

Delta Design THE DESIGN TASK

Structural Engineer's Primer appended.

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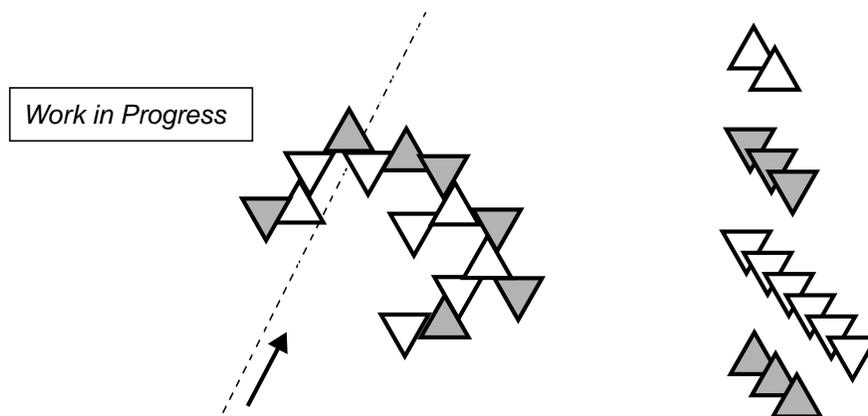
1.1 Introduction

Congratulations! You are now a member of an expert design team. Your collective task will be to design a new residence suitable for inhabitants of the imaginary Deltoid plane. These written materials, provided to help you prepare for this task, are organized in four sections.

The next section provides an overview of life on the Deltoid plane, DeltaP as it is known to the natives. The following section describes your team, and the final, your design task. A second handout, different for each team member, provides the specific information you will need to perform the role you have been assigned within your team. Each team member will contribute different expertise to the project, and each has different design responsibilities to fulfill. All must work together for your team to create a first-rate design.

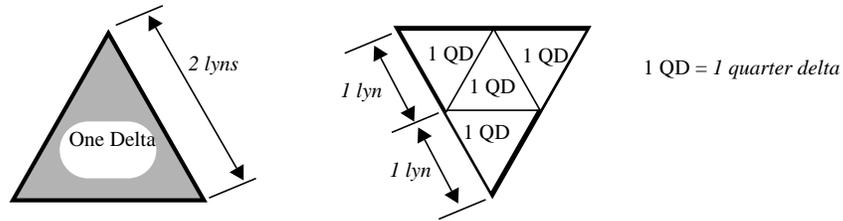
1.2 Life on DeltaP

Life on DeltaP, residential and otherwise, is quite different from what you have grown accustomed to here on Earth. First off, DeltaP is a plane, not a planet, so your team will be designing in two-dimensional rather than three-dimensional space. If your design "meets spec" and is considered attractive and functional by your Deltan clients, one view on a single sheet of paper will convey to those responsible for constructing it all the information they need to do so.



The view on this single sheet may not be quite what you expect, however, because in addition to lacking a z axis, Deltoid space has unfamiliar relations between the x and y axes as well. What we think of as “perpendicular” is hopelessly skewed to a Deltan, and vice-versa. In our units, a right angle on DeltaP measures 60° or $\pi/3$ radians. Thus all sides of an equilateral triangle form lines considered perpendicular to all others. If there were such a thing as a “circle” on DeltaP, it would be composed of only $4\pi/3$ radians.

But there is no such thing as a “circle” on DeltaP, nor even the concept of continuity embodied therein. In this flat though angular world, residents construct their artifacts strictly with discrete triangular forms. Of these, the equilateral triangle -- with its three perpendicular sides (!)-- is considered the most pleasing. Accordingly, your team will design the residence by assembling into a



cluster the most prized building materials on DeltaP, equilateral triangular components called “deltas.” Deltas come in red and blue versions and always measure *2 lynes* per side. Four “quarter-deltas”, *QDs*, triangular units of area measure with sides of *1 lyn*, fit within a delta.

