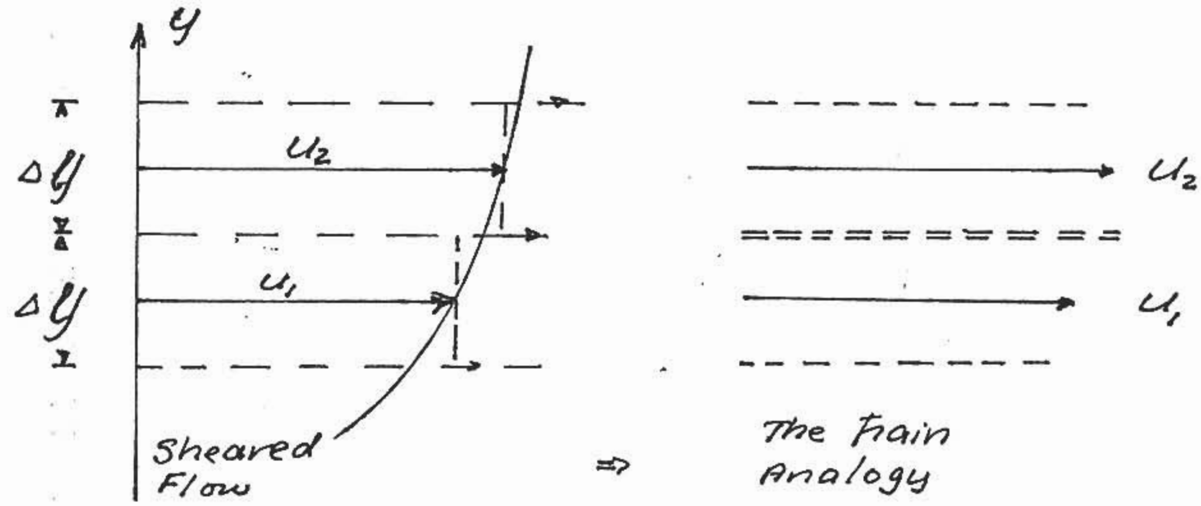
LECTURE #15

1.060 ENGINEERING MECHANICS II

The Nature of Shear Stresses in a Fluid



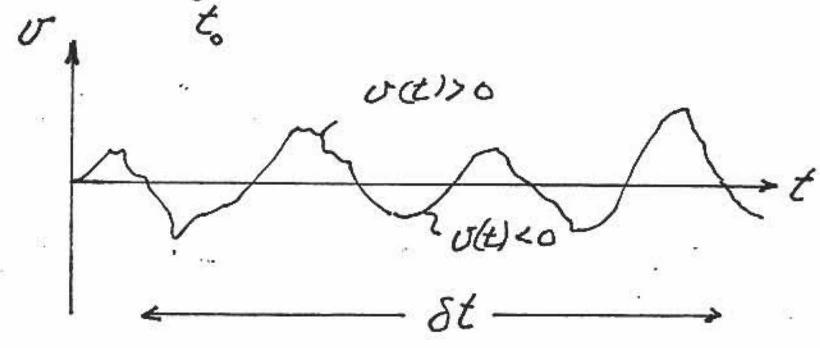
Depicted above is a shear flow of a fluid, $u = u(y)$, with streamlines (in the x-direction) based on the mean flow characteristics ($u(y)$ is the mean velocity).

We may conceptualize this flow, by considering two adjacent layers, ① & ②, as if they were two trains traveling along parallel tracks at slightly different velocities, u_1 and u_2, where

$$u_2 - u_1 \simeq \frac{\partial u}{\partial y} \Delta y$$

The common boundary between the two trains is a streamline for the mean flow. Thus, there is no mean velocity in the y-direction, i.e.

$$\int_{t_0}^{t_0+\delta t} v(t)\,dt = \bar{v}(t_0) = 0$$

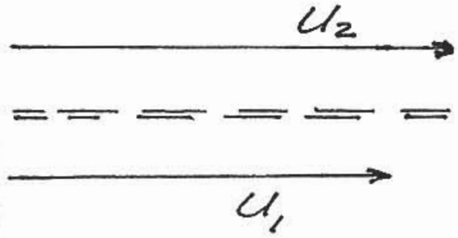


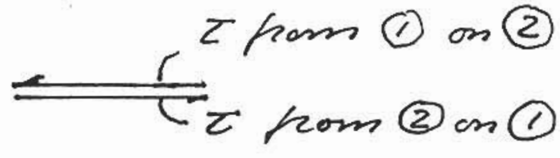
However, on a time scale less than δt $v(t)$ is sometimes positive (fluid goes from ① to ②) and sometimes nega-tive (fluid goes from ② to ①). It is only on the average (over δt) that $v = 0$.

Now, when $v(t) > 0$, a small amount of fluid, δV, passes from ①, with a veloci-ty u_1 in the x-direction, and arrives in ② where the velocity is $u_2 > u_1$. This small amount of fluid, δV, now becomes part of ② and must therefore be accelerated up to speed u_2. To achieve this a positive force must be supplied by train ② on the volume δV that arrived from ①. Conversely, δV arriving from ① exerts a force in the negative x-direction on train ②.

