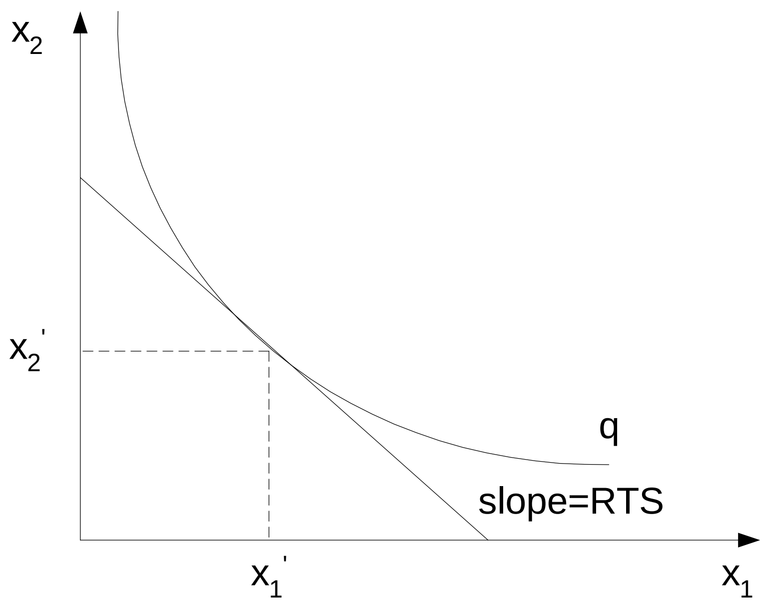
- Input-Output:

$$q = \min(ax_1, bx_2)$$



Rate of Technical Substitution (RTS)

- Substitution rates for inputs
  - Replace a unit of input 1 with RTS units of input 2 keeping the same level of production



$$RTS = \left.\frac{\partial x_2}{\partial x_1}\right|_q = -\frac{\partial q / \partial x_1}{\partial q / \partial x_2}$$

RTS: An Example

- Cobb-Douglas Technology $q = \alpha x_1^{\ a} x_2^{\ b}$

$$\frac{\partial q}{\partial x_1} = \alpha a x_1^{\ a-1} x_2^{\ b}$$

$$\frac{\partial q}{\partial x_2} = \alpha b x_1^{\ a} x_2^{\ b-1}$$

$$RTS = \left.\frac{\partial x_2}{\partial x_1}\right|_q = -\frac{a}{b}\frac{x_2}{x_1}$$

Elasticity of Substitution

- The elasticity of substitution measures the percentage change in factor proportion due to 1 % change in marginal rate of technical substitution

$$s = \frac{\partial(x_2 / x_1)}{\partial[RTS]} \frac{RTS}{(x_2 / x_1)}$$

$$s = \frac{\partial \ln(x_2 / x_1)}{\partial \ln(RTS)}$$

- For Cobb-Douglas:

$$\frac{\partial(x_2 / x_1)}{\partial[RTS]} = 1 \Big/ \frac{\partial[RTS]}{\partial(x_2 / x_1)}$$

$$= 1 \Big/ \frac{-a}{b} = -\frac{b}{a}$$

$$\longrightarrow \quad s = (-b/a) \frac{(-a/b)/(x_2 / x_1)}{(x_2 / x_1)} = 1$$

Other Production Functions

- Constant Elasticity of Substitution (CES):