Lyns? QDs? Not surprisingly, Deltan systems of measurement are as unfamiliar as that for spatial coordinates. Table 1 summarizes the measurement schemes on DeltaP that you will need to know to carry out your design task.

All of DeltaP’s units of measure share the divisibility and extensibility conventions of the metric

TABLE 1. Measurements on DeltaP

Measurement	Unit of Measurement	Symbol
Time	Wex	wx
Distance	Lyn	ln
Area	Quarter-Delta	qd
Heat	Deltan Thermal Unit	DTU
Temperature	Degrees Nin	$^\circ\text{Nn}$
Force	Din	Dn
Moment	Lyn-Din	LD
Currency	Zwig	!

system; in the measure of time, for example, there are both microwex (μwx) and megawex ($M\text{wx}$). In relation to the attention-and life-spans of Deltans, these units are roughly equivalent to seconds and years, respectively, here on Earth.

As building components, deltas have functional and aesthetic characteristics that are more complex than their simple form and even dimensions would suggest. Especially when assembled into a cluster, as you will be doing, they behave in interesting ways. Deltas conduct heat among themselves, radiate heat to outer space, melt if too hot, and grow if too cool. Red deltas produce heat. All deltas are subject to DeltaP’s two-dimensional gravity (which is itself subject to axial shifts during DeltaP’s not-infrequent gravity waves). Three different kinds of cement are needed to join them together, and joint alignment with respect to gravity affects ease of production as well as

structural integrity. Different colors and different quantities of deltas cost different amounts of money per delta, and can be assembled in clusters that are either exceedingly ugly or very attractive to the Deltans. Your task will be to create a design that meets prescribed goals for all of these characteristics.

1.3 Design Team Roles & Responsibilities

Your design team is organized such that each of you will be responsible for a subset of the design goals. One of you will be PROJECT MANAGER. Your main concerns will be with cost and schedule, the interpretation and reconciliation of performance specifications, and negotiations with the contractor and client. You want to keep costs and time-to-build at a minimum, but not at the expense of quality. When your team submits its final design, the project manager must report the estimated cost (in zwigs) and the time (in wex) that it will take to build.

Another of you will be the STRUCTURAL ENGINEER. Your main concern will be to see that the design “holds together” as a physical structure under prescribed loading conditions. You must see to it that the two points at which your structure is tied to ground are appropriately chosen and that continuity of the structure is maintained. When your team submits its final design, the structural engineer must attest to its integrity by identifying the strongest and weakest joints, and estimating the average load on all joints expressed as a percentage of the failure load.

Another of you will be the THERMAL ENGINEER. You will want to insure that the design meets the “comfort-zone” conditions specified in terms of an average temperature. You must also ensure that the temperature of all individual deltas stays within certain bounds. When your team submits its final design, the thermal engineer must estimate internal temperature and identify the hottest and coldest deltas.

Finally, one of you will be the ARCHITECT. Your concern is with both the form of the design in and of itself and how it stands in its setting. You must see to it that the interior of the residence takes an appropriate form and that egress is convenient. You should also develop a design with character. When your team submits its final design, the architect should be prepared to present a sketch and discuss generally how and why the Deltans will find the residence attractive and functional. The architect will also be asked to estimate a few more quantitative measures of architectural performance.

The following section describes the specifications that your design must meet to be accepted by your clients on DeltaP. Familiarize yourself with these specifications. Then, for schooling in your specialty, turn to the separate primer you have received that discusses the science and technology of your domain. The primer contains the knowledge and heuristics you will need to estimate the design parameters for which you are responsible. If you have questions that it does not answer, do not hesitate to ask. You should be expert in your role before your team begins the design phase.

1.4 The Design Task

Your Deltan clients have cleared the space shown on the site map and come to your team with their need for the design of a new residential cluster. The cluster itself must meet the following specifications.

The client wants the cluster to provide a minimum interior area of 100 QDs (Each diamond on your girded site map defines an area of two QDs). The shape of this space, which can of course exceed the minimum, is a matter of design. The client has expressed enthusiasm for the newer

The Design Task

mode of segmenting interior space, a mode that breaks with the two-equal-zone tradition and values the suggested privacy of nooks and crannies. Still the space must be connected, i.e. no interior walls can cut the space into completely separate spaces. There must be one and only one entrance/exit.

