

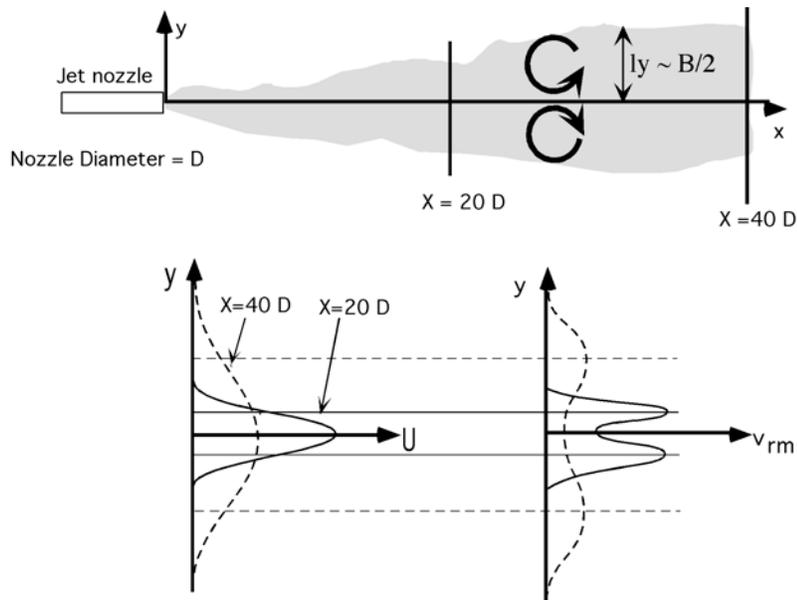
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1.061 / 1.61 Transport Processes in the Environment  
Fall 2008

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### Answer 7.1

- 1) Profiles of  $v_{rms}$  are sketched below, with  $x=20D$  shown as a solid line and  $x = 40D$  shown as a dashed line. The strongest turbulent fluctuations occur at the points of maximum mean shear. Because the maximum shear is weaker at the downstream station, the fluctuations are weaker here as well.
- 2) First, let  $B(x)$  be the width of the jet at position  $x$ . While many eddy scales coexist in a turbulent jet, it is the biggest scales that dominate mixing. The biggest eddies have the scale of the flow, which for a jet is  $B$ . We find the sign of rotation of these eddies by imagining the motion of a rigid stick placed in the flow. Because the sign of the shear (and thus vortex rotation) reverses at mid-jet, a single eddy cannot grow larger than the half-width of the jet. Thus, we can say more specifically that the largest eddies in the jet will be the scale of the half-width,  $B/2$ .



### Answer 7.2

- a) The logarithmic region is fitted with a red line. From this line we estimate the value of  $u_*$ .

$$u_* = \frac{0.4}{2.3} \frac{1.13 - 0.54 \text{ cm/s}}{\log_{10}(1/0.1)} = 0.1 \text{ cm/s}$$

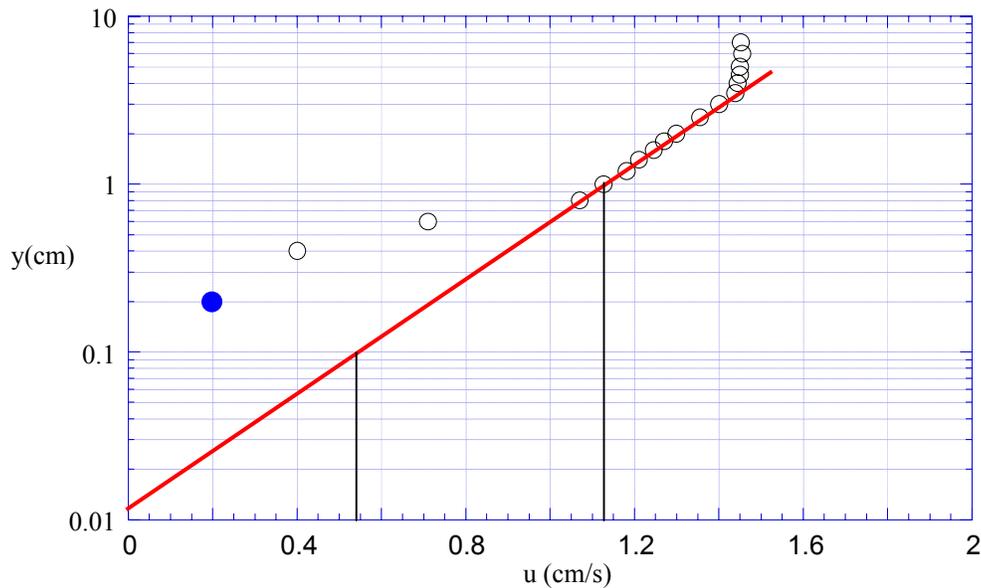
- b) The bed stress is  $\tau_w = \rho u_*^2 = 0.001 \text{ Pa} (= \text{Nm}^{-2})$