$$q = (ax_1^{\ t} + bx_2^{\ t})^{s/t}$$

  - Elasticity of Substitution = *1/(1-t)*

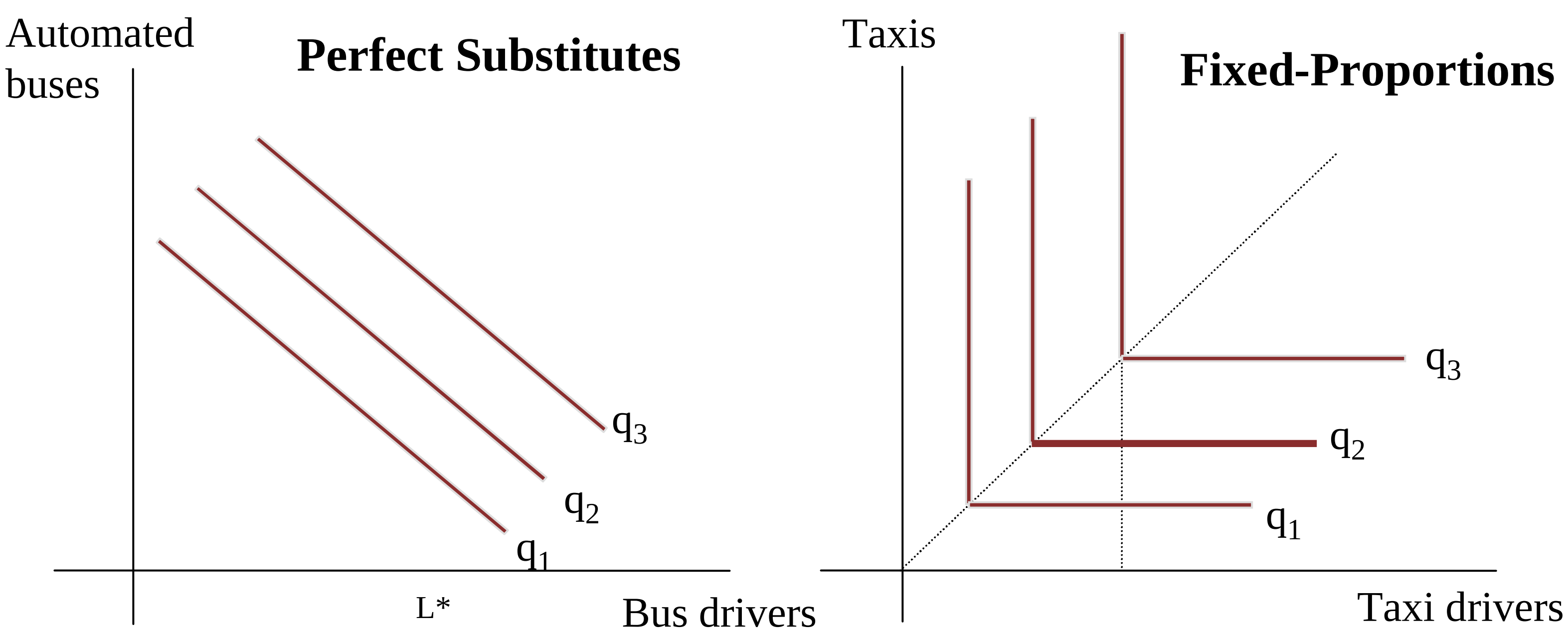
- Translog
  - State of the Art
  - Variable Elasticities, Interaction Terms
  - More in Next Lecture

Transit Production

- Inputs: capital, labor, fuel, maintenance
- Rejected the Cobb-Douglas form (Viton 1981, Berechman 1993, and others)
  - Implying significant interactions among inputs
- Low substitution rates among inputs
  - In particular capital and labor (one-vehicle-one-driver operations)
  - Suggests fixed proportions type of technology (Input-Output)

Perfect Substitutes and Fixed Proportions

- Does the technology allow substitution among inputs or not?



Joint Production Function

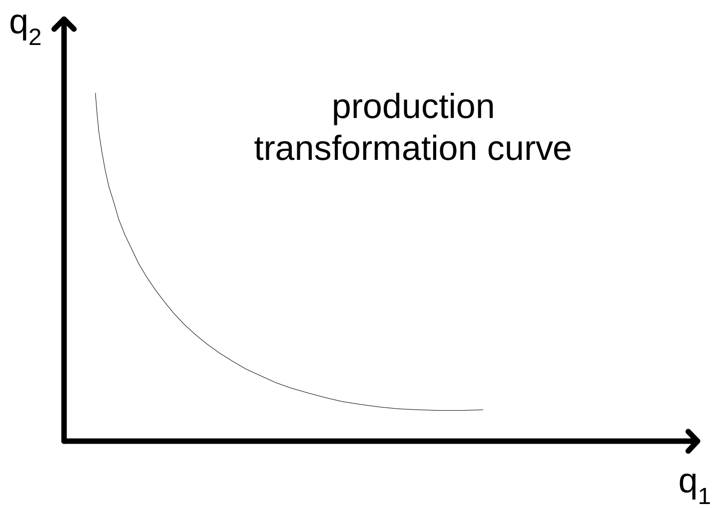
- Describes the production of several outputs
- Railroad companies:
  - Klein 1974: $(q_1)^r(q_2)^s = \alpha K^a L^b F^c$
  - Hasenkamp 1976:

$$\left[b_1(q_1)^r + b_2(q_2)^r\right]^{s/r} = \left(a_1K^t + a_2L^t + a_3F^t\right)^{u/t}$$

