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1.061 / 1.61 Transport Processes in the Environment  
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### Answer 8.1

**Diffusion** is a specific term referring to the net flux resulting from zero-mean, random motions. The random motions may be Brownian motion – resulting in molecular diffusion – or may be due to turbulent eddies. Strictly, the size of the eddies must be small compared to the patch of particles in order for impact of the eddies on the patch to be truly random, and thus the net flux to be purely diffusive.

**Advection** is the transport of particles (concentration) by the mean current.

**Dispersion** is a broad term used to describe all processes, except diffusion, that act to disperse a patch, and thus diminish concentration. Typically, dispersion coefficients reflect a combination of advection and diffusion processes that are difficult to model separately. For example, the coupled effect of differential longitudinal advection and cross-channel diffusion creates shear dispersion. The existence of multiple discrete flow paths, e.g. through a porous medium, also creates a dispersion of particles, because the time to traverse each path differs with its geometry and the degree of bending/turning – often called the tortuosity. Thus, many particles released together but which take different pore pathways will get separated (dispersed) in the longitudinal direction. Finally, the lateral diffusion of particles into a region of zero velocity (a pore space with no outlet, a side-pool in a river, a wake behind an obstruction) will create longitudinal dispersion. The particles that enter the zones of zero velocity get held back relative to the particles that do not get trap. When the particles are eventually released from the ‘dead-zone’, they are separated in space (dispersed) from the rest of the cloud.

### Answer 8.2

The expression for longitudinal dispersion in a wide channel is  $K_x = 5.93u_* h$ , where  $u_*$  is the shear velocity and  $h$  the water depth. We wish to make the comparison,

$$\frac{K_{X1}}{K_{X2}} = \frac{u_{*1} h_1}{u_{*2} h_2} \quad (1)$$

First consider how  $u_*$  will change between section 1 and 2. For steady uniform flow driven by bed slope, the momentum balance requires that

$$u_* = \sqrt{ghS} \quad \text{Equation 23, Chapter 7.}$$

Such that

$$u_{*1}/u_{*2} = \sqrt{(h_1 S_1)/(h_2 S_2)} \quad (2)$$

**If the channel depth is constant**, then from (1)

$$\frac{K_{X1}}{K_{X2}} = \frac{u_{*1}}{u_{*2}} = \sqrt{\frac{S_1}{S_2}} > 1.$$

The dispersion is greater in the steeper channel.

**If the channel width is constant**, then from continuity the channel depth must change inversely with the depth-averaged velocity,  $U$ . Specifically,  $U_1 h_1 = U_2 h_2$ , or

$$h_1/h_2 = U_2/U_1. \quad (3)$$

If the substrate is the same, we can infer that  $U/u_*$  is constant. More formally, using the drag coefficient for the bed,  $C_b$ , we can write

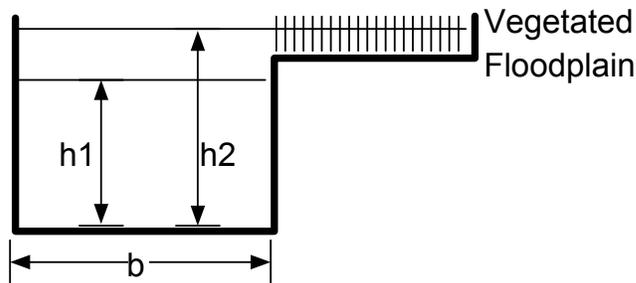
$$\tau_{bed} = \rho C_b U^2 = \rho u_*^2,$$

which also indicates that for constant  $C_b$  the ratio  $U/u_*$  is constant, *i.e.*

$$U_1/u_{*1} = U_2/u_{*2}. \quad (4)$$

Combining (3) and (4),  $h_1/h_2 = u_{*2}/u_{*1}$ . Using this ratio in (1),  $K_{X1}/K_{X2} = 1$ . That is, for a constant channel width, the longitudinal dispersion is the same in both sections.

