Introduction to Transportation Demand Analysis and Overview of Consumer Theory

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1.201 / 11.545 / ESD.210
Transportation Systems Analysis: Demand & Economics

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Outline

I. Introduction to Transportation Demand Analysis
- Choices
- Complexity
- Sample statistics
- Roles of demand models

II. Overview of Consumer Theory

Choices Impacting Transport Demand

- Decisions made by Organizations
  - Firm locates in Boston or Waltham
  - Firm invests in home offices, high speed connections
  - Developer builds in downtown or suburbs
- Decisions made by Individual/Households
  - Live in mixed use area in Boston or in residential suburb
  - Do not work or work (and where to work)
  - Own a car or a bike
  - Own an in-vehicle navigation system
  - Work Monday-Friday 9-5 or work evenings and weekends
  - Daily activity and travel choices:
    what, where, when, for how long, in what order, by which mode and route, using what telecommunications

Complexity of Transport Demand

- Valued as input to other activities (derived demand)
- Encompasses many interrelated decisions
  - Very long-term to very short-term
- Large number of distinct services differentiated by location and time
- Demographics & socioeconomic matter
- Sensitivity to service quality
- Supply and demand interact via congestion

Complexity and *Variety* → wide assortment of models to analyze transportation users' behavior.

Mode Share Statistics

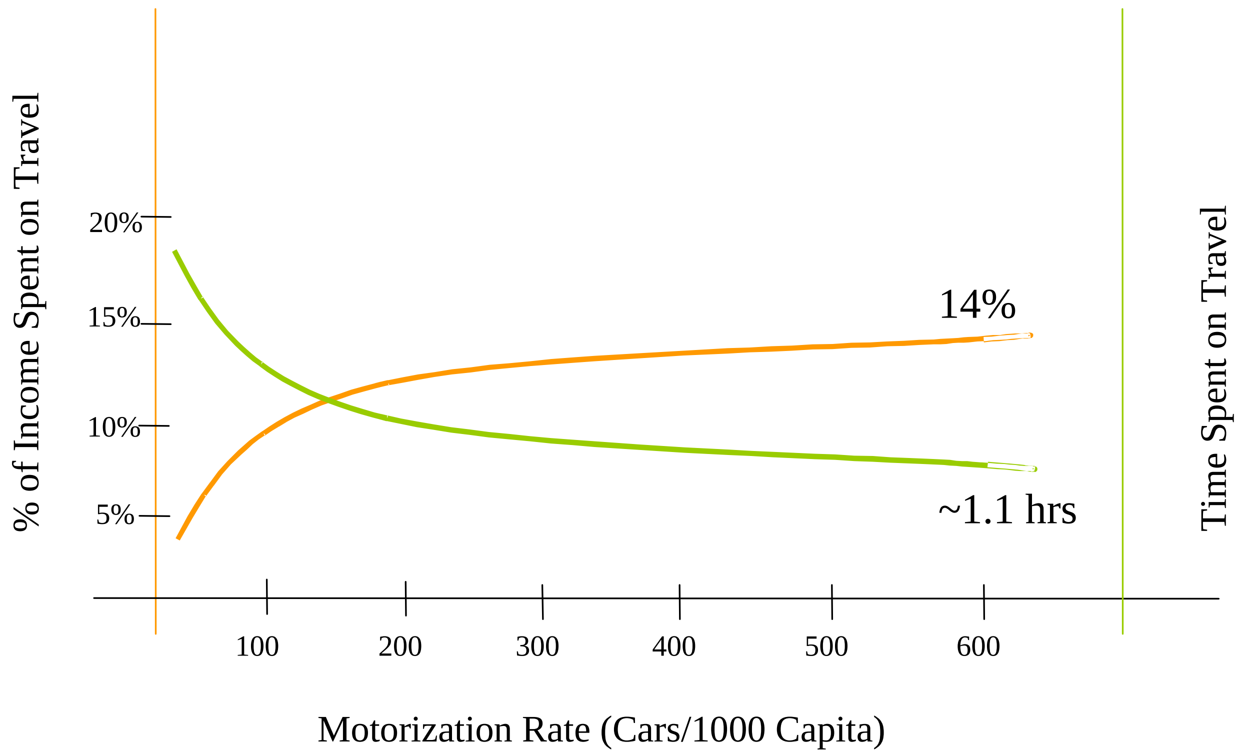
Transit Shares for Work Trips in Selected U.S. Cities

City	Year	Transit Mode Share (%)
Boston, MA	1990	10.64
	2000	9.03
Chicago, IL	1990	13.66
	2000	11.49
New York, NY	1990	26.57
	2000	24.90
Houston, TX	1990	3.78
	2000	3.28
Phoenix, AZ	1990	2.13
	2000	2.02

Source: US Census, 1990, 2000

Travel Expenditures

- The average generalized cost (money and time) per person in developed countries is very stable.