  - q_1, q_2 - passenger-miles, freight ton-miles
  - K, L, F - capital input, labor, fuel respectively
  - Constant elasticity of marginal rate of substitution among inputs
  - Constant elasticity of transformation among outputs

Production Transformation Curve

- Convex shape: economies of specialization
- Firm can produce a relatively large amount of one (passenger or freight) service or a limited amount of both.
- Conforms with industry trends



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- **Profit maximization and cost minimization**
- Average and marginal costs

Profit Maximization

- Joint choice of input and production levels
  - For a single product:

$$\left.\begin{aligned} &\max_{x} \pi = pq - WX \\ &s.t. \quad q = q(X) \end{aligned}\right\} \Rightarrow \max_{x} \; pq(X) - WX$$

  - *W* – input prices
  - *p* – output price
- Assume *W* and *p* are fixed

The Competitive Firm

- Price taker does not influence input and output prices
- Applies when:
  - Large number of selling firms
  - Identical products
  - Well informed customers

Optimal Production

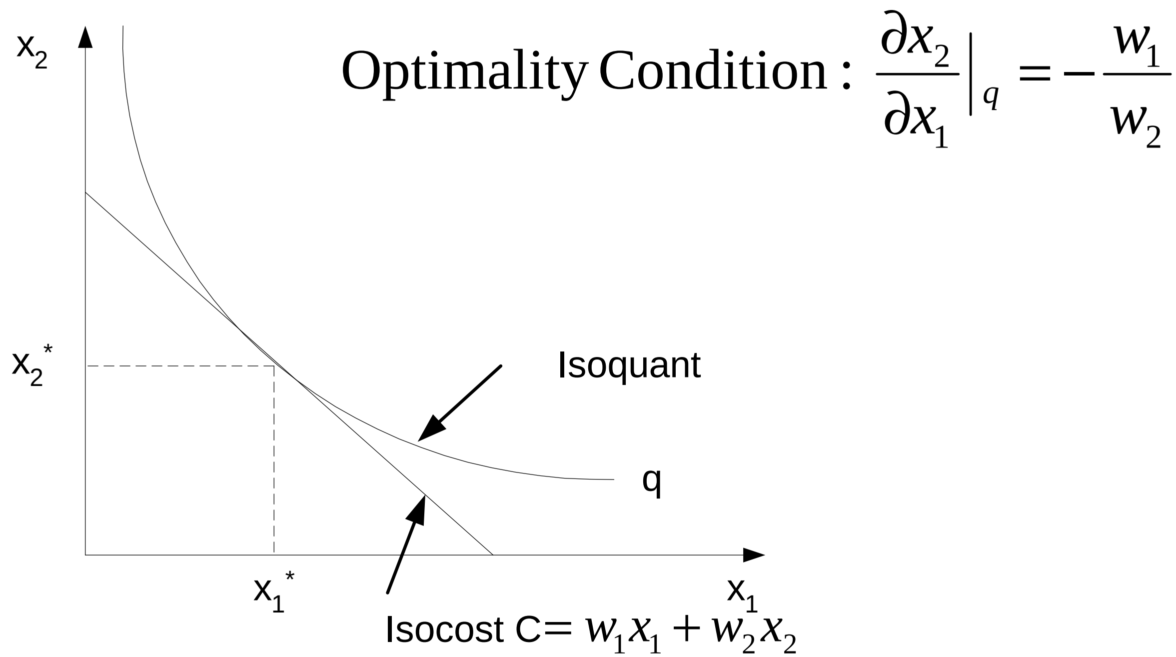
- At the optimum:

$$\underbrace{p\frac{\partial q(X^*)}{\partial x_i}}_{\substack{\text{marginal revenue}\\\text{from input unit}}} = \underbrace{w_i}_{\text{marginal cost}} \qquad \forall i$$

- Marginal revenue = Marginal cost
  - If marginal cost is lower, the firm would profit from using 1 extra unit of input *i*.
  - If marginal cost is higher, the firm would profit from using 1 less unit of input *i*.

