MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

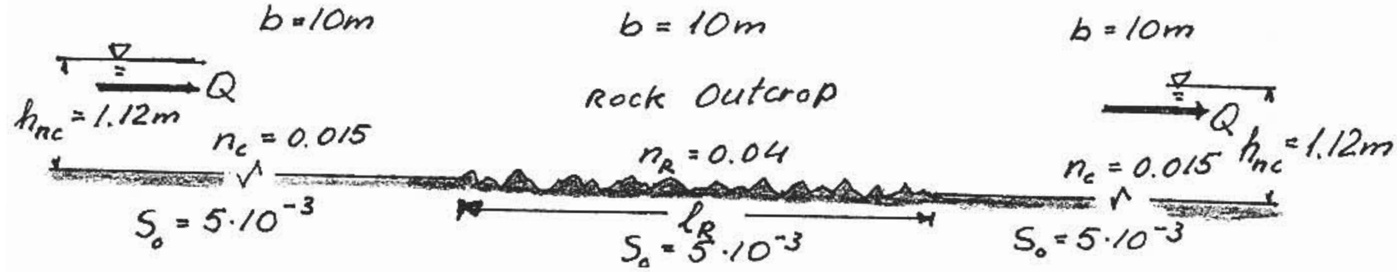
1.060 Engineering Mechanics II

In-Class Examination 12 May, 2006

The test consists of 6 questions related to one problem. To enable you to continue from one question to the next, default values are provided.

You must use default values in subsequent questions if your answers differ from default values by more than 10%.

The Problem



A very long rectangular channel of width $b = 10$ m carries a steady flow of water and has a constant slope $S_o = 5 \cdot 10^{-3}$. Near its midpoint, the channel cuts through a rock outcrop of length l_R. Upstream and downstream of the rock outcrop the channel walls are lined with unfinished concrete (Manning's $n = n_c = 0.015$ in SI-units). For the stretch cutting through the rock outcrop the channel is constructed by blasting the rock and the channel walls are left unfinished corresponding to a Manning's $n = n_R = 0.04$ (SI). The length of the blasted section of the channel, l_R, is sufficiently long to reach normal depths in this section.

Question No: 1 (10)

In the concrete-lined channel, far upstream (and downstream) of the rock outcrop, the depth of flow is measured to be $h_{nc} = 44" = 1.12$ m. Determine the discharge, Q, in the channel, show that the concrete-lined channel slope is *steep*, and find the average boundary shear stress acting on the concrete-lined walls, τ_{sc}, corresponding to the depth h_{nc}.

Regardless of your answer in Question No: 1 take **$Q = 50$ m^3/s**, *when answering the following questions, and remember to use default values if your answers differ from these by more than 10%.*

Question No: 2 (15)

Determine the normal depth, h_{nR}, velocity, V_{nR}, and Froude Number, Fr_{nR}, for the blasted rock channel section. (Default values: $h_{nR} = 2.1$ m; $V_{nR} = 2.4$ m/s; $Fr_{nR} = 0.53$)

Question No: 3 (10)

Determine critical depth in the concrete and blasted rock channel sections, h_{cc} and h_{cR}, respectively. (Default values: $h_{cc} = h_{cR} = 1.4$ m)

Question No: 4 (15)

Determine the conjugate and alternate depths, $h_{nR,,conj}$ and $h_{nR,alt}$, respectively, to normal depth, h_{nR}, in the blasted rock channel section. (Default values: $h_{nR,\ conj} = 0.8$ m; $h_{nR,alt} = 0.9$ m)

Question No: 5 (40)

Sketch the surface profile from far upstream to far downstream of the rock outcrop. Identify all gradually varied flow profiles, the depths at beginning and end of the blasted rock channel section, and the depths upstream and downstream of hydraulic jumps (if present).

Question No: 6 (10)

Estimate the necessary length of the blasted rock section, l_R,. in order to assure that normal depth, h_{nR}, is reached in this channel.

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SOLUTIONS

Question No: 1

Manning's equation for normal (steady, uniform) flow (since "far" upstream/downstream of outcrop)

$$Q = V_{nc} A_{nc} = \frac{1}{n_c} R_h^{2/3} \sqrt{S_o} A_{nc} = \frac{\sqrt{S_o}}{n_c} \frac{(bh_{nc})^{5/3}}{(b+2h_{nc})^{2/3}} = \underline{\underline{49.8 \, m^3/s}}$$

$$V_{nc} = \frac{Q}{bh_{nc}} = 4.45 \frac{m}{s} \Rightarrow \underline{Fr_{nc}} = \frac{V_{nc}}{\sqrt{g h_{nc}}} = \underline{1.36 > 1} : \underline{\underline{\text{Slope is STEEP}}}$$