Source: Schäfer A., 1998, "The Global Demand for Motorized Mobility", Transportation Research A, 32(6): 455-477.

Transport Demand Elasticities

- Elasticity: % change in demand resulting from 1% change in an attribute
- Derived from demand models:

Work Trips (San Francisco)	Auto	Bus	Rail
Price	-0.47	-0.58	-0.86
In-vehicle time	-0.22	-0.60	-0.60

Vacation Trips (U.S.)	Auto	Bus	Rail	Air
Price	-0.45	-0.69	-1.20	-0.38
Travel time	-0.39	-2.11	-1.58	-0.43

Source: Schäfer A., 1998, "The Global Demand for Motorized Mobility", Transportation Research A, 32(6): 455-477.

Value of Time

- The monetary value of a unit of time for a user.

Work Trips (San Francisco)	Auto	Bus	
In-vehicle time	140	76	*Percentage of after tax wage*
Walk access time		273	
Transfer wait time		195	

Vacation Trips (U.S.)	Auto	Bus	Rail	Air	
Total travel time	6	79-87	54-69	149	*Percentage of pretax wage*

Freight	Rail	Truck	
Total transit time	6-21	8-18	*Percentage of shipment value per day*

Source: Jose Gómez-Ibañez, William B. Tye, and Clifford Winston, editors, Essays in Transportation Economics and Policy, page 42. Brookings Institution Press, Washington D.C., 1999.

Role of Demand Models

- Forecasts, parameter estimates, elasticities, values of time, and consumer surplus measures obtained from demand models are used to improve understanding of the ramifications of alternative investment and policy decisions.
- Many uncertainties affect transport demand and the models are about to do the impossible

Role of Demand Models: Examples

- From Previous Lecture:
  - High Speed Rail: Works in Japan/Europe, how about in US?
  - Traffic Jams: Build more, manage better, or encourage transit use?
  - Truck Traffic: evaluate tradeoffs of environmental protection.

Outline

- Basic concepts
  - Preferences
  - Utility
  - Choice
- Additional details important to transportation
- Relaxation of assumptions
- Appendix: Dual concepts in demand analysis

Preferences

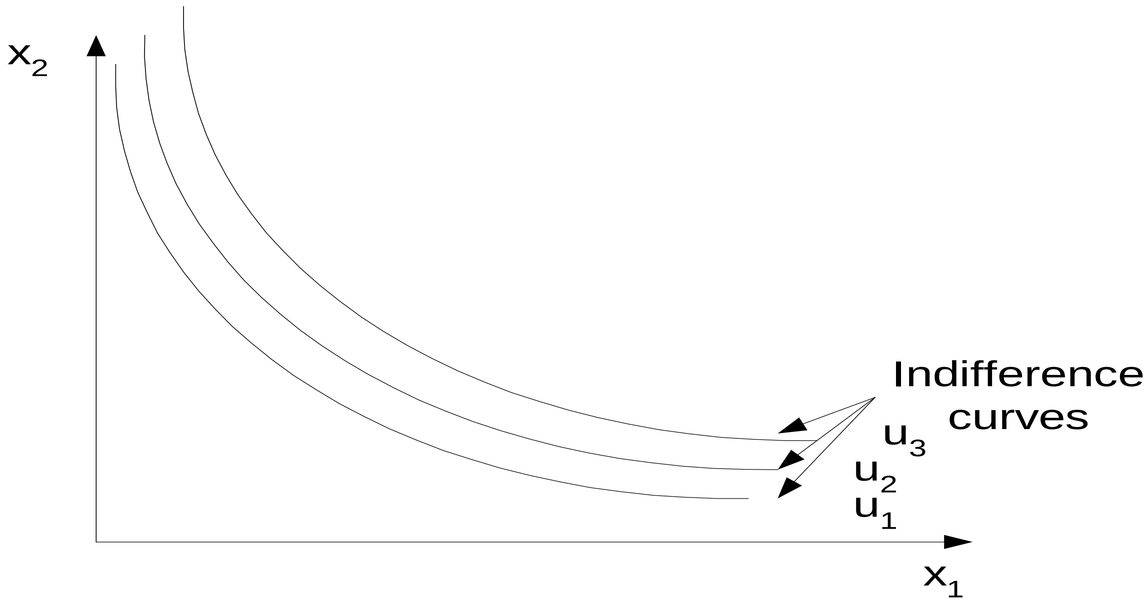
- The consumer is faced with a set of possible consumption bundles
  - Consumption bundle: a vector of quantities of different products and services $X = \{x_1,\ldots,x_i,\ldots,x_m\}$
  - A bundle is an array of consumption amounts of different goods
- Preferences: ordering of the bundles
  - $X \succ Y$: Bundle *X* is preferred to *Y*
  - Behavior: choose the most preferred consumption bundle
  - Transitivity, Completeness, and Continuity