Cost Minimization

- Given input prices and a required level of production, the firm chooses amounts of inputs that will minimize its cost



Mathematical Formulation for Cost Minimization

$$\min_{X} C = WX$$

$$s.t. \;\; q(X) = q$$

- Solution: $X^* = X(W, q)$ defined as the conditional factor demand function
- Substitute in and obtain the cost function:

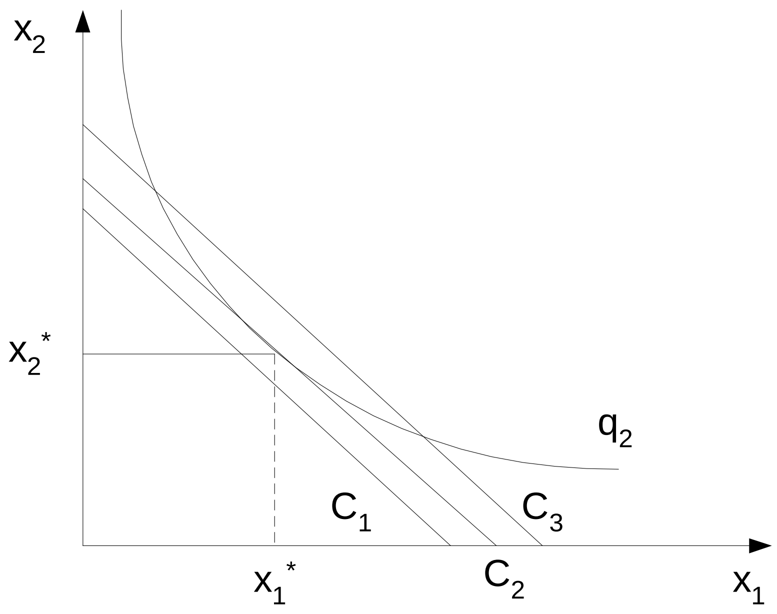
  $C = WX^* = C(W, q)$

Dual Views of the Same Problem

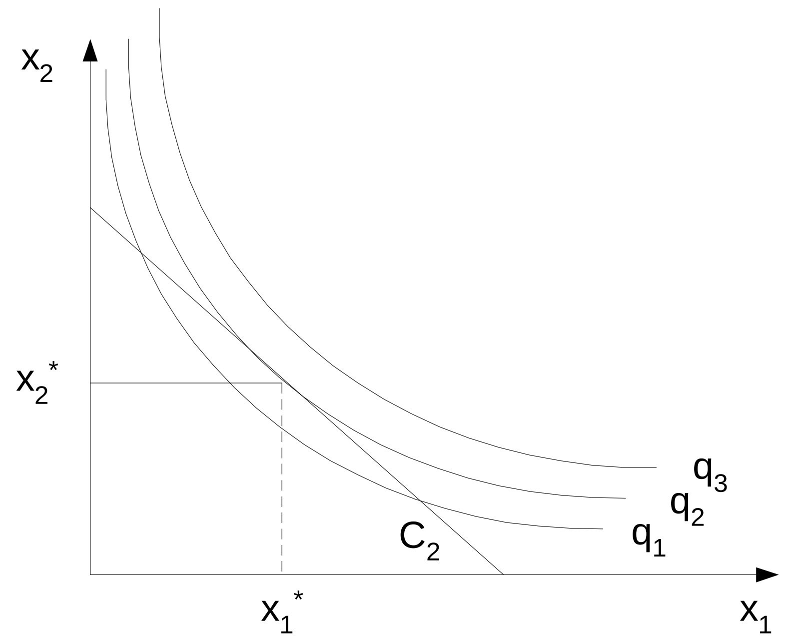
- Production problem: maximize production given level of cost
- Cost problem: minimize cost given a desired level of production.