- c) The laminar sub-layer thickness is  $\delta_s = 5\nu/u_* = 5 (0.01 \text{ cm}^2\text{s}^{-1})/0.1 \text{ cm s}^{-1} = 0.5 \text{ cm}$

- d) The blue point ( $u = 0.2 \text{ cm/s}$ ,  $y = 0.2 \text{ cm}$ ) is within the laminar sub-layer. Because turbulence is excluded from the laminar sub-layer, the wall stress can be found from the molecular viscosity and the shear,  $\tau_w = \mu \partial u / \partial y = \mu u / y = (0.001 \text{ kgm}^{-1}\text{s}^{-1})(0.2 \text{ cm s}^{-1}) / (0.2 \text{ cm}) = 0.001 \text{ Pa}$ , consistent with the estimate based on  $u_*$

e)  $y_o = 0.011$  cm from the graph. For smooth turbulent flow,  $y_o$  would be  $\nu/9u_* = 0.011$  cm. Since this agrees with the observed  $y_o$ , the flow must be smooth turbulent. Alternatively, for a sand grain the Nikuradse roughness is the sand diameter,  $\epsilon = d = 0.1$  cm. Thus, the roughness Reynolds number is  $\epsilon u_* / \nu = (0.1 \text{ cm})(0.1 \text{ cm s}^{-1}) / (0.01 \text{ cm}^2 \text{ s}^{-1}) = 1 < 5$ , also indicating that the boundary is Smooth Turbulent.

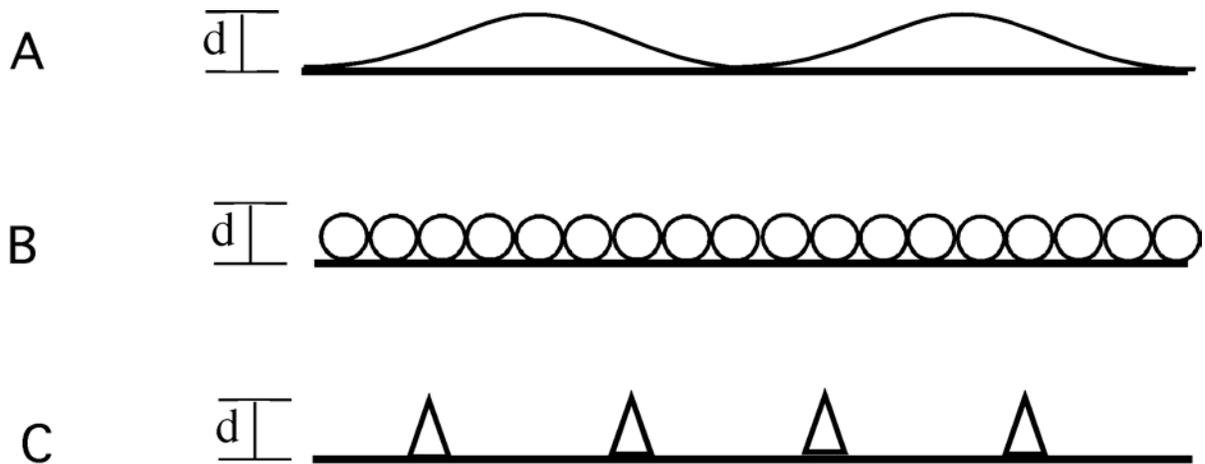
f) Dye released at  $y = 3$  mm will be within the laminar sub-layer. Its trace will be a straight line, as the dye will tracer a single streamline without deviation. Dye released at  $y = 3$  cm will be above the laminar sub-layer, and so will feel the effects of turbulence. The dye tracer will be squiggly, as the dye is distorted by the action of many tiny eddies.