The Utility Function

- A function that represents the consumer's preferences ordering $X \succ Y \Leftrightarrow U(X) > U(Y)$
  - Utility function is not unique
    - $U(x_1, x_2) = ax_1 + bx_2$
    - $U(x_1, x_2) = x_1^a x_2^b$
  - Unaffected by order-preserving transformation
    - $10 \times U(x_1, x_2) + 10$?
    - $\exp(U(x_1, x_2))$?
    - $(U(x_1, x_2))^2$?

Indifference Curves

- Constant utility curve
- The consumer is indifferent among different bundles on the same curve



Marginal Utility and Trade-offs

- Marginal utility

$$MU_i = \partial U(X) \big/ \partial x_i$$

$$U(x_1, x_2) = 4x_1 + 2x_2$$

$$MU_1 = 4$$

$$U(x_1, x_2) = x_1^{0.5} x_2^{0.5}$$

$$MU_1 = 0.5 x_1^{-0.5} x_2^{0.5}$$

- Marginal rate of substitution (MRS)

$$MRS = \frac{\partial U(X) \big/ \partial x_1}{\partial U(X) \big/ \partial x_2} = \frac{MU_1}{MU_2}$$

$$MRS = 4 \big/ 2$$

$$MRS = x_2 \big/ x_1$$

Consumer Behavior

- Utility (preference) maximization
- Bounded by the available income

$$\max U(X)$$

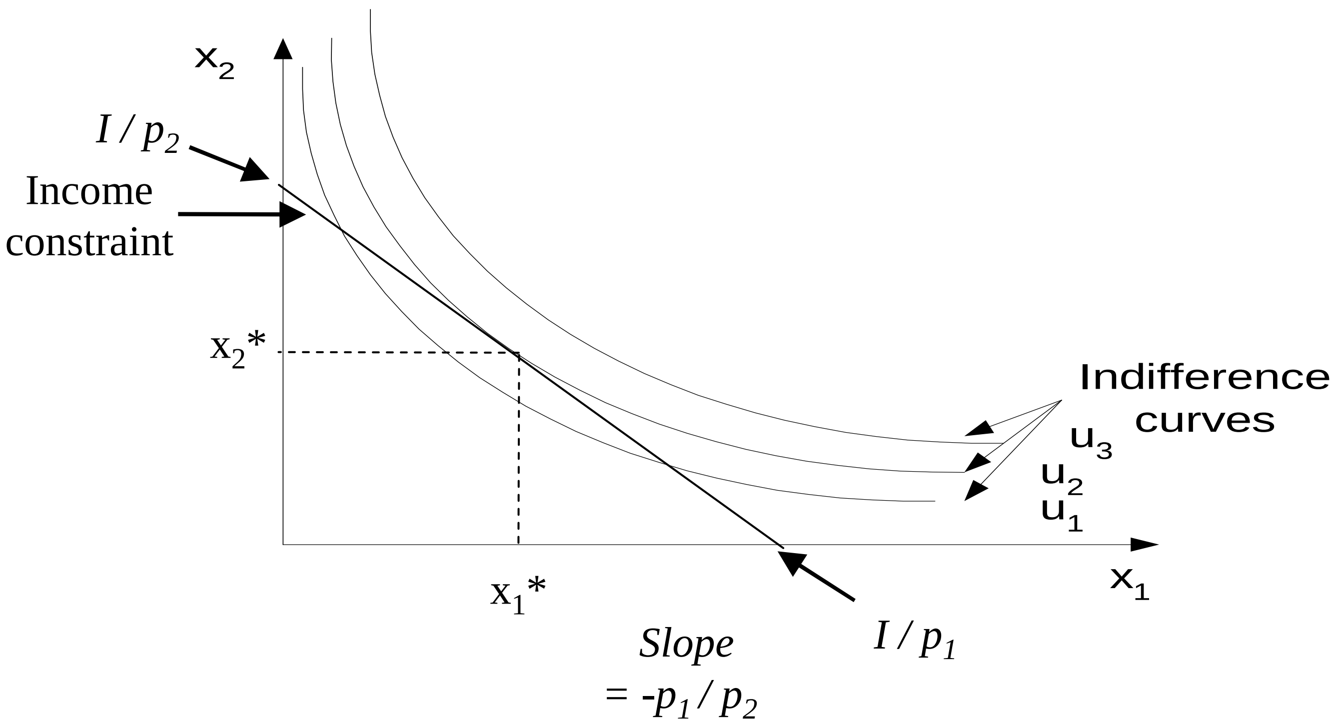
$$s.t. \quad PX \le I \qquad (\sum_{i=1}^{m} p_i x_i \le I)$$