The client is known to be color sensitive blue; too much blue brings on the blues, so to speak. No more than 60% blue ought to be allowed; certainly blue deltas are not to exceed 70% of the cluster.

The residence, as all clusters, must be anchored at two points and two points only. There is a limit to the amount of force each anchor can support, as well as to the amount of internal moment each joint can withstand. Exceeding either limit would cause catastrophic failure and send the unwary residents tumbling into the void. The cluster should be designed for a life of thirty megawex. Gravity waves, rare but always possible, should be considered.

The average interior temperature must be kept within the Deltan comfort zone, which lies between 55 and 65 °Nn. The temperature of the elements themselves must be kept above the growth point of 20 °Nn and below the melt-down point of 85 °Nn. Delta temperatures outside of this range will result in catastrophic structural failure with little more warning than excessive load.

All of this -- design, fabrication and construction -- must be done under a fixed budget and within a given time period. At your team meeting you are to develop a conceptual design that meets or exceeds all design goals. When each team submits their design, individual members will be asked to report design performance on parameters for which they are responsible.

TABLE 2.

Summary of Design Specifications

Functional Internal Area	100 qd
Maximum Cool Deltas (% Total)	60-70%
Average Internal Temperature Range	55-65 °Nn
Individual Delta Temperature Range	20-85 °Nn
Maximum Load at Anchor Points	20 Dn
Maximum Internal Moment	40 LD
Overhead Factor -K	(varies)
Total Budget	! 1400.00

Delta Design STRUCTURAL ENGINEER PRIMER

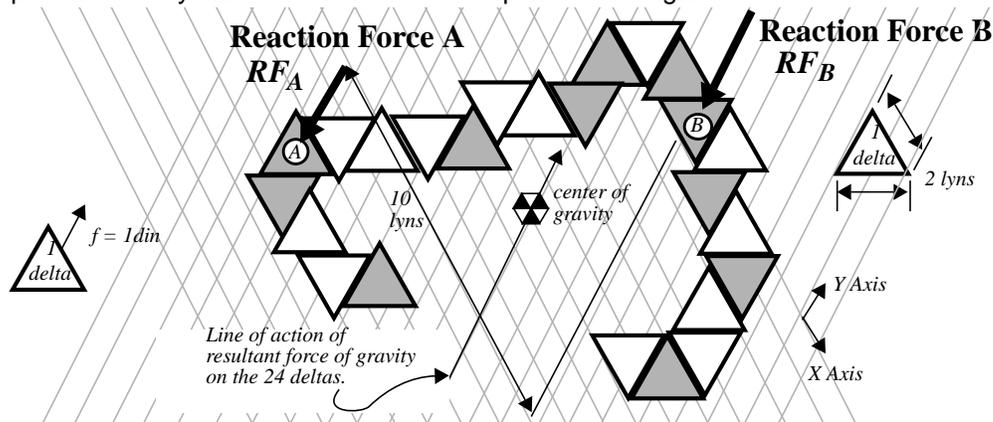
1.1 Introduction

As structural engineer, you are responsible for the physical integrity and robustness of your team's design. You must insure that the residence you propose will hold together under prescribed loading conditions. You should see to it that the two points at which your structure is anchored to the plane are appropriately chosen, that all joints are sufficiently strong, and that the overall shape of the cluster does not violate sound structural engineering practice. You should also strive for an elegant and efficient design, one that provides the requisite strength and durability with minimum costs and materials.

When your team submits its final design, you will be asked to attest to its quality by explaining the location of the anchors, identifying the strongest and weakest joints, and estimating, as a measure of robustness, the average load on all joints expressed as a percentage of failure loads. You may be asked to predict what will happen to your design during the next gravity wave. This primer will give you the tools, essentially the methods of static equilibrium analysis, with which to do your work. It assumes you have read the introduction to the Delta design exercise.