### Answer 7.3

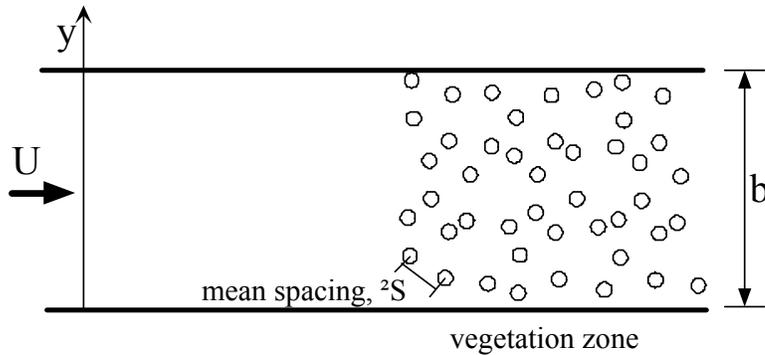
If  $d \ll \delta_s$ , every shape will be buried within the laminar sub-layer. The outer flow will not feel any of the shapes and the flow will be Smooth Turbulent. Under these conditions all cases have the same  $y_o = \nu/9u_*$ .

If  $d \gg \delta_s$  the flow is Rough Turbulent and the flow feels each shape. The shape that disturbs the flow the most, *i.e.* creates the most separation and flow distortion, will create the greatest momentum sink and will feel the roughest. This is shape C. Shape A is the most streamlined, and thus produces the least disturbance and separation. Based on this we expect  $y_{oC} > y_{oB} > y_{oA}$ .



#### Answer 7.4

- a) In the unvegetated zone the lateral eddy scales are set by the channel width,  $l_y \sim b$ . But, in the vegetated zone eddies of scale  $b$  are destroyed by the physical obstruction of the plant stems. In this zone the lateral eddy scale is set by the vegetation spacing and diameter, such that  $l_y \sim d$  or  $\Delta S$ . Because of the confining walls, the mean channel velocity is approximately the same in both regions. Because the eddy scale is so much reduced in the vegetated zone ( $\Delta S, d \ll b$ ) we expect the lateral diffusivity to be reduced in this region, relative to the unvegetated zone.
- b) If  $Ud/\nu = 1$ , the wakes behind each stem will be laminar and will not contribute additional eddies to the flow. If  $Ud/\nu = 1000$ , the stems contribute stem-scale eddies to the flow through separation and the generation of Kelvin-Helmholtz vortices. So, for the later case we expect that  $v'$  will be augmented by the wake-vortices, and that  $D_{t,y}$  will be greater with the higher Reynolds number.
- c) If the flow is unconfined the vegetative drag will lead to a reduction of velocity within the vegetated zone as flow is redirected around the patch of vegetation. Then, the velocity in the vegetated zone,  $U_V$ , will be less than the velocity in the free stream,  $U$ . If  $U_V < U$  and the stem wakes are not contributing additional turbulence ( $U_V d/\nu \ll 100$ ) we would expect that the turbulence intensity in the vegetated zone is also less,  $v'_V < v'$ . In addition, the turbulence scales will be smaller in the vegetated zone, as discussed in a). With both  $v'$  and  $l_y$  smaller in the vegetated zone, then  $D_{t,y}$  will also be smaller in the vegetated zone. Even if the stem Reynolds' number,  $U_V d/\nu$ , is sufficiently high that the stems contribute additional eddies, their scale ( $l_y \sim d$ ) is too small compared to those in the free stream,  $l_y > b$ , and still the diffusivity in the vegetated zone will remain small compared to the unvegetated zone.



### Answer 7.5

a) The logarithmic region is fitted with a red line. From this line we estimate the value of  $u_*$ .

$$u_* = \frac{0.4 (15 - 0 \text{ cm/s})}{2.3 \log_{10}(4/0.2)} = 2.0 \text{ cm/s}$$

b) The thickness of the laminar sub-layer is  $5\nu/u_* = 5 (0.01 \text{ cm}^2\text{s}^{-1})/(2.0\text{cm s}^{-1}) = 0.025 \text{ cm}$ .

c) Considering that the gravel diameter, 1 cm, is much larger than the laminar sub-layer, it is most likely that the gravel does contribute to the bed resistance, i.e. the flow feels the gravel roughness. More formally, we must determine if the flow is smooth or rough turbulent. From the graph,  $y_o = 0.2 \text{ cm}$ . For smooth turbulent flow  $y_o = \nu/9u_* = 5.6 \times 10^{-4} \text{ cm}$ , which is smaller than the observed  $y_o$ , so the flow is not smooth. If the flow were rough turbulent, the equivalent roughness,  $\epsilon = 30y_o = 6 \text{ cm}$ , and the roughness Reynolds' Number would be,  $\epsilon u_*/\nu = 1200$ , which is much higher than the limit for rough turbulent flow (70-100). So we conclude the flow is rough turbulent, and the gravel contributes to the roughness of the flow.

d) A covering of uniform sand grains of 6-cm diameter would be needed to produce the same flow resistance.