$$X \quad \text{feasible} \quad (\text{e.g. non} - \text{negativity})$$

  - P - vector of prices $P = \{p_1, \ldots, p_i, \ldots, p_m\}$
  - I – income
- When considering two goods, the constraint would be:

$$p_1x_1 + p_2x_2 \le I$$

Geometry of the Consumer's Problem

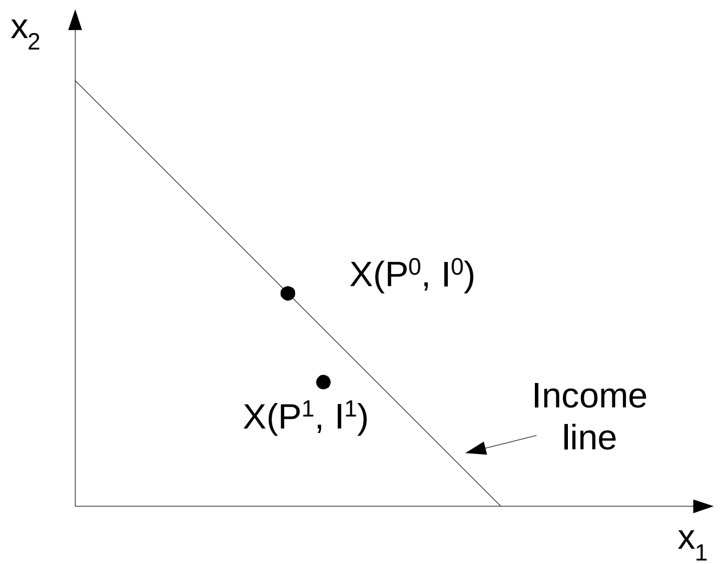


Revealed Preferences

- A chosen bundle is revealed preferred to all other feasible bundles:

  $X(P^0, I^0)$ - the demanded bundle at Point 0

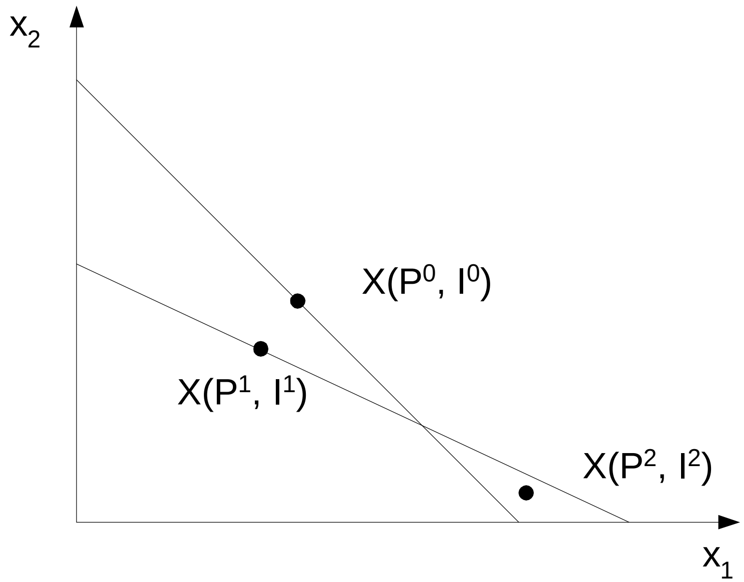
$$X(P^0, I^0) \succ X(P^1, I^1) \quad if \quad P^0 X(P^0, I^0) \geq P^0 \; X(P^1, I^1)$$



Indirect Revealed Preferences

- Transitivity of preferences:
  - $X(P^0, I^0)$ is indirectly revealed preferred to $X(P^2, I^2)$ if:

