MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

