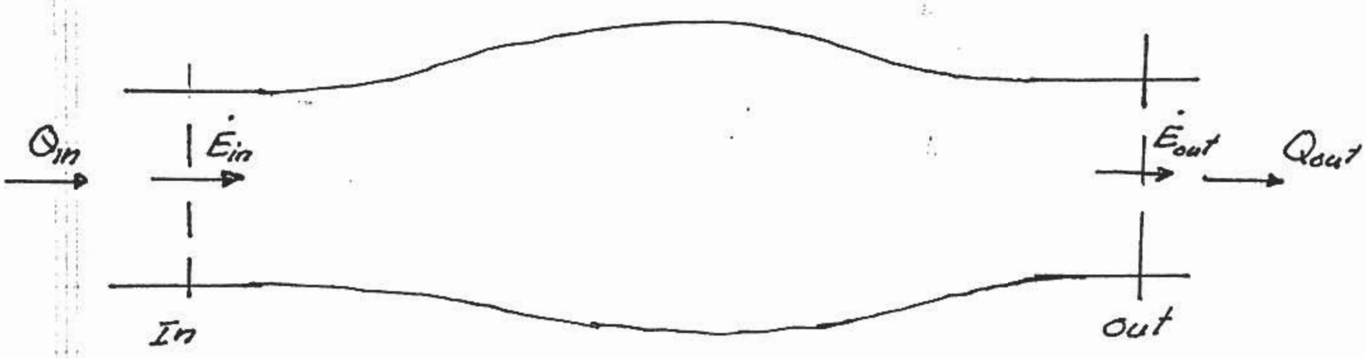
LECTURE # 13

1.060 ENGINEERING MECHANICS II

CONSERVATION OF MECHANICAL ENERGY



$\dot{E}$ = flow rate of mechanical energy across flow area A where flow is well behaved = $\rho g Q H$

$$\left.\begin{array}{l}\sum \dot{E}_{in} - \sum \dot{E}_{out} \\ \sum(\rho g Q_{in} H_{in}) - \sum(\rho g Q_{out} H_{out})\end{array}\right\} = \text{Net rate of Dissipation of Mechanical Energy within CV}$$

Similar to momentum principle: knowledge of in- and outflow conditions quantifies what's going on inside without us knowing the details

If only one inflow & one outflow area, then $Q_{in} = Q_{out}$ from volume conservation (Steady)

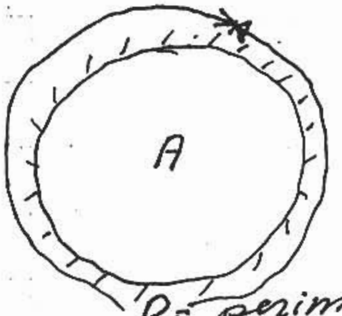
$$\rho g Q H_{in} - \rho g Q H_{out} = \dot{E}_{diss}$$

or

$$H_{in} - H_{out} = \frac{\dot{E}_{diss}}{\rho g Q} = \text{Head Loss} = \Delta H_{1-2}$$

If loss is due to friction along streamtube walls, we have from Momentum (Lecture #12)

$$H_1 - H_2 = H_{in} - H_{out} = \int_{s_1}^{s_2} \frac{\tau_s P}{\rho g A}\, ds = \Delta H_f$$



P = perimeter of solid-fluid contact

$$\tau_s P = \int_P \tau \, dP$$

but if τ_s = constant, as it would be for a circular pipe, and if A is uniform, then τ_s, P, and A are constants – and

$$\Delta H_f = \text{frictional head loss} = \frac{\tau_s P (s_2 - s_1)}{\rho g A}$$