$X(P^0, I^0) \succ X(P^1, I^1)$ and $X(P^1, I^1) \succ X(P^2, I^2)$



Optimal Consumption

- The consumer's problem

$$\max U(X)$$
$$s.t. \quad PX \leq I$$

- Assuming U(X) increases with X

$$PX = I$$

- The Lagrangean

$$L(X,\lambda) = U(X) + \lambda(I - PX)$$

λ: Lagrange multiplier of budget constraint

Optimal Consumption

- Optimality conditions:

$$\frac{\partial L}{\partial x_i} = \frac{\partial U(X^*)}{\partial x_i} - \lambda p_i = 0 \qquad \forall i = 1,\ldots,m$$

$$\Rightarrow \quad \frac{\partial U\left(X^*\right)}{\partial x_i} = \lambda p_i \qquad \forall i$$

- Dividing conditions: $\underbrace{\frac{\partial U\left(X^*\right)\big/\partial x_i}{\partial U\left(X^*\right)\big/\partial x_j}}_{MRS} = \frac{p_i}{p_j} \qquad \forall i \neq j$

Example: Cobb-Douglas Utility

- The consumer's problem:

$$\max U(X) = x_1^{a} x_2^{b}$$

$$s.t. \quad p_1 x_1 + p_2 x_2 \le I$$

- The Lagrangean: $L(X,\lambda) = x_1^{a} x_2^{b} + \lambda(I - p_1 x_1 - p_2 x_2)$
- Optimal solution:

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = a x_1^{*a-1} x_2^{*b} - \lambda p_1 = 0 \\ \dfrac{\partial L}{\partial x_2} = b x_1^{*a} x_2^{*b-1} - \lambda p_2 = 0 \\ \dfrac{\partial L}{\partial \lambda} = I - p_1 x_1^{*} - p_2 x_2^{*} = 0 \end{cases}$$

$$\frac{x_1^{*}}{x_2^{*}} = \frac{a}{b}\frac{p_2}{p_1}$$

$$x_1^{*} = \frac{a}{a+b}\frac{I}{p_1}$$

$$x_2^{*} = \frac{b}{a+b}\frac{I}{p_2}$$

Review

Basic concepts

- Preferences
- Utility functions $U(X)$
- Optimal consumption $max\ U(X)\ s.t.\ PX \leq I$
- Demand functions $X^* = X(P,I)$

Next…

- Indirect utility
- Complements and Substitutes
- Elasticity
- Consumer surplus

Indirect Utility Function

- Recall

  X^* - the demanded bundle

  $X(P, I)$ - the consumer's demand function

- Indirect utility function
  - The maximum utility achievable at given prices and budget:

$$V(P,I) = \max \; U(X)$$
$$s.t. \;\; PX = I$$

  - Substitute the solution $X(P, I)$ back into the utility function to obtain:

maximum utility = $V(P, I) = U(X(P, I))$

Example: Cobb-Douglas Utility

- Recall our earlier problem: $$\max U(X) = x_1^{a} x_2^{b}$$ $$s.t. \quad p_1 x_1 + p_2 x_2 \le I$$

- We found: $$x_1^* = \frac{a}{a+b}\frac{I}{p_1} \qquad x_2^* = \frac{b}{a+b}\frac{I}{p_2}$$

- So V(p,I) = … = $$\frac{I^{a+b}}{p_1^{a} p_2^{b}}\left[\left(\frac{a}{a+b}\right)^{a}\left(\frac{b}{a+b}\right)^{b}\right]$$

- What is $\dfrac{\partial V(p,I)}{\partial I}$?

Complements and Substitutes

- Gross substitutes

$$\frac{\partial x_j(P,I)}{\partial p_i} > 0$$

- Gross complements

$$\frac{\partial x_j(P,I)}{\partial p_i} < 0$$

Demand Elasticity

- Percent change in demand resulting from 1% change in an attribute.