Balance of gravity and boundary shear stress gives:

$$\rho g A_{nc} S_o = \tau_s P_{nc} \Rightarrow \underline{\underline{\tau_s}} = \rho g \frac{A_{nc}}{P_{nc}} S_o = \rho g \frac{b h_{nc}}{b+2h_{nc}} S_o = \underline{\underline{44.8 \frac{N}{m^2}}}$$

Question No: 2

$$Q/b = q = Vh = \frac{1}{n} R_h^{2/3} \sqrt{S_o}\, h = \frac{1}{n}\left(\frac{bh}{b+2h}\right)^{2/3} \sqrt{S_o}\, h$$

$$q = \frac{\sqrt{S_o}\, h^{2/3}}{n(1+2h/b)^{2/3}} h \Rightarrow \underline{h_{nR} = \left(\frac{q n_R}{\sqrt{S_o}}\right)^{0.6}\left(1+\frac{2h_{nR}}{b}\right)^{0.4}}$$

$$q = Q/b = 50/10 = 5 \frac{m^2}{s}; \quad n = n_R = 0.04; \quad S_o = 5 \cdot 10^{-3}$$

$$h_{nR} = 1.866\,(1+0.2\,h_{nR})^{0.4} \Rightarrow h_{nR}^{(0)} = 0 \Rightarrow h_{nR}^{(1)} = 1.87m \Rightarrow h_{nR}^{(2)} = 2.12m \Rightarrow$$

$$\underline{h_{nR}^{(3)} = 2.15m = h_{nR}^{(4)}} \Rightarrow \underline{h_{nR} = 2.15 \, m}$$

$$\underline{\underline{V_{nR}}} = q/h_{nR} = 5/2.15 = \underline{\underline{2.33 \, m/s}}$$

$$\underline{\underline{Fr_{nR}}} = \frac{V_{nR}}{\sqrt{g h_{nR}}} = \frac{2.33}{\sqrt{9.8 \cdot 2.15}} = \underline{\underline{0.51}}$$

Question No: 3

Critical depth corresponds to $Fr = \frac{V_c}{\sqrt{g h_c}} = 1$

or $V_c = \sqrt{g h_c} \Rightarrow q = V_c h_c = \sqrt{g h_c}\, h_c = \sqrt{g}\, h_c^{3/2}$

$$\underline{h_c} = \left(q/\sqrt{g}\right)^{2/3} = \left(5/\sqrt{9.8}\right)^{2/3} = \underline{1.37m}$$

$Fr = 1$ does not depend on n-value: $\underline{h_{cc} = h_{cR} = h_c = 1.37m}$

Notice: $h_{nc} = 1.12m < h_c = 1.37m < h_{nR} = 2.15m$

in agreement with $Fr_{nc} > 1$ [concrete channel is "steep"] and $Fr_{nR} < 1$ [blasted rock channel is "mild"]

Question No: 4

Conjugate depth is when $MP_1 = MP_2$. Cheat Sheet gives, since $h_{nR} > h_c$ is subcritical:

$$h_1 = \underline{h_{nR,conj}} = \frac{h_{nR}}{2}\left(-1 + \sqrt{1 + 8 Fr_{nR}^2}\right) = \frac{2.15}{2}\left(-1 + \sqrt{1 + 8(0.51)^2}\right) = \underline{0.81m}$$

(Much simpler than solving $\left(\rho\left(\frac{q}{h_1}\right)^2 + \frac{1}{2}\rho g h_1\right)h_1 = \left(\rho V_{nR}^2 + \frac{1}{2}\rho g h_{nR}\right)h_{nR}$ by iteration)

Alternate depth is when $E_1 = E_2 = E_{nR}$

$$E_1 = h_1 + \frac{V_1^2}{2g} = h_1 + \frac{q^2}{2 g h_1^2} = h_1 + \frac{1.28}{h_1^2} = h_{nR} + \frac{V_{nR}^2}{2g} = 2.42$$

Since we are looking for the supercritical solution (h_{nR} is subcritical) we write this as

$$h_1 = \left(\frac{1.28}{2.42 - h_1}\right)^{1/2}$$ and iterate: $h_1^{(0)} = 0 \Rightarrow h_1^{(1)} = 0.73m \Rightarrow h_1^{(2)} = 0.87m;$

$h_1^{(3)} = 0.91m \Rightarrow h_1^{(4)} = \underline{0.92m \Rightarrow h_1^{(5)} = 0.92m = h_1 = h_{nR,alt}}$