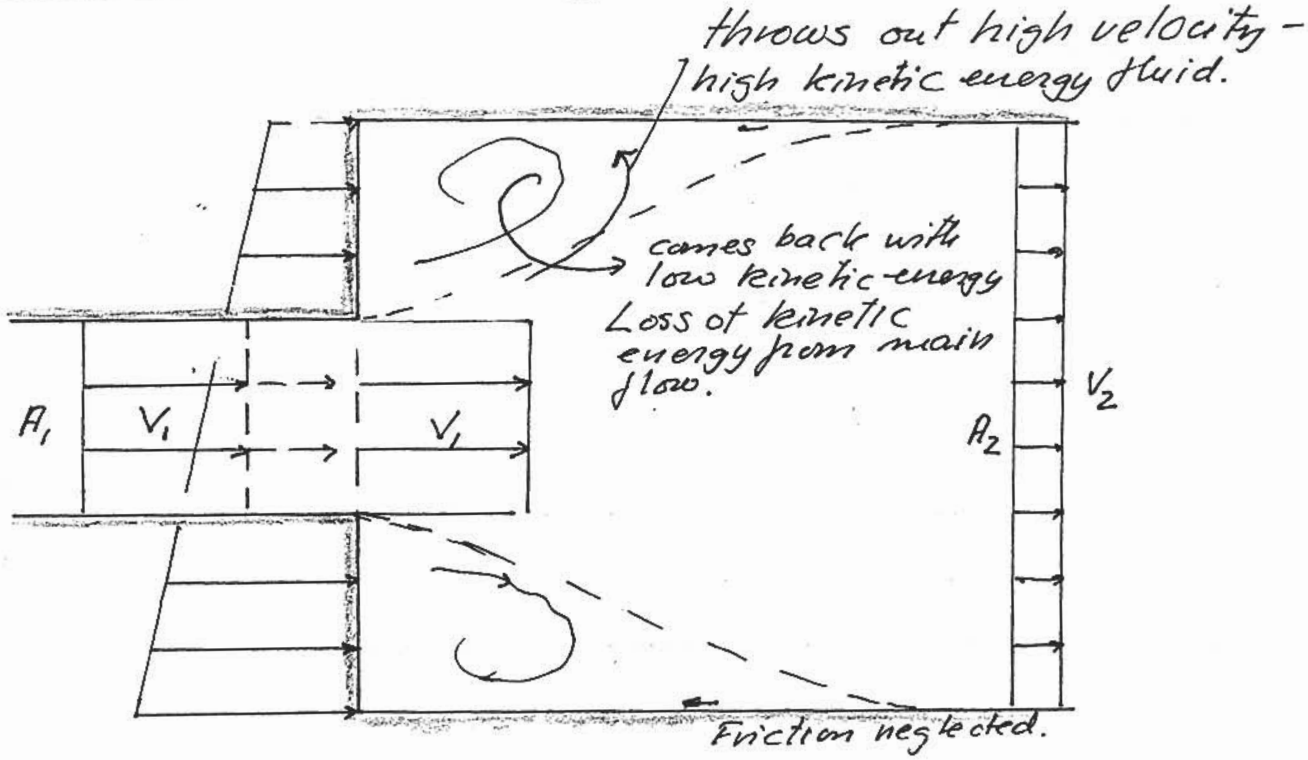
This has a very "nice" physical interpretation.

$\rho g Q \Delta H_f$ = rate of dissipation of Mech. Energy due to friction along walls over a length of pipe $(s_2 - s_1)$ $= \rho g Q \tau_s P (s_2 - s_1) / (\rho g A)$.

but $V = Q/A$, so

$$\dot{E}_{diss,f} = \underbrace{\Big[\underset{\text{shear stress}}{\tau_s}\ \underset{\text{area upon which } \tau_s \text{ acts}}{P(s_2 - s_1)}\Big]}_{\left\{\substack{\text{Total frictional force} \\ \text{resisting the movement}}\right\}} \cdot \underbrace{\underset{\text{average velocity}}{V}}_{\{\text{Velocity}\}} = \text{Rate of Work Done to overcome FRICTION.}$$

EXPANSION HEADLOSS



Continuity : $\quad Q = V_1 A_1 = V_2 A_2$

$$+\rho\, 0^2 (A_2 - A_1)$$

Momentum : $\quad \rho V_1^2 A_1 + p_1[A_1 + (A_2 - A_1)] =$

$$\rho V_1^2 A_1 + p_1 A_2 = \rho V_2^2 A_2 + p_2 A_2 \qquad \text{or}$$

$$(p_1 - p_2)\cancel{A_2} = \rho(V_2 A_2)V_2 - \rho(V_1 A_1)V_1 = \rho V_2 \cancel{A_2}(V_2 - V_1)$$

$$Q = V_1 A_1 = Q$$

$$H_1 - H_2 = \frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + \underbrace{z_1 - z_2}_{0} = \Delta H_{exp}$$

$$\underline{\Delta H_{exp}} = \frac{2V_2(V_2 - V_1)}{2g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{V_2^2 - 2V_2V_1 + V_1^2}{2g} = \underline{\frac{(V_1 - V_2)^2}{2g}}$$

GENERALIZED BERNOULLI EQUATION

$$H = \text{Total Head} = K_e \frac{V^2}{2g} + \frac{p}{\rho g} + z$$

is defined only for flow that is well behaved, i.e.