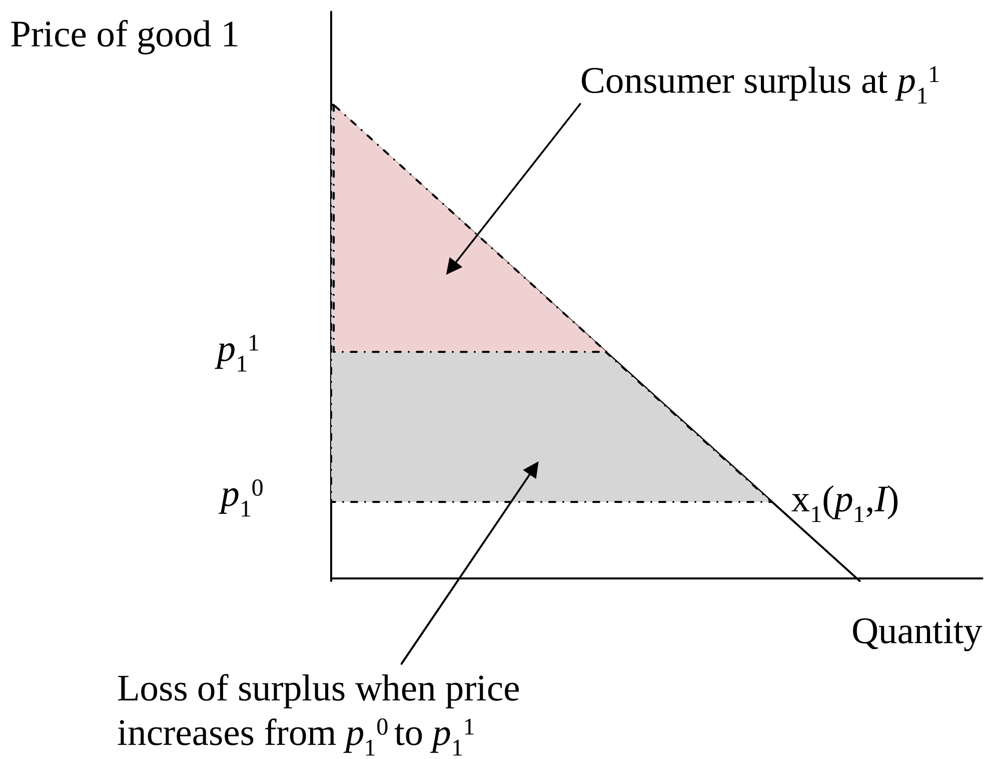
e.g. price-elasticity $$\frac{\%\ \text{change in}\ X^*}{\%\ \text{change in}\ P} = \frac{\Delta X^* / X^*}{\Delta P / P}$$

– Own elasticity $$\varepsilon_{p_i}^{x_i} = \frac{p_i}{x_i(P,I)} \frac{\partial x_i(P,I)}{\partial p_i}$$

– Cross elasticity $$\varepsilon_{p_i}^{x_j} = \frac{p_i}{x_j(P,I)} \frac{\partial x_j(P,I)}{\partial p_i}$$

Consumer Surplus (or Welfare)

- Consumer surplus
  - difference between the total value consumers receive from the consumption of a good and the amount paid



Consumer Surplus

- Consumer surplus is key for evaluating public policy decisions:
  - Building transportation infrastructure
  - Changing regulations (e.g., emissions)
  - Determining fare and service structures

Review

- Basic concepts
  - Preference, Utility, Rationality
- Other useful details
  - Indirect utility
  - Complements and Substitutes
  - Elasticity
  - Consumer Surplus

Next… Discussion of assumptions

Assumptions

- Impact of consumption of one good on utility of another good?
- Income is the only constraint
- Utility is a function of quantities
  (a good is a good)
- Demand curves are for an individual
- Behavior is deterministic
- Goods are infinitely divisible (continuous)

Separable Utility

- The consumption of one good does not affect the utility received from some other good

$$U(X) = \sum_{i=1}^{m} U_i(X_i)$$

- Separability into groups of goods

$$U(X) = U\left(U_1(X_{g1}),\ldots,U_k(X_{gk})\right)$$

X_{gk} - group *k* of goods

- Allows allocating budget in 2 stages:
  - Between groups
  - Within a group

Attributes of Goods

Classic: Only quantities matter.

Quantities → Attributes

- The consumer receives utility from attributes of goods rather then the goods themselves
- Example: (dis)utility of an auto trip depends on travel time, cost, comfort etc.

$$U = U(A)$$
$$A = A(X)$$

  - X – Goods
  - A – Attributes
  - U – Utility

"Household Production" Model

- Consumers purchase goods in order to "produce" utility from them, depending on their attributes
- Classic example: Households purchase food in order to obtain calories and vitamins (could also include enjoyment of taste), which then produce "utility"

Utility of a Transportation Mode

- Consumption bundles: auto, bus, train, etc.
- Utility function

  $$U_{bus} = \beta_0 + \beta_1 WT_{bus} + \beta_2 TT_{bus} + \beta_3 C_{bus}$$

  – WT_{bus} –waiting time (minutes)
  – TT_{bus} – total travel time (minutes)
  – C_{bus} – total cost of trip (dollars)
- Parameters β represent tastes, and vary by education, gender, trip purpose, etc.