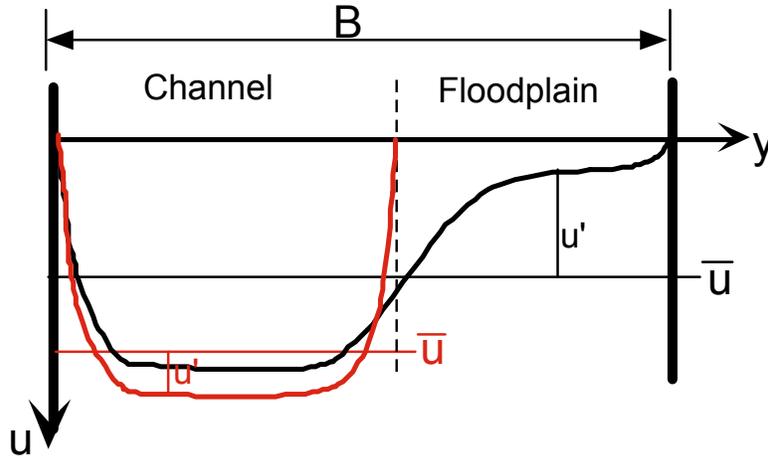
**Answer 8.3**



When the water depth increases from  $h_1$  to  $h_2$ , flow enters the floodplain. Vegetative drag and the shallow depth combine to retard flow on the floodplain relative to the channel. This produces strong lateral shear that augments dispersion. The lateral profile of depth-averaged velocity is shown below for flow depth  $h_1$  (red) and  $h_2$  (black). The velocity fluctuations,  $u'$ , are deviations from the channel mean velocity,  $\bar{u} = Q/A_{total}$ . Adapting the expression for shear dispersion (eq. 16, chap. 8) to the lateral shear,

$$K_x = -\frac{1}{BD} \int_0^B \int_0^y u' \int_0^y u' dy dy,$$

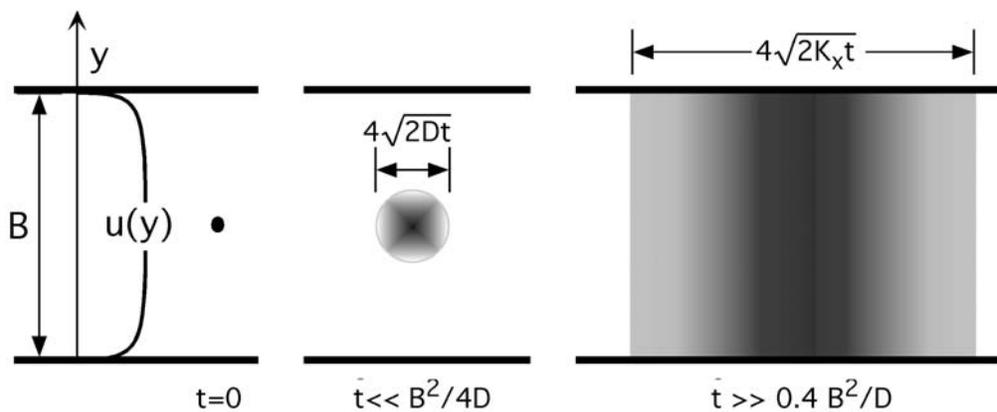
where  $B$  is the total channel width with the flood plain. From the sketch below, it is clear that the sum of spatial fluctuations,  $u'$ , is greater when the water depth permits flow on the floodplain, and so we expect  $K_x$  to increase at the greater water depth. This is a greater effect than the increase lateral diffusivity that might occur with increasing width, which according to the above equation would decrease  $K_x$ . In particular the lateral diffusivity cannot increase significantly through the floodplain because of obstruction by vegetation.



**Answer 8.4**

**Time  $t_1 \ll B^2/4Dy$ :** When the slug is initially released, it is very small compared to the width of the channel. Released in the center of the channel, the variation in velocity (shear) across the patch is negligible, and the entire patch advects at the same speed. Because the patch is not experiencing differential advection, the spreading of the cloud in the longitudinal direction is due to longitudinal diffusion only. The cloud's longitudinal length scale is  $4\sqrt{2Dt}$ . More specifically, letting the release point be  $(x, y, z) = (0, 0, 0)$ , the concentration field evolves as,

$$C(x, y, z, t) = \frac{M}{(4\pi Dt)^{3/2}} \exp\left(-\frac{(x-ut)^2}{4Dt} - \frac{y^2}{4Dt} - \frac{z^2}{4Dt}\right)$$



**Time  $t_2 \gg 0.4 B^2/Dy$ .** By this time the patch has grown to uniformly fill the lateral dimension of the channel. In addition, sufficient time has passed for the longitudinal dispersion due to the lateral shear to reach Fickian behavior. The longitudinal length-scale of the patch is now  $4\sqrt{2K_x t}$ , i.e. the patch growth rate is dictated by the dispersion coefficient,  $K_x$ . In the vertical direction the cloud continues to grow via vertical diffusion, such that the concentration field evolves as,

$$C(x, z, t) = \frac{M}{B 4\pi \sqrt{DK_x t}} \exp\left(-\frac{(x - ut)^2}{4K_x t} - \frac{z^2}{4Dt}\right).$$