1.2 The Gravitational Field — The Center of Gravity

A uni-directional, gravitational force field acts on each delta in the plane. The direction of this force is parallel to the y axis shown on the site map and in the figures.



Each delta experiences a force of one *din*. Thus for the cluster of 24 elements shown in the figure, we can say

- that it has a total weight of 24 *dins*, and
- that the resulting force due to Deltan gravity acts in the plane along a line parallel to the *y* axis and running through the cluster's center of gravity, as shown.

The structure is kept stationary despite this force by offsetting reaction forces at the anchors, marked in the figure as points *A* and *B*.

The first step in structural analysis is to locate the cluster's center of gravity (CG). For our initial purposes, we actually only need the CG's *x* coordinate, which gives us the line of action of the gravity force shown on the previous page. We do not need to know the *y* coordinate until we consider DeltaP's recurrent gravity waves, which flip gravity between axes, and when the time comes, you can determine it by similarly flipping the following moment equilibrium calculation. You may also use the moment equilibrium technique to locate the CG of any subsection of a cluster.

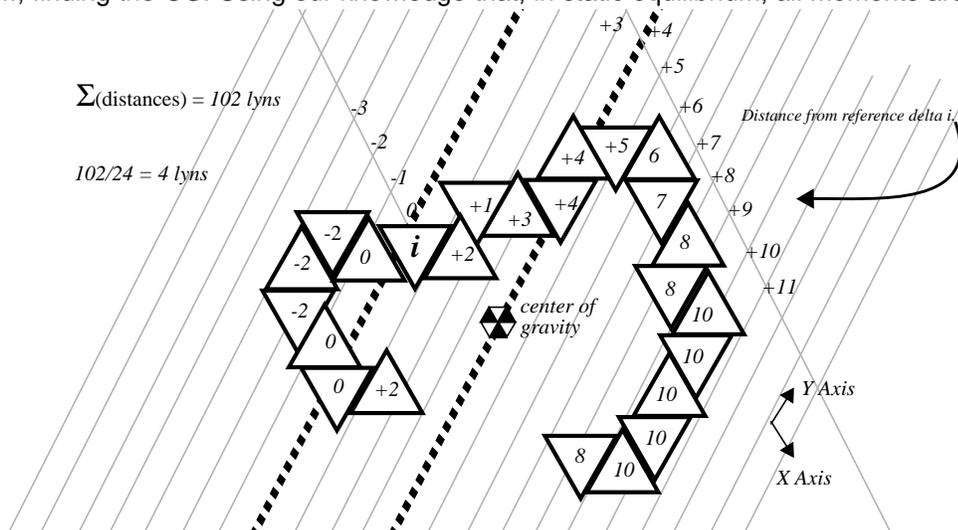
There are two things to keep in mind throughout your calculations. First, keep them as simple as possible. Work only in integers, always rounding up or down and estimating distances, forces and moments to the nearest *lyn*, *din*, or *lyn-din* respectively.

Second, keep in mind the peculiarities of Deltan space, where "perpendicular" describes an arc measuring only 60 degrees or $\pi/3$ radians in our units, and where distance measurements are made only along lines parallel to the axes. On DeltaP, the distance between anchors *A* and *B*, for example, measures 10 *lyns*, as shown in the previous figure.

This distinction is critical in the calculation of *moment*, the turning effect of a force about a point. As on Earth, moment is still the product of the force and its distance from the point, but the distance must be measured in Deltan space. The moment that force RF_A exerts about point *B*, for example, is the product of the distance in *lyns*, measured parallel to the *x* axis, from the line of action of RF_A to anchor *B* (10 *lyns*), and force RF_A measured in *dins*. Not surprisingly, moment, M , is measured in *lyn-dins*, abbreviated *LD*:

$$M (RF_A \text{ about } B), \text{ in } \textit{lyn-dins} = 10 \textit{ lyns} \times RF_A \textit{ dins}$$

Now, finding the CG. Using our knowledge that, in static equilibrium, all moments around any



given point will sum to zero, we can find a cluster's CG in reference to any delta, call it the i^{th} , by

