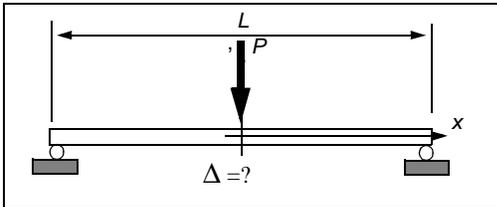
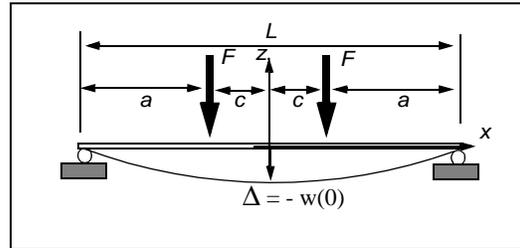


Class Exercise #3

1.101 Structures Work

The figure shows our simply supported beam. The displacement of the centroidal axis ($z=0$) was given to you as

$$w|_{\text{midspan}} = -\left(\frac{Fa}{24EI}\right) \cdot (3L^2 - 4a^2) = -\Delta$$



To deduce an expression for the midspan displacement of the beam shown at the left - load P is concentrated at mid-span, we set $a = L/2$, and $F = P/2$. (Think of moving the two loads, each of magnitude F , in toward the midspan, shrinking c to zero).

Making these substitutions, we have:

$$w|_{\text{midspan}} = -\left(\frac{PL^3}{48EI}\right)$$

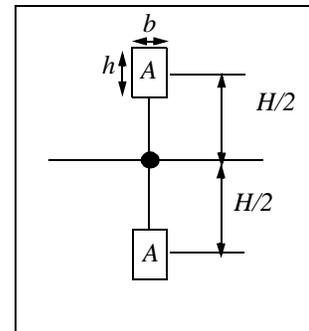
For the beam two “flanges” of equal area, A , located at $\pm H/2$ from the centroid of the section and with “web” infinitely thin, the second moment of area, with respect to the centroid, is approximately

$$I = 2 \cdot \left(\frac{H}{2}\right)^2 \cdot A$$

A more accurate result comes from integrating over the two sections. Doing one section is all we need do because of symmetry. Thus

$$I = 2 \cdot \int_{z=(H-h)/2}^{z=(H+h)/2} z^2 \cdot b dz \text{ which evaluates to}$$

$$I = 2 \cdot \left\{ \left(\frac{H}{2}\right)^2 \cdot bh + \frac{bh^3}{12} \right\} = 2 \cdot \left\{ \frac{H^2}{2} \cdot A + \frac{bh^3}{12} \right\}$$



The first term within the bracket is just what we had as our approximation. The second term we recognize is just the “local” second moment of area of the top bit. Our approximation is good if $(h/H)^2$ is small for we can write:

$$I = 2 \cdot \left\{ \left(\frac{H}{2}\right)^2 \cdot bh + \frac{bh^3}{12} \right\} = 2 \cdot \left(\frac{H}{2}\right)^2 \cdot bh \left\{ 1 + \frac{1}{3} \cdot \left(\frac{h}{H}\right)^2 \right\}$$

e.g., if $h = 0.1 \cdot H$ then I is less than 1% more than our approximate result.