Cost and Production Duality

Minimizing cost for a given level of production



Maximizing production level for a given level of cost



Deriving Cost Functions from Production Functions: Example

- Production function: $q = \alpha x_1^{\;a} x_2^{\;b}$
- Cost minimization problem:

$$\left.\begin{aligned} &C(w,q) = \min\; w_1 x_1 + w_2 x_2 \\ &s.t.\quad \alpha x_1^{\;a} x_2^{\;b} = q \end{aligned}\right\} \Rightarrow \min\; w_1 x_1 + w_2 \alpha^{-1/b} q^{1/b} x_1^{\;-a/b}$$

- First order conditions:

$$w_1 - \frac{a}{b} w_2 \alpha^{-1/b} q^{1/b} x_1^{\;-(a+b)/b} = 0$$

Example (cont)

- Conditional demand function for input 1:

$$x_1(w_1, w_2, q) = \alpha^{-1/a+b} \left[\frac{aw_2}{bw_1} \right]^{b/a+b} q^{1/a+b}$$

- Conditional demand function for input 2:

$$x_2(w_1, w_2, q) = \alpha^{-1/a+b} \left[\frac{aw_2}{bw_1} \right]^{-a/a+b} q^{1/a+b}$$

- Cost function:

$$C(w_1, w_2, q) = w_1 x_1(w_1, w_2, q) + w_2 x_2(w_1, w_2, q)$$

$$= \alpha^{-1/a+b} \left[\left(\frac{a}{b} \right)^{b/a+b} + \left(\frac{a}{b} \right)^{-a/a+b} \right] w_1^{a/a+b} w_2^{b/a+b} q^{1/a+b}$$

Example: TL vs. LTL Carriers

Truckload (TL) Carriers

		Labor	Capital	Fuel	Purchased Transportation
Own-Price Elasticity		-0.566	-0.683	-0.582	-1.920
Elasticity of Substitution					
Labor			0.590	0.177	2.300
Capital		0.590		0.514	2.190
Fuel		0.177	0.514		2.780
Purchased Transportation		2.300	2.190	2.780	

Less-Than-Truckload (LTL) Carriers

		Labor	Capital	Fuel	Purchased Transportation
Own-Price Elasticity		-0.372	-0.762	-0.724	-0.973
Elasticity of Substitution					
Labor			0.968	0.766	0.947
Capital		0.968		0.762	1.440
Fuel		0.766	0.762		0.856
Purchased Transportation		0.947	1.440	0.856	

1988 and 1990 Case Studies in:

McCarthy, P. Transportation Economics: Theory and Practice: A Case Study Approach. Blackwell Publishers, 2001

- LTL has more reliance on purchased transport
- LTL has greater substitution between factors (eg. Replace warehouse workers with logistics systems)

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- Basic Concepts
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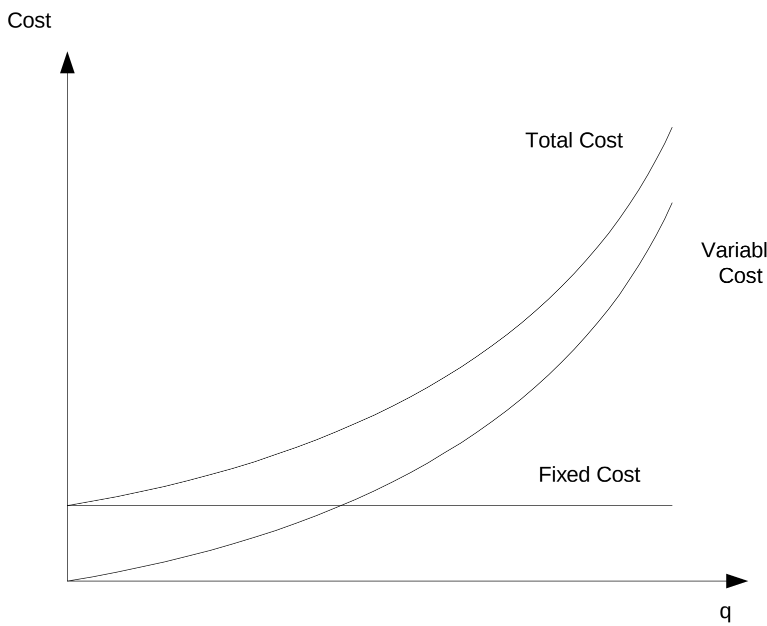
- **Average and marginal costs**