Estimating Support Loads

equating the sum of moments around it generated by gravity acting on each individual delta to the moment of the entire cluster around it:

$$\sum_{j=1}^N (\mathbf{f} \times D_{ij}) = N \times \mathbf{f} \times D_{i, cg}$$

where:

- N is the number of deltas in the cluster;
- \mathbf{f} is the gravity force experience by each delta, equal to 1;
- D_{ij} is the distance between the i^{th} and each other delta;
- $D_{i, cg}$ is the distance between the i^{th} delta and the CG's line of action due to gravity.

Simplifying and solving for $D_{i, cg}$ gives us:

$$D_{i, cg} = \frac{\sum_{i=1}^N D_{ij}}{N}$$

So finding the CG's x coordinate is as simple as summing up the distances between any delta and all others, dividing by the total number of deltas, and adding the result to the x coordinate of the delta used as a reference. Just be sure to adhere to Deltan measurement technique, and express distances to the left and right of the reference as negative and positive numbers respectively.

In the example shown in the figure, the distances from the i^{th} delta sum to 102 *lyns*. Dividing by $N=24$ gives us 4 *lyns*, which we then count over, and *viola*, we have the location of the CG's *line of action* of the force due to gravity. Call this the *CG LOA*.

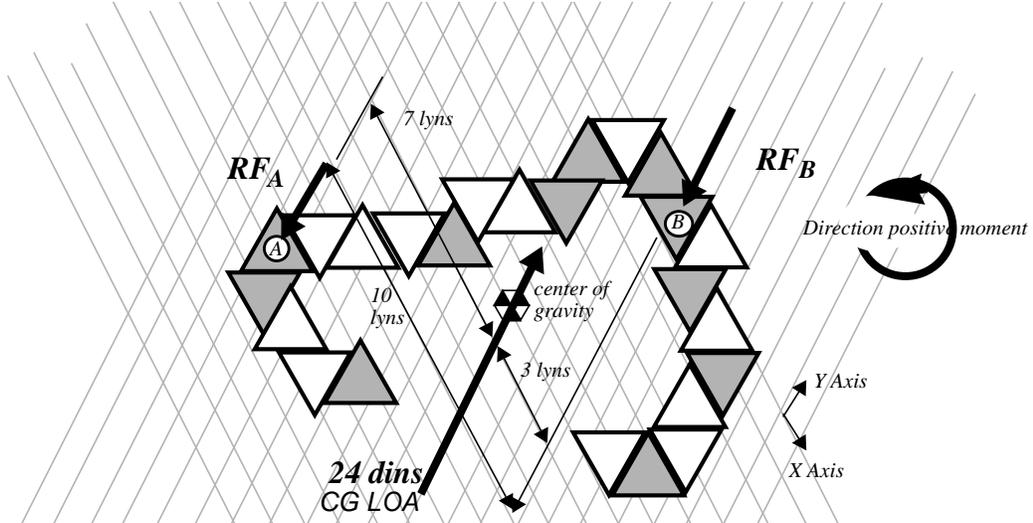
1.3 Estimating Support Loads

Each of the two anchors has sufficient strength to support a load of 20 *dins*. They can resist no twisting effect, no moment, about their axes, i.e. they act as frictionless pins holding the cluster in place. To estimate the support loads that they will be subject to, first sum up the total force due to gravity acting on all deltas in the cluster. We know that this force, measured in *dins* as discussed above, is equal to the total number of elements N since each element experiences a force of 1 *din*. We also know that, for equilibrium, the two reaction forces at the anchors, marked on the figure as \mathbf{RF}_A and \mathbf{RF}_B must also sum to this total force:

$$\mathbf{RF}_A + \mathbf{RF}_B = N \times \mathbf{f} = 24 \text{ dins}$$

We need a second equation to solve for the reaction forces, which we can get by calculating the moment equilibrium of the entire cluster with reference to either anchor. We will use anchor B

as a reference. In Deltan equilibrium analysis lingo — we take moments about point B. Referring



to the figure, assume that a positive moment acts to rotate the cluster counter-clockwise. Measure the distance from the CG LOA to anchor B: these 3 lynes are the distance over which the 24 din gravity force acting on the entire cluster creates a moment about B.