Time Budgets and Value of Time

- Along with *income constraint*, there is also a *time constraint* (e.g., 24 hours in a day)

  → Gives time *value*.

- Value of time is the marginal rate of substitution between time and cost

$$U_{bus} = \beta_0 + \beta_1 WT_{bus} + \beta_2 TT_{bus} + \beta_3 C_{bus}$$

$$VOT = \frac{MU_{TT}}{MU_C} = \frac{\beta_2}{\beta_3} \ \$/\text{min}$$

Aggregate Consumer Demand

- Aggregate demand is the sum of demands of all consumers

$$X(P, I_1, \ldots, I_N) = \sum_{n=1}^{N} X_n(P, I_n)$$

  - *n* – An individual consumer
  - *N* - # of consumers
- Is this viable? Are individuals sufficiently similar to simply "add up"?

Heterogeneity

- Mobility rates, São Paulo, 1987:

Family income	Share in population	Mobility rate (all trips)	Mobility rate (motorized trips)
<240	20.8%	1.51	0.59
240-480	28.1%	1.85	0.87
480-900	26.0%	2.22	1.24
900-1800	17.2%	2.53	1.65
>1800	7.9%	3.02	2.28

Source: Vasconcellos, 1997, "The demand for cars in developing countries", Transportation Research A, 31(A): 245-258.

Introducing Uncertainty

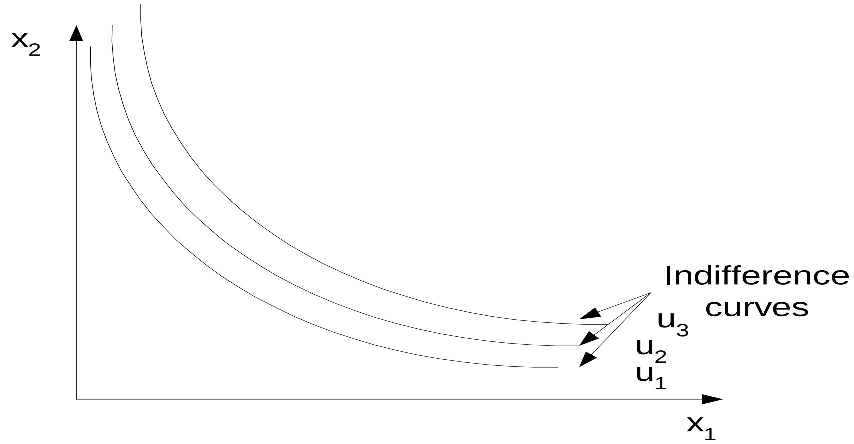
- Random utility model

  U_i = V(attributes of i; parameters) + $epsilon_i$

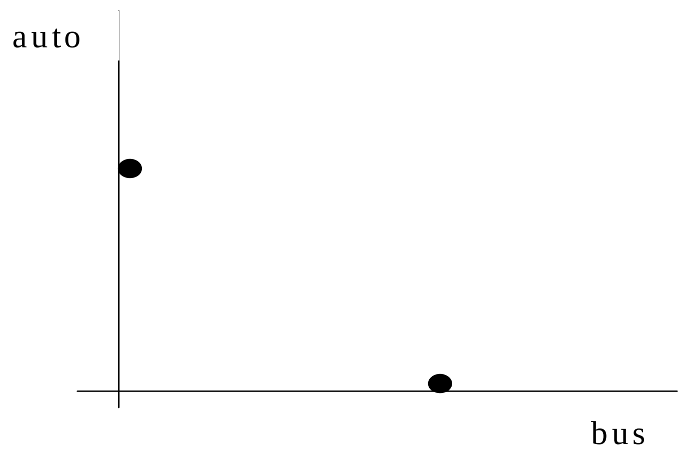
- Decision maker deterministic, but analyst imperfect due to:
  - Unobserved attributes
  - Unobserved taste variations
  - Measurement errors
  - Use of proxy variables

Continuous vs. Discrete Goods

- Continuous goods



- Discrete goods



Summary

- Basic concepts
  - Preference, Utility, Rationality
- Other useful details
  - Indirect utility
  - Complements and Substitutes
  - Elasticity, Consumer Surplus
- Relaxing the assumptions and working towards practical, empirical models

