

**Application Example 9**  
**(Independent Random Variables)**

**RELIABILITY OF A BUILDING UNDER EXTREME WIND  
LOADS - CHOOSING THE DESIGN WIND SPEED**

Consider the problem of designing a tall building for a certain level of reliability against wind loads. The building has a planned life of  $N$  years.

Let  $V_i$  be the maximum wind speed in year  $i$  at the site of the building (for simplicity we ignore wind direction). Although the values of wind speed at closely spaced times are probabilistically dependent random variables, yearly maximum values may be considered independent. Moreover, the distribution of  $V_i$  is the same for all years  $i$ . Therefore, the variables  $V_1, \dots, V_N$  may be considered independent and identically distributed (iid), with some common cumulative distribution  $F_V(v)$ .

Empirical data indicate that, at most locations, the distribution  $F_V(v)$  is either Extreme Type I or Extreme Type II. The latter has the form

$$F_V(v) = e^{-(v/u)^{-k}} \quad (1)$$

where  $u$  and  $k$  are positive parameters. In Boston, plausible values of  $u$  and  $k$  are  $u = 49.4$  mph and  $k = 6.5$ . Therefore, for Boston

$$F_V(v) = e^{-(v/49.4)^{-6.5}} \quad (2)$$

where  $V$  is in mph.

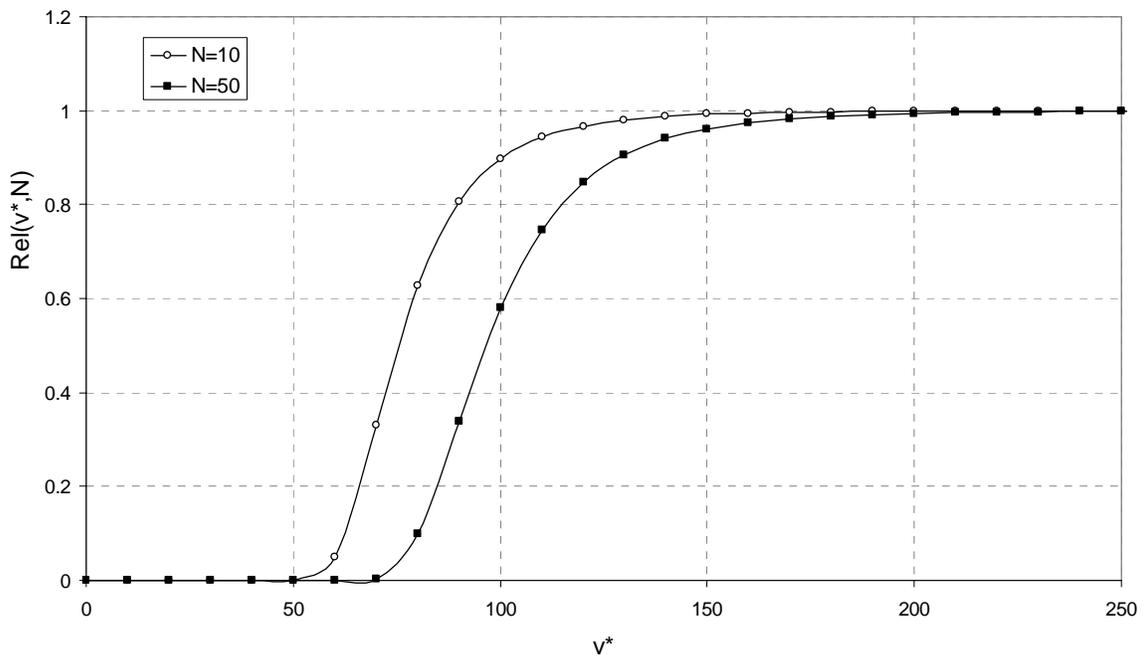
Consider now the problem of choosing the design wind speed  $v^*$  for a building in Boston, such that the probability of non-exceeding  $v^*$  during the design life of the building equals a target reliability value. The probability that any given  $v^*$  is not

exceeded in N years (the reliability of the building) is  $\text{Rel}(v^*, N) = P\left[\max_{i=1, N} V_i \leq v^*\right]$  and

may be calculated as a function of  $v^*$  and N as:

$$\begin{aligned} \text{Rel}(v^*, N) &= P[(V_1 \leq v^*) \cap (V_2 \leq v^*) \cap \dots \cap (V_N \leq v^*)] \\ &= \{F_V(v^*)\}^N \\ &= e^{-N(v^*/49.9)^{-6.5}} \end{aligned} \quad (3)$$

Plots of  $\text{Rel}(v^*, N)$  against  $v^*$  are shown in Figure 1 for N = 10 and N = 50.