Since we know that, in equilibrium, the (positive) moment of RF_A about anchor B must balance the (negative) moment of the entire cluster about B, we have that:

$$10 \text{ lynes} \times RF_A - (24 \text{ dins} \times 3 \text{ lynes}) = 0$$

We solve for RF_A

$$RF_A = 72 \text{ lyn-dins} / 10 \text{ lynes} = 7 \text{ dins}$$

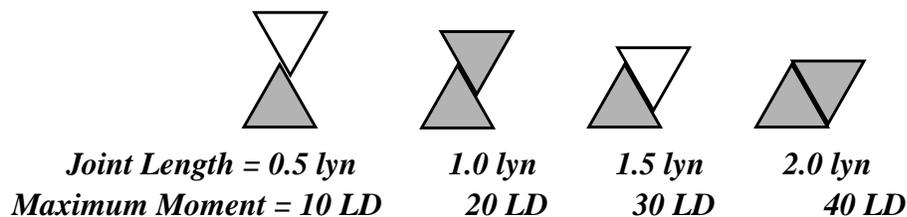
And, with the first equation, for RF_B

$$RF_B = 24 - RF_A = 17 \text{ dins}$$

Since anchors can withstand 20 dins, anchor A is in fine shape. Anchor B, however, only has a safety margin of 20/17, or roughly 1.2. This is ok, but you won't win awards for robustness. Can you see how to relocate the anchors for a better balance of reaction forces?

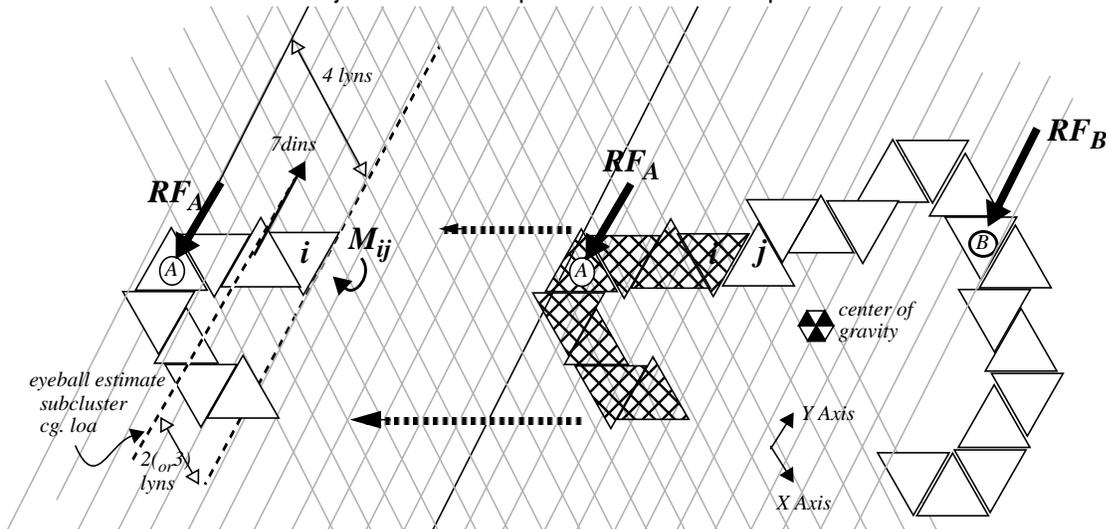
1.4 Internal Moments and Fastener Requirements

Adjacent elements are held together by cement. The cement is necessary because otherwise the structure could not support the internal forces and moments, again due to gravitational loading. The internal moment is the most critical of these. Although the *strength of the cement* varies by supplier, your source has certified hers at 20 LD per lyn of contact, resulting in the linear relationship between length and strength shown in the figure below. A fully overlapped joint fastened with



this cement will have a maximum allowable internal moment of 40 lyn-dins.