Next Lecture… Discrete Choice Analysis

Appendix

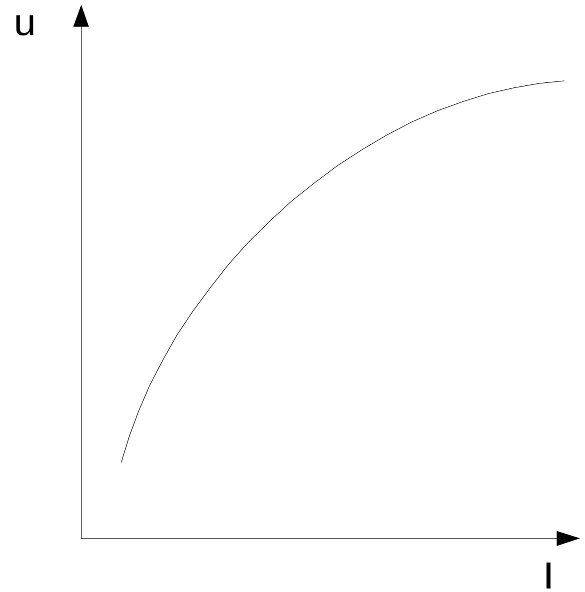
Dual concepts in demand analysis

Expenditure Function

- The minimal income needed to achieve any level of utility at given prices
- Solution to the problem:

$$E(P,u) = \min PX$$
$$s.t. \quad U(X) \geq u$$

- The dual to the utility maximization problem



Cobb-Douglas Utility

Min $p_1 x_1 + p_2 x_2$

Subject to $x_1^a x_2^b \geq u$

- Similar formula for the Lagrangean and solve…
- We find:

$$x_1^* = \left[\frac{p_2}{p_1} \cdot \frac{a}{b} \right]^{b/(a+b)} u^{1/(a+b)}$$

- So:

$$E(p,u) = p_1 x_1^* + p_2 x_2^*$$

Consumer Surplus

- Directly related to the expenditure function
- Let $E_1 = E(p_1^1, p_2, ..., p_m, u_0)$
  and $E_0 = E(p_1^0, p_2, ..., p_m, u_0)$

- Change in consumer surplus = $E_0 - E_1$

Compensated Demand Function

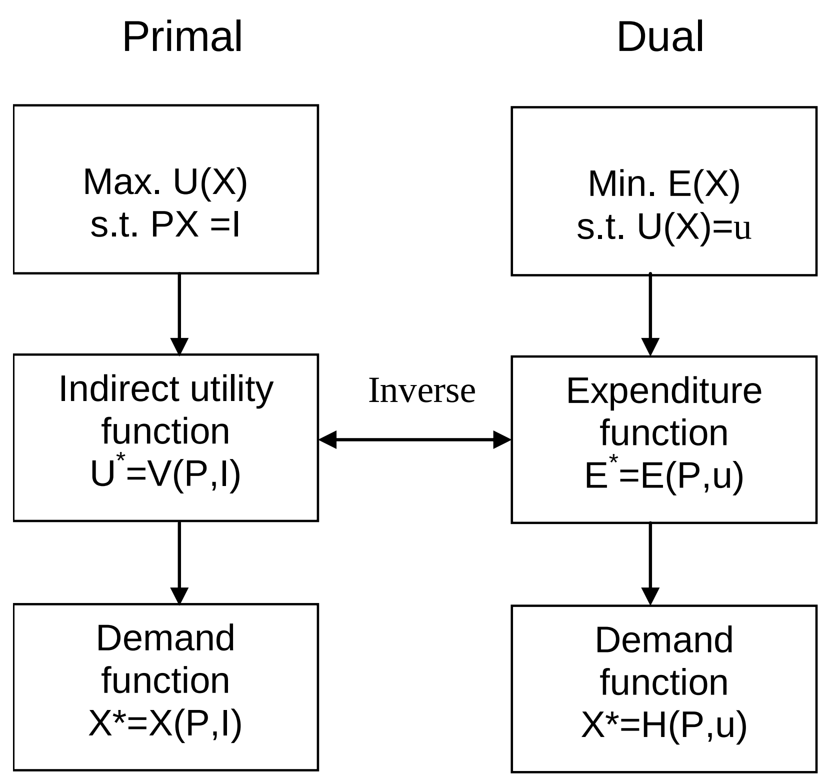
- The expenditure problem solution:

$$\left.\begin{aligned} E(P,u) &= \min PX \\ &s.t. \quad U(X) \geq u \end{aligned}\right\} \Rightarrow \quad X(P,u) = H(P,u)$$

- $H(P,u)$ is the compensated demand function
- Shows the demand for a good as a function of prices assuming utility is held constant.
- Substituting back into the objective function:

$$E(P,u) = \sum_i p_i x_i(P,u) = PX(P,u)$$

Relations Among Demand Concepts



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