LECTURE # 23

1.060 ENGINEERING MECHANICS II

OPEN CHANNEL FLOW

or

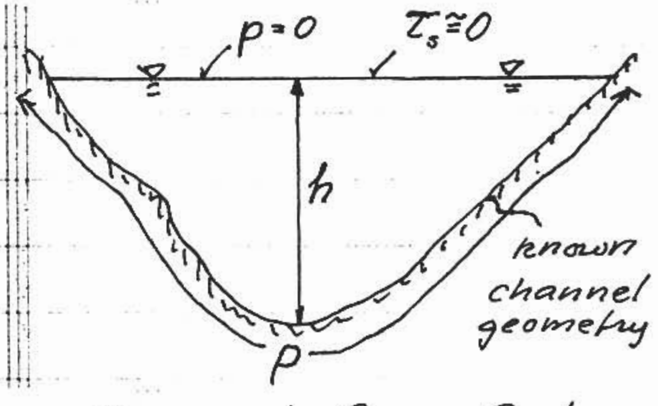
FREE SURFACE FLOW

Whatever name is given to this aspect of Hydraulics, it is the Hallmark of Fluid Mechanics in CEE!

Up to this point we have primarily been concerned with flows in (man-made) closed conduits (pipes, ducts), although we, from time to time, have used examples of flows with a free surface, e.g. flow under gates and hydraulic jumps, to illustrate the fundamental principles of fluid mechanics (Conservation of mass/volume, energy (Bernoulli), and momentum). We now turn our attention to the analysis of flows in natural "conduits" such as rivers and streams, i.e. channels in which the fluid flows under the influence of gravity and has a free surface in contact with air as its upper boundary.

The main differences between flows in closed conduits and open channels are:

1) The area of flow is no longer prescribed, but a function of the location of the free surface, i.e. the depth, h, of flow in the "open" channel, and this depth is a priori unknown – <u>h = depth of flow is a variable</u>



Channel Cross-Section.

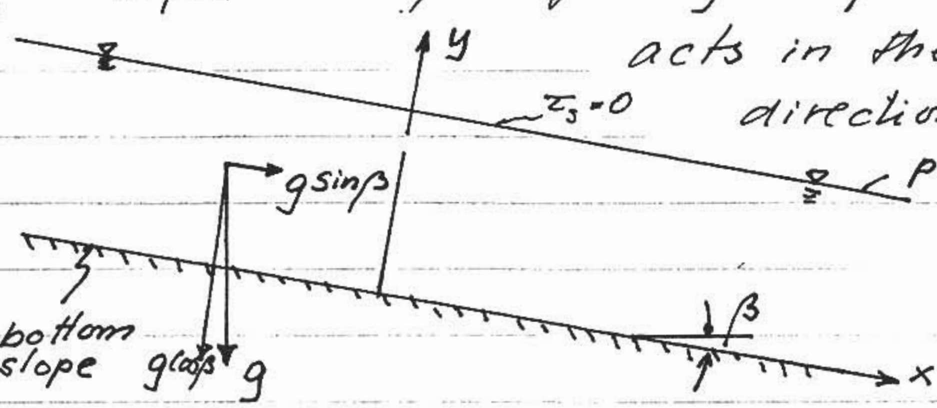
For a known channel geometry we must know h in order to specify flow area-related quantities:

A = flow area = $A(h)$

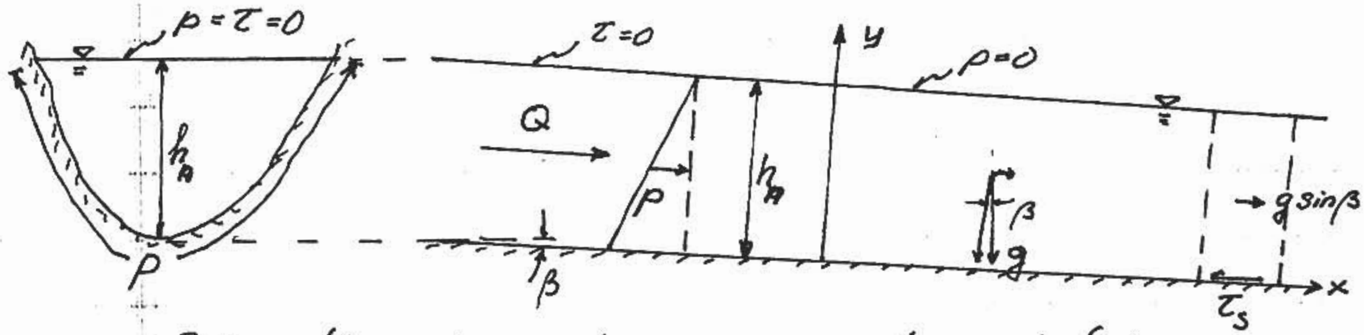
P = wetted perimeter = length of channel surface with fluid-solid contact = $P(h)$

R_h = hydraulic radius = $A(h)/P(h) = R_h(h)$

2) Since $p = p_{atm} = 0$ at the free surface it is not possible to impose an external pressure gradient on the flow (as we could in closed conduits). The pressure gradient is replaced by a gravity component that acts in the down-slope direction



UNIFORM, STEADY FLOW



Prismatic channel – cross-section $\neq f(x)$
Constant bottom slope – β = angle w. horizontal
Uniform flow : $\partial/\partial x = 0$; Steady flow : $\partial/\partial t = 0$

Straight parallel streamlines (parallel to bottom, i.e. x)