Given this length-strength relationship, how do we estimate the actual internal moments that joints will experience and must withstand? Simply treat each joint as the boundary of a subcluster, and derive the moment at the joint from the equation for moment equilibrium of this subcluster. For



the subcluster shown in the figure the moment at the intersection of deltas i and j, denoted M_{ij} is found in the following steps:

- find the sub-cluster's CG LOA and its distance from the joint: here, about 2 or 3 lynes;
- find the force exerted due to gravity by the subcluster, a quantity equal to the number of deltas in t: here 8 dins;
- find the product, which gives us the moment of the subcluster about the joint;
- if the subcluster contains any anchor points, figure the moments of resulting reaction forces around the joint; in this case, we know that $RF_A = 7\text{ dins}$, and measure the distance as 4 lynes;
- taking moments about the joint as positive if they tend to rotate the section clockwise, express the fact that for equilibrium the sum of all the moments must be zero:

$$M_{ij} + (\text{moment of subcluster about joint}) - (\text{moment of } RF_A \text{ about joint}) = 0$$

- solve for M_{ij}

$$M_{ij} = (\text{moment of } RF_A \text{ about joint}) - (\text{moment of subcluster about joint})$$

So in our case, we get

$$M_{ij} = (7\text{ din} \times 4\text{ lyn}) - (8\text{ dn} \times 2\text{ or } 3\text{ lyn}) = 12\text{ or } 4\text{ LD.}$$

This sounds pretty tame, since the joint is fully overlapped and can withstand 40 LD. In fact, it sounds like a waste of cement, because a joint 25% shorter would still provide a hefty safety margin.

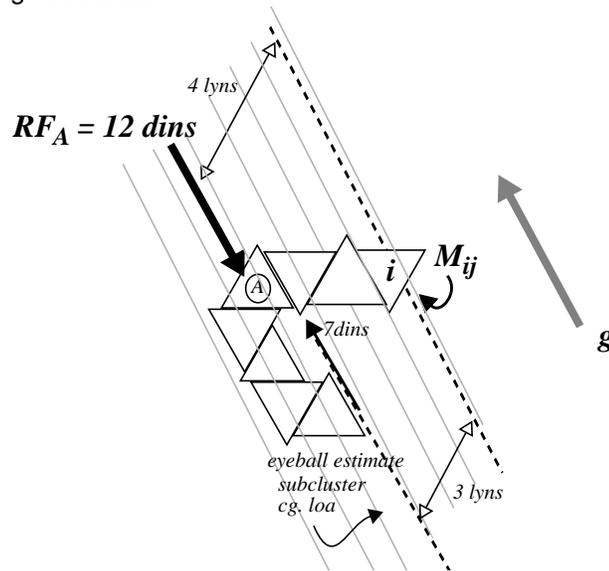
With these same six steps, we can estimate the internal moment at any joint we choose to examine, and compare it to the strength of the joint.

1.5 Gravity Waves

Finally, although the gravitational force across the Deltoid plane is forever constant in magnitude, the rare but inevitable gravity wave will, when it comes, instantaneously shift its direction orthogonally; from that of the y axis shown on the site map to that of the x axis. Gravity will then retain that direction until the next wave arrives and causes it to shift back. The figure shows the change in internal moment that the joint just discussed would undergo in the event of such a wave. We now obtain

$$M_{ij} = (12 \text{ dins} \times 4 \text{ lyns}) - (7 \text{ dins} \times 3 \text{ lyns}) = 27 \text{ LD}$$

which is significantly greater than the value previously obtained. Note that the reaction force at the anchor point has changed as well.



The last gravity wave passed through about 40 megawex (Mwx) ago, causing widespread destruction. The time period between waves is a random variable — a Poisson process with a mean arrival time of 80 Mwx . The statistics are generally considered “good” only for the past gigawex or so, and myths of old suggest that but ten Mwx separated two waves at the time of the flood. Should you worry about it?