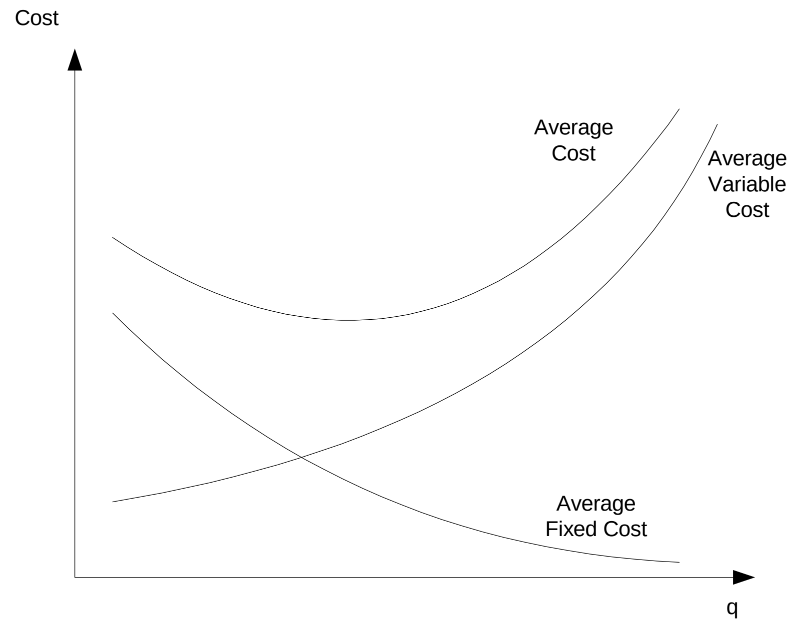
Average and Marginal Costs

- Total cost: $C(q) = WX(W, q)$
- Average cost: $AC(q) = \dfrac{C(q)}{q}$
- Marginal cost: $MC(q) = \dfrac{\partial C(q)}{\partial q}$

Geometry of Cost Functions



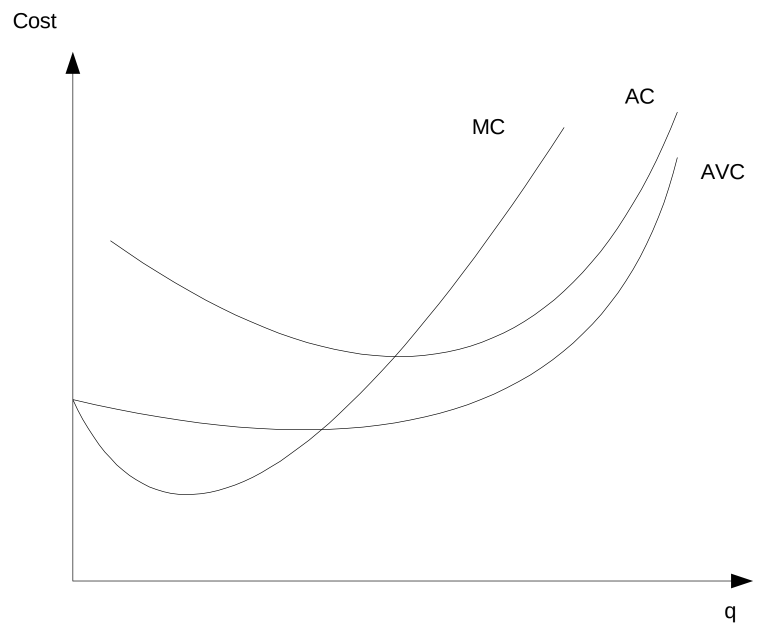
Total Cost



Average Cost

Geometry of Cost Functions

- AC = MC at min AC point
- AVC = MC at min AVC point



Examples of Marginal Costs

- One additional passenger on a plane with empty seats
  – One extra meal
  – Extra terminal possessing time
  – Potential delays to other passengers
- 100 additional passengers/day to an air shuttle service
  – The costs above
  – Extra flights
  – Additional ground personnel

Using Average and Marginal Costs

- Profitability/Subsidy Requirements
  - Compare average cost and average revenue
- Profitability of a particular trip
  - Compare marginal cost and marginal revenue
- Economic efficiency
  - Price = MC
- Regulation
  - Declining average cost

Summary

- Basic Concepts
- Production functions
  - Isoquants
  - Rate of technical substitution
- Profit maximization and cost minimization
  - Dual views of the same problem
- Average and marginal costs

Next lecture… Transportation costs

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