Pressure variation $\perp$ streamlines, i.e. in y-direction, is hydrostatic.

$$\frac{\partial p}{\partial y} = -\rho g \cos\beta \Rightarrow p = \rho g \cos\beta \,(h_n - y)$$

$p = 0$ at $y = h_n$ = free surface
β generally so small that $\cos\beta \simeq 1$, i.e. $y \sim z$
h_n – depth (normal depth) = const. , i.e. $h_n \neq h_n(x)$
Free surface is parallel to bottom

Force balance parallel to bottom
Gravity force $= \rho\, \delta\forall\, g_x = \rho(A\,\delta x)\, g \sin\beta$
Pressure force : complete balance $\Rightarrow$ No net press. force
Boundary shear force $= \tau_s A_\tau = \tau_s(P\,\delta x)$, where
P = wetted perimeter = length of cross-section with fluid-solid contact – $\tau_{free\,surf} \cong 0$.

$$\tau_s P\,\delta x = \rho g A \sin\beta\,\delta x$$

or

$$\underline{\tau_s} = \rho g \frac{A}{P} \sin\beta = \underline{\rho g R_h S_o} = \underline{\gamma R_h S_o}$$

τ_s = average shear stress along wetted perimeter
$R_h = A/P$ = hydraulic radius
S_o = bottom "slope" = $\sin\beta$
(Note: slope normally $\tan\beta$, but $\sin\beta \sim \tan\beta$ when β is small and $\cos\beta \simeq 1$. However, there is a difference between $\sin\beta$ and $\tan\beta$ if $\beta > \sim 10^\circ = 0.15$ rad., e.g. for flow down a spilway of a dam).

<u>BASIC HYDRAULIC FORMULA</u>

$$\tau_s = \underline{\tau_b = \rho g R_h S_o}$$

In a river - open channel - if it is of approximately constant cross-section, i.e. ~ prismatic, and you know h ($\sim h_n$) and the bottom slope, e.g. from topographic map $S_o \sim (\Delta\,\text{elevation})/(\Delta\,\text{length})$, you can use the basic <u>Hydraulic Formula</u> to predict the average boundary shear stress τ_b along the wetted perimeter of the channel.

<u>BUT THAT IS NOT WHAT YOU WANT</u>! YOU WANT Q - the discharge in the river.

DARCY-WEISBACH FORMULA

$$\tau_s = \tau_b = \frac{1}{8}\rho f V^2 = \rho g R_h S_o$$

so

$$V = \sqrt{\frac{8g}{f}}\,\sqrt{R_h}\,\sqrt{S_o}$$

so, if you knew f, you could get V and then

$$Q = AV$$

but

$$f = f\left(Re = \frac{V(4R_h)}{\nu},\ \frac{\varepsilon}{4R_h}\right) \quad : \underline{\text{MOODY}}$$

so you would need to know $\underline{\varepsilon = \text{channel roughness}}$.

Same scenario as pipe flow analysis: Guess value of $f = f^{(1)}$, get $V = V^{(1)}$ and then $Re = Re^{(1)} \Rightarrow$ Go to MOODY to update $f = f^{(2)}$ etc.

If you want the river stage for a given Q, things get a bit more involved. For example, guess value of $h = h^{(1)}$ and $f = f^{(1)}$, obtain $V^{(1)}$ and then $A^{(2)} = Q/V^{(1)}$. From $A^{(2)} = A(h^{(2)})$ obtain new value of $h^{(2)}$ etc.

This is a very long procedure since you have to iterate on both h and f.

Alternative approach is presented later. (Lecture #24) based on Manning's Equation.

CHEZY FORMULA

$f \approx 0.02$ gives

$$V = C\sqrt{R_h}\sqrt{S_o} \qquad C = \text{Chezy's "}C\text{"} = \sqrt{\frac{8g}{f}} \approx \underline{60\,\frac{m^{1/2}}{s}}$$

MANNING'S EQUATION

Fully rough turbulent flow $\Rightarrow f = f\left(\frac{\varepsilon}{4R_h}\right)$. Turns out that

$$f = 0.113\,(\varepsilon/R_h)^{1/3}$$

$$\sqrt{\frac{8g}{f}} = \sqrt{\frac{8g}{0.113\,\varepsilon^{1/3}}}\,R_h^{1/6} = \frac{1}{n}\,R_h^{1/6}$$

$$V = \frac{1}{n}\,R_h^{2/3}\sqrt{S_o}$$

$$n = \text{Manning's "}n\text{"} = \sqrt{\frac{0.113\,\varepsilon^{1/3}}{8g}} = 0.038\,\varepsilon^{1/6} \quad [SI]$$

ε = channel roughness in [m]; $[n] = [m^{-1/3}\,s]$

Notice weak dependency of "n" on $\varepsilon \Rightarrow$ n increases by factor of 2 if ε increases by $2^6 = 64$!

Values for Manning's "n" always given in SI-units - EVEN IN TEXTS EXCLUSIVELY USING BRITISH UNITS! Therefore, in British Units the Manning-Equation reads

$$V = \frac{1.49}{n}\,R_h^{2/3}\sqrt{S_o} \quad \text{in fps when } R_h \text{ in ft and "}n\text{" in } m^{-1/3}s$$

$[1.49 \Rightarrow [m = 3.28\,ft]^{1/3} = 1.49\,ft^{1/3}]$

How rediculous can it get in terms of mixing up units? Manning's Equation takes one of the top prizes!