$$K_e \approx K_m \approx 1$$

$$V = Q/A$$

p = pressure at center of gravity of A

z = elevation of center of gravity of A

$$H_1 - H_2 = \Delta H$$

H_1 = "upstream", H_2 = "downstream" total head

Flow is from ① → ②

$\Delta H \geq 0$ = headloss from ① to ②

$$\Delta H = \Delta H_f + \Delta H_{exp}$$

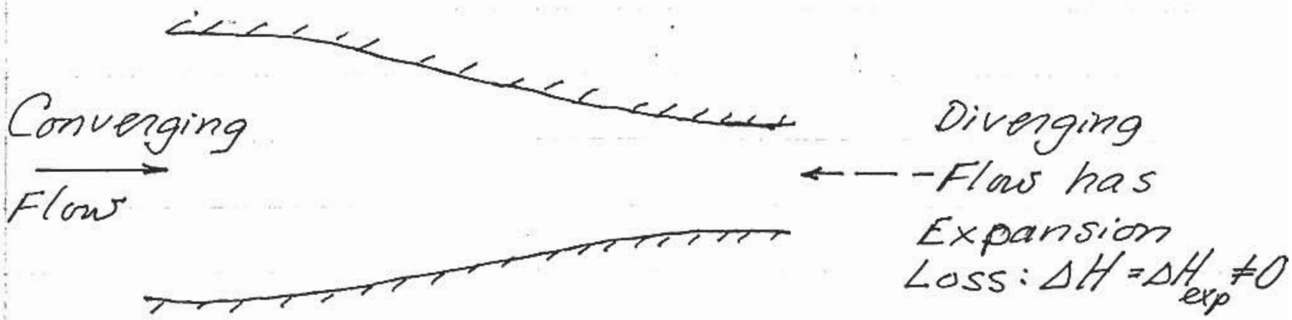
ΔH_f = frictional head loss $= \int_1^2 \frac{\tau_s P}{\rho g A} ds$

ΔH_{exp} = expansion head loss $= \frac{(\Delta V)^2}{2g}$

When does $\Delta H \cong 0$, i.e. $H_1 = H_2$ Work?

"Short" Transition of "Converging" Flow

"Converging" flow means that the velocity in the direction of flow is increasing.



$\Delta H \approx 0$ if the convergence (transition) is "short". "Short" means that frictional headloss can be considered negligible.

$$\Delta H_f \approx \frac{f}{4}\left(\frac{P\ell_{12}}{A}\right)\frac{V^2}{2g} \ll \frac{V^2}{2g} \quad \text{then } \Delta H_f \approx 0$$

With $f \approx 0.02$ and "circular" crossections this translates to an approximate condition of

ℓ_{12} = length of streamline ≈ length of transition $\ll$ 50 D

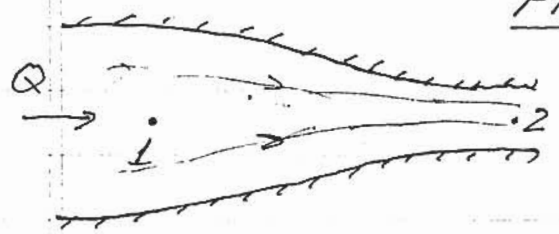
where D is linear dimension of the flow area.