Similarly, when $v(t) < 0$ fluid is transferred from ② to ①, arrives in ① with an excess velocity, $u_2 > u_1$, and exerts a force in the positive x-direction on train ①

Thus, the exchange of fluid between the two trains (on a time scale less than the one we are interested in resolving) acts to produce a force between the two trains such that the slower train holds back the faster train and the faster train tries to speed up the slower one. This interaction is analogous to a shear force acting along the common boundary between the two trains





Per unit area of the common boundary of ① and ② the mean rate of volume transfer from ① to ② [same as from ② to ① since it is a mean streamline is

$$v^+_{1\to 2} = \frac{1}{2}\frac{1}{\delta t}\int_{\delta t} |v(t)|\, dt = v^-_{2\to 1} = v_\tau$$

and we have

$$\underline{\tau} = v_\tau \, \rho (u_2 - u_1) = \rho \underline{(v_\tau \Delta y)} \underline{\frac{\partial u}{\partial y}}$$

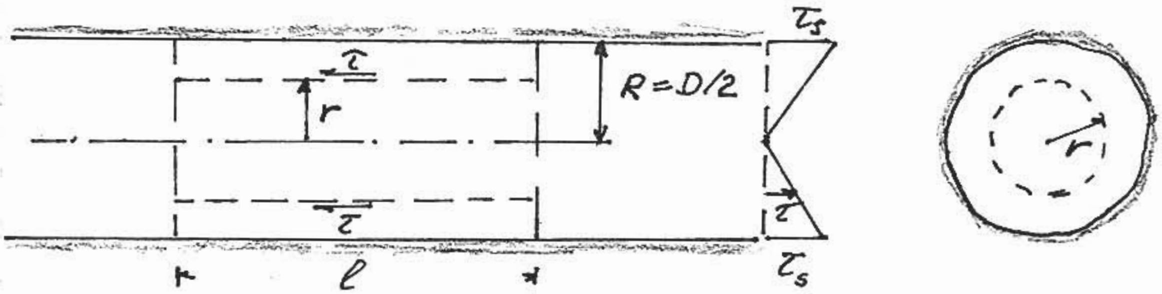
For a laminar flow the exchange of fluid across streamlines is on a molecular scale, i.e.

v_τ and Δy are material properties and $v_\tau \cdot \Delta y = \nu$ = kinematic viscosity

For a turbulent flow the exchange of fluid across ~~mean~~ mean streamlines is associated with the chaotic nature of the flow and far more vigorous than molecular motions, i.e.,

v_τ and Δy are related to the nature of the turbulent flow (<u>not</u> a material property) and $v_\tau \cdot \Delta y = \nu_t$ = turbulent eddy viscosity $\gg \nu$ = kinematic viscosity.

Shear Stress Distribution



Force equilibrium:

$$\pi r^2 \left(-\frac{\partial p}{\partial x}\, \ell\right) = 2\pi r\, \tau\, \ell$$

or

$$\tau = -\frac{r}{2}\frac{\partial p}{\partial x} = \left(-\frac{R}{2}\frac{\partial p}{\partial x}\right)\frac{r}{R} = \tau_s \frac{r}{R}$$

Shear stress varies linearly with r

Note: This is so whether the flow is laminar or turbulent!

Velocity Distribution for Laminar Flow

$$\tau = -\rho \nu \frac{\partial u}{\partial r} \qquad (\text{"}-\text{" since } u \text{ increases in } -r- \text{ direction})$$

$$-\frac{\tau_s}{\rho\nu}\frac{r}{R} = \frac{\partial u}{\partial r} \Rightarrow u = -\frac{\tau_s}{2\rho\nu}\frac{r^2}{R} + C$$

No-slip condition @ $r = R$: $u(R) = 0$

$$u = \frac{\tau_s R}{2\rho\nu}\left(1 - \left(\frac{r}{R}\right)^2\right) = U_0 \left(1 - \left(\frac{r}{R}\right)^2\right)$$

i.e.

u varies parabolically with r, with maximum $U_{max} = U_0$ at center line, $r = 0$

From $u(r)$ we obtain the average velocity

$$V = \frac{Q}{A} = \frac{1}{\pi R^2} \int_0^R u_0 \left(1 - \frac{r^2}{R^2}\right) \underbrace{2\pi r\,dr}_{dA}$$

or

$$\underline{V = \frac{u_0}{2}}$$

Also, we have

$$\tau_s = -\rho\nu \left.\frac{\partial u}{\partial r}\right|_{r=R} = \rho\nu 2 \frac{u_0}{R} = 8\rho\nu\frac{V}{D}$$

and expressing τ_s by the friction factor relationship, we have

$$\tau_s = \frac{1}{8}\rho f V^2 = 8\rho\nu\frac{V}{D}$$

or

$$\underline{f = \frac{64}{Re} \quad ; \quad Re = \text{Reynolds Number} = \frac{VD}{\nu}}$$

is the expression for the Darcy-Weisbach Friction Factor for laminar flow in a circular, smooth pipe, i.e. for $Re < Re_{crit} \sim 2\cdot 10^3$

For turbulent flows, we must resort to experiments in order to obtain the Darcy-Weisbach Friction Factor.