**Figure 1: Reliability of a building in Boston against wind as a function of design wind speed  $v^*$ , for exposure periods of 10 and 50 years.**

Notice that:

1. For any given  $N$ , the reliability approaches 1 quite slowly, meaning that one must design for very high wind speeds in order to attain high safety levels. This is due to the fact that the distribution in Eq. 2 has a long upper tail. Had one chosen an Extreme Type I distribution for  $V$ , one would have found less extreme design demands;
2. Viewing the probability in Eq. 3 as the reliability of the building against wind loads is conservative, because exceeding  $v^*$  does not necessarily imply serious damage to the building. Typically, the actual strength of a building is much greater than the nominal strength, due to built-in conservatism in design codes, nominal material properties, and good construction practice. Therefore, the actual reliability may be much higher than the probability in Eq. 3;
3. Interestingly, if the yearly maximum winds  $V_i$  are iid with the Extreme Type II distribution in Eq. 1, then the maximum wind in  $N$  years,  $V_{\max,N} = \max_{i=1,N} V_i$  has itself an Extreme Type II distribution, with the same parameter  $k$  but different parameter  $u_N$ . In fact,

$$\begin{aligned}
 F_{V_{\max,N}}(v) &= e^{-N(v/u)^{-k}} \\
 &= e^{-(v/u_N)^{-k}}
 \end{aligned}
 \tag{4}$$

where  $u_N = uN^{1/k}$ .

***Problem 9.1***

*To better understand various aspects of the problem,*

(i) plot the cumulative distribution function in Eq. 1 and its derivative (the probability density function of  $V$ ) for different values of  $u$  and  $k$ , to see how these parameters affect the distribution of yearly wind speed (you might use the four combinations  $u = 40$  or  $60$  mph and  $k = 4$  or  $7$ );

(ii) make plots analogous to those in Figure 1 for the case when yearly maximum winds have Extreme Type I distribution, with the same mean value  $m$  and variance  $\sigma^2$  as implied by Eq. 2, i.e. with  $m = u\Gamma\left(1 - \frac{1}{k}\right) = 55.1$  mph and  $\sigma^2 = u^2\left[\Gamma\left(1 - \frac{2}{k}\right) - \Gamma^2\left(1 - \frac{1}{k}\right)\right] = 157.5$  (mph)<sup>2</sup> where  $\Gamma$  is the gamma function [on these expressions for the mean and variance of the Extreme Type II distribution, see for example Benjamin and Cornell (1970), p. 279]. The cumulative distribution of the Extreme Type I distribution is given by

$$F_V(v) = 1 - e^{-e^{\alpha(v-u')}} \quad (5)$$

where  $\alpha$  and  $u'$  are parameters. The values of  $\alpha$  and  $u'$  for the above mean value and variance are  $\alpha = \frac{\pi}{\sqrt{6}} \frac{1}{\sigma} = 0.102$  and  $u' = m - \frac{0.577}{\alpha} = 43.93$  mph. Comment on the results, in comparison with Figure 1.

### Problem 9.2

The following data on maximum yearly wind speed were recorded at Great Falls, Montana, 10 m above ground, during the period 1944-1977 (from Simiu and Scanlan, 1996).

year	max speed (mph)	year	max speed (mph)
1944	57	1961	60
1945	65	1962	66

1946	62	1963	55
1947	58	1964	51
1948	64	1965	60
1949	65	1966	55
1950	59	1967	60
1951	65	1968	51
1952	59	1969	51
1953	60	1970	62
1954	64	1971	51
1955	65	1972	54
1956	73	1973	52
1957	60	1974	59
1958	67	1975	56
1959	50	1976	52
1960	74	1977	49

- (i) Calculate the sample mean and sample variance of this dataset.
- (ii) Find the parameters of the Extreme Type I distribution for which the theoretical mean and variance match the sample mean and sample variance (use formulas given above as part of Problem 9.1).
- (iii) Produce reliability plots analogous to those in Figure 1.

Benjamin, J. R. and C. A. Cornell, *Probability, Statistics and Decision for Civil Engineers*, McGraw-Hill, 1970.

Simiu, E. and R. Scanlan, *Wind Effects on Structures*, 3<sup>rd</sup> ed., Wiley, 1996.