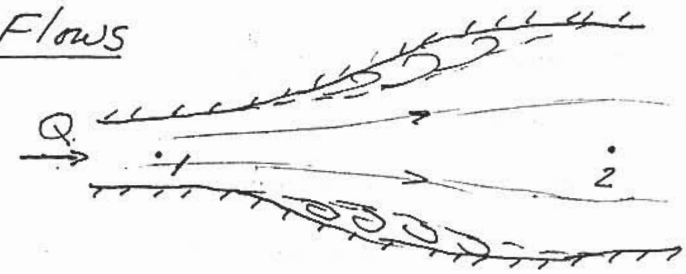
SOME EXAMPLES

$H_1 = H_2$ OK | $H_1 \neq H_2$

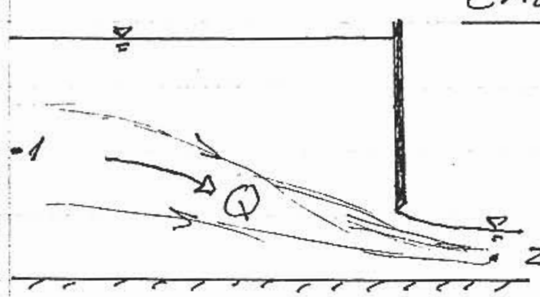
Converging Flows | Diverging Flows

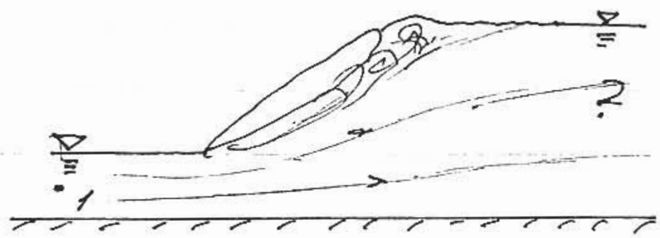
Pipe Flows





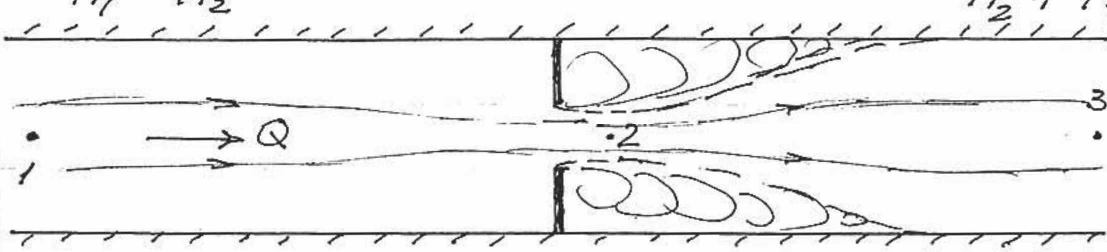
Channel Controls





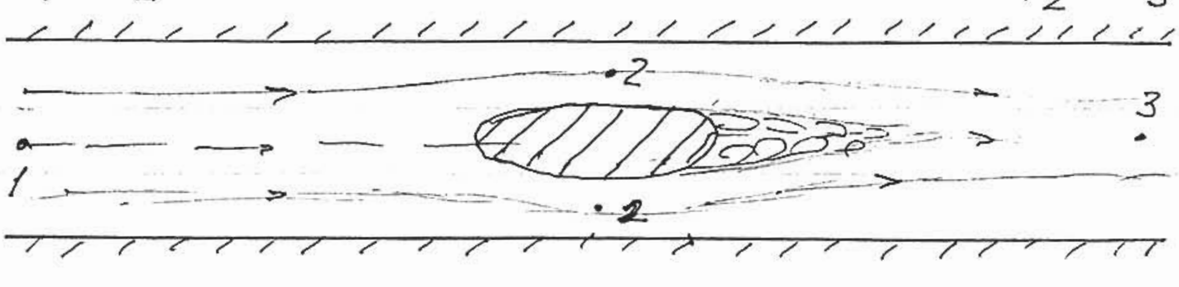
Orifice Meter in Pipe

$H_1 = H_2$ | $H_2 \neq H_3$



Flow around Body

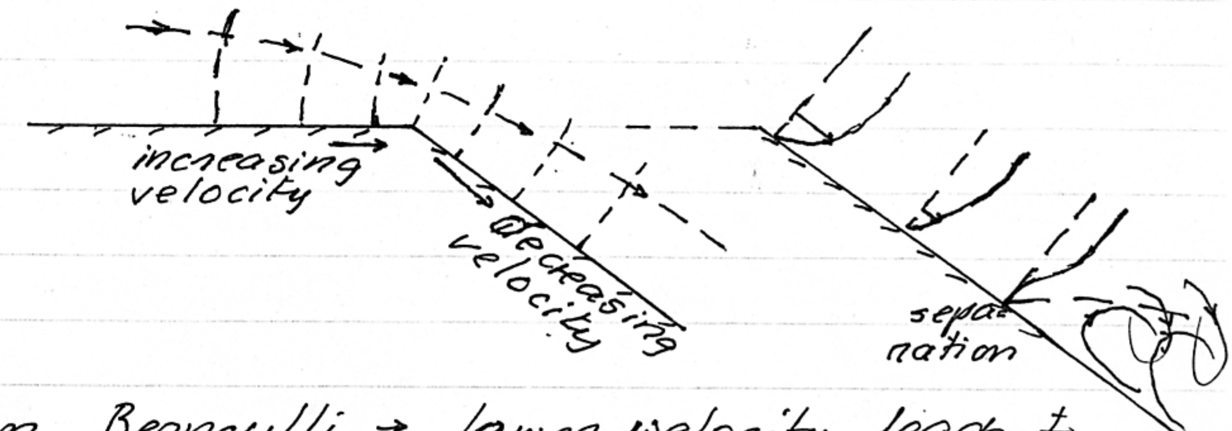
$H_1 = H_2$ | $H_2 \neq H_3$



When does $\Delta H \cong 0$ NOT Work?

"Expanding" Flow, even if Transition is "Short"

"Expanding" flow mean that the velocity in the direction of flow is decreasing.



From Bernoulli → lower velocity leads to higher pressure!

As fluid particle is moving "against" a pressure gradient that slows it down the particle will be stopped and turned around unless it had enough momentum when it encountered the "adverse" pressure gradient. For a real fluid the velocity near a solid boundary is very low (no-slip condition) and its momentum is therefore low and it does not take much to turn it around: The flow separates from the boundary creating an eddy of low velocity swirling fluid that extracts energy from the main flow and causes a

HEADLOSS