Brief Notes #2
Random Variables: Discrete Distributions

- **<u>Discrete Distributions</u>**

  - Probability Mass Function (PMF)

    $$P_X(x) = P(X = x) = \sum_{\text{all } O:\ X(o)=x} P(O)$$

  - Properties of PMF's

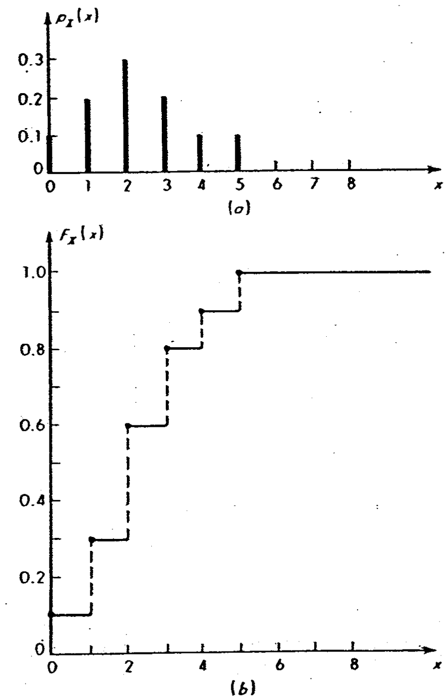
    1. $0 \le P_X(x) \le 1$
    2. $\sum_{\text{all } x} P_X(x) = 1$

  - Cumulative Distribution Function (CDF)

    $$F_X(x) = P(X \le x) = \sum_{u \le x} P_X(u)$$

  - Properties of CDF's

    1. $0 \le F_X(x) \le 1$
    2. $F_X(-\infty) = 0$
    3. $F_X(\infty) = 1$
    4. if $x_1 > x_2$, then $F_X(x_1) \ge F_X(x_2)$

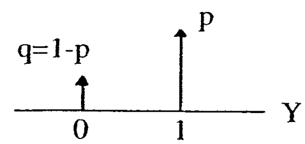


Discrete distributions
(a) Probability Mass Function PMF
(b) Cumulative Distribution Function CDF

• Examples of discrete probability distributions

• Bernoulli distribution

$$Y = \begin{cases} 1 \text{ if an event of interest occurs (success)} \\ 0 \text{ if the event does not occur (failure)} \end{cases}$$



Y is called a Bernoulli or indicator variable

$$P_Y(y) = \begin{cases} p, & y = 1 \\ q = 1 - p, & y = 0 \end{cases}$$

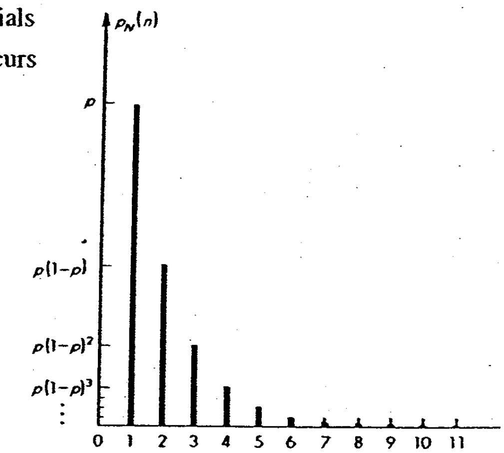
• Geometric distribution - sequence of Bernoulli trials

N = number of trials at which first success occurs

$N = 1, 2, 3, \ldots$

$$P_N(n) = P(N = n) = (1-p)^{n-1} p$$

$$F_N(n) = \sum_{i=1}^{n} P_N(i) = \sum_{i=1}^{n} (1-p)^{i-1} p = 1 - (1-p)^n$$



Geometric Distribution

• Binomial distribution

Consider a sequence of Bernoulli trials.
Let M = number of successes in n trials
$M = 1, 2, 3, \ldots, n$

$$P_M(m) = \frac{n!}{m!(n-m)!} p^m q^{n-m}, \text{ where } \frac{n!}{m!(n-m)!} = \binom{n}{m} = \text{binomial coefficient}$$

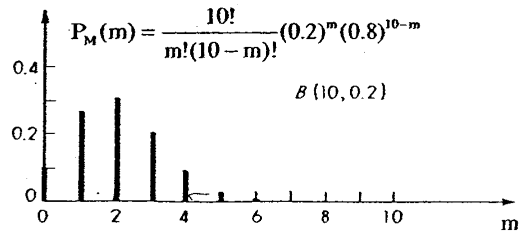
where p and q = 1 - p are the probabilities of success and failure in individual Bernoulli trials. In particular, the probability of no success is:

$P_M(0) = q^n = (1-p)^n$

$P_M(0) \approx 1 - pn$ if $pn << 1$

and the probability of all successes is

$P_M(n) = p^n$



Binomial distribution B(n,p)

- <u>Poisson distribution</u>

Assumptions:

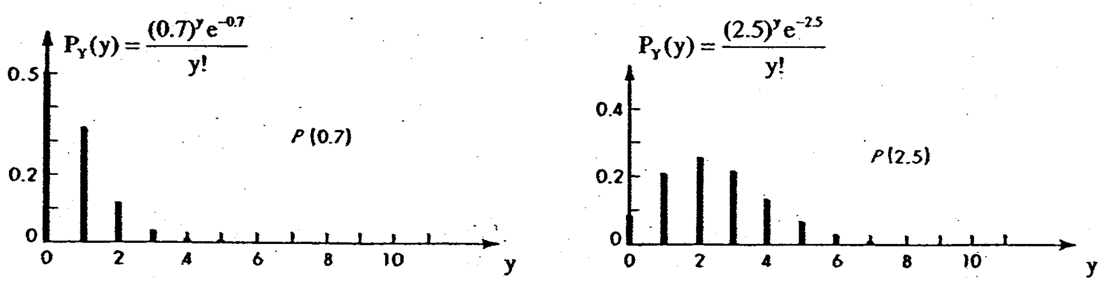
1. In a time interval of short duration Δ, the probability of one occurrence is $\lambda\Delta$, where λ = occurrence rate (expected number of occurrences per unit time).
2. The probability of two or more occurrences in Δ is negligible.
3. The occurrences in non-overlapping intervals are independent.

Under these conditions, the number of occurrences in each interval of duration Δ is either 0 or 1, with probability $p = \lambda\Delta$ of being 1. Let Y = no. of occurrences in [0, t], where $t = n\lambda$. Then Y has binomial distribution with probability mass function

$$P_Y(y) = \binom{n}{y} p^y q^{n-y}, \text{ where } p = \lambda\Delta = \lambda\frac{t}{n}$$

As $n \to \infty$,

$$P_Y(y) = \frac{(\lambda t)^y e^{-\lambda t}}{y!} \qquad \text{(Poisson PMF)}$$



Poisson distribution P(λt)