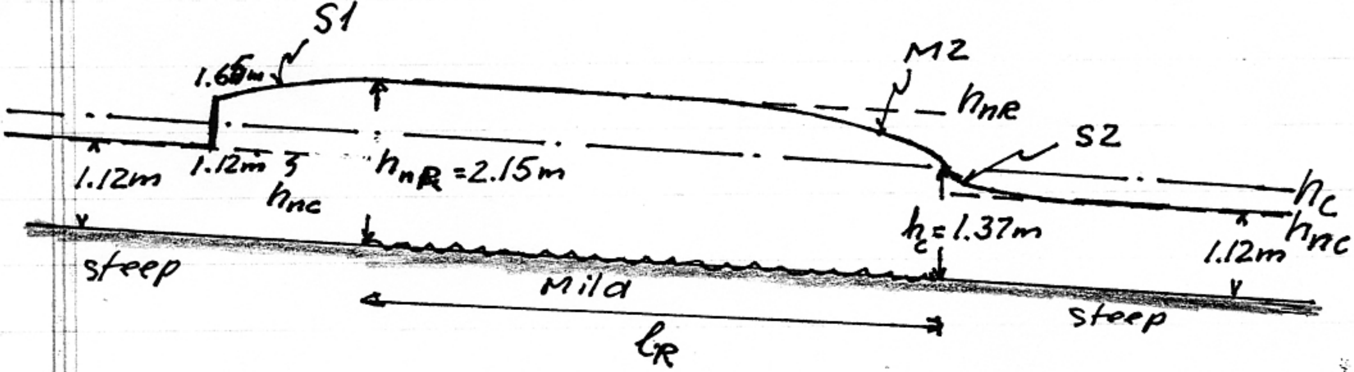
Question No:5

Concrete Channel has steep slope: $Fr_{nc} > 1$, $h_{nc} < h_c$
Blasted Rock channel has mild slope: $Fr_{nR} < 1$, $h_{nR} > h_c$
Flow goes from supercritical (upstream) to subcritical (in rock channel): There must be a hydraulic jump somewhere!
Flow goes from subcritical in blasted rock channel to supercritical in downstream concrete channel. This can be done! A draw-down M2-curve in rock channel hits critical depth at downstream transition to concrete channel and continues as an S2-curve until $h_{nc} < h_c$ is reached.
If hydraulic jump in rock channel it must be from $h_{nR,conj}$ to h_{nR}. But $h_{nR,conj} = 0.81\,m < h_{nc} = 1.12\,m$ = normal depth in concrete channel. $h_{nc} < h_c$ and profile must be M3 ($h_{nc} < h_c < h_{nR}$) if continuing into rock channel. But M3-curve has $dh/dx > 0$, i.e. we can't get down to $h_{nR,conj}$. Only way to go, is to have jump in upstream concrete channel from h_{nc} to $h_{nc,conj}$, where (Cheat Sheet)

$$\underline{h_{nc,conj}} = \frac{h_{nc}}{2}\left(-1+\sqrt{1+8Fr_{nc}^2}\right) = \frac{1.12}{2}\left(-1+\sqrt{1+8(1.35)^2}\right) = \underline{1.65\,m}$$

$(Fr_{nc} = V_{nc}/\sqrt{gh_{nc}} = (q/h_{nc})/\sqrt{gh_{nc}} = (5/1.12)/\sqrt{9.8\cdot 1.12} = 1.35$
and a S1 (backwater) curve connecting $h_{nc,conj}$ and h_{nR}, which must be "hit" right at the entrance to the blasted rock channel since $Fr_{nR} < 1$, i.e. rock channel has "mild" slope.

Sketch of Gradually Varied Flow Profile



Question No: 6

$$\frac{dh}{dx} \simeq \frac{h_c - h_{nR}}{l_{Rmin}} = \frac{S_o - \overline{S_f}}{1 - \overline{Fr}^2}$$

$$\overline{Fr}^2 = \left(Fr_{nR}^2 + Fr_c^2\right)/2 = \left(0.51^2 + 1^2\right)/2 = 0.63$$

$$\overline{S_f} = \left(S_{fnR} + S_{fc}\right)/2 = \left(S_o + \frac{n_R^2 q^2}{h_c^{10/3}/(1+\frac{2h_c}{b})^{4/3}}\right)/2 =$$

$$S_o\left(1 + \left(\frac{h_{nR}}{h_c}\right)^{10/3}\left(\frac{1+2h_c/b}{1+2h_{nR}/b}\right)^{4/3}\right)/2 = 2.43\,S_o$$

$$\frac{h_c - h_{nR}}{l_{Rmin}} \simeq \frac{S_o - \overline{S_f}}{1 - \overline{Fr}^2} = \frac{(1-2.43)S_o}{1-0.63} = -3.86\,S_o$$

or

$$l_{Rmin} \simeq \frac{h_{nR} - h_c}{3.86 \cdot S_o} = \frac{2.15 - 1.37}{3.86 \cdot 5 \cdot 10^{-3}} = 40m \;!$$

If $l_R > l_{Rmin}$ it would be possible for the M2-curve to reach normal depth before the transition to the upstream concrete channel. Say: $\underline{\underline{l_R > \sim 100m}}$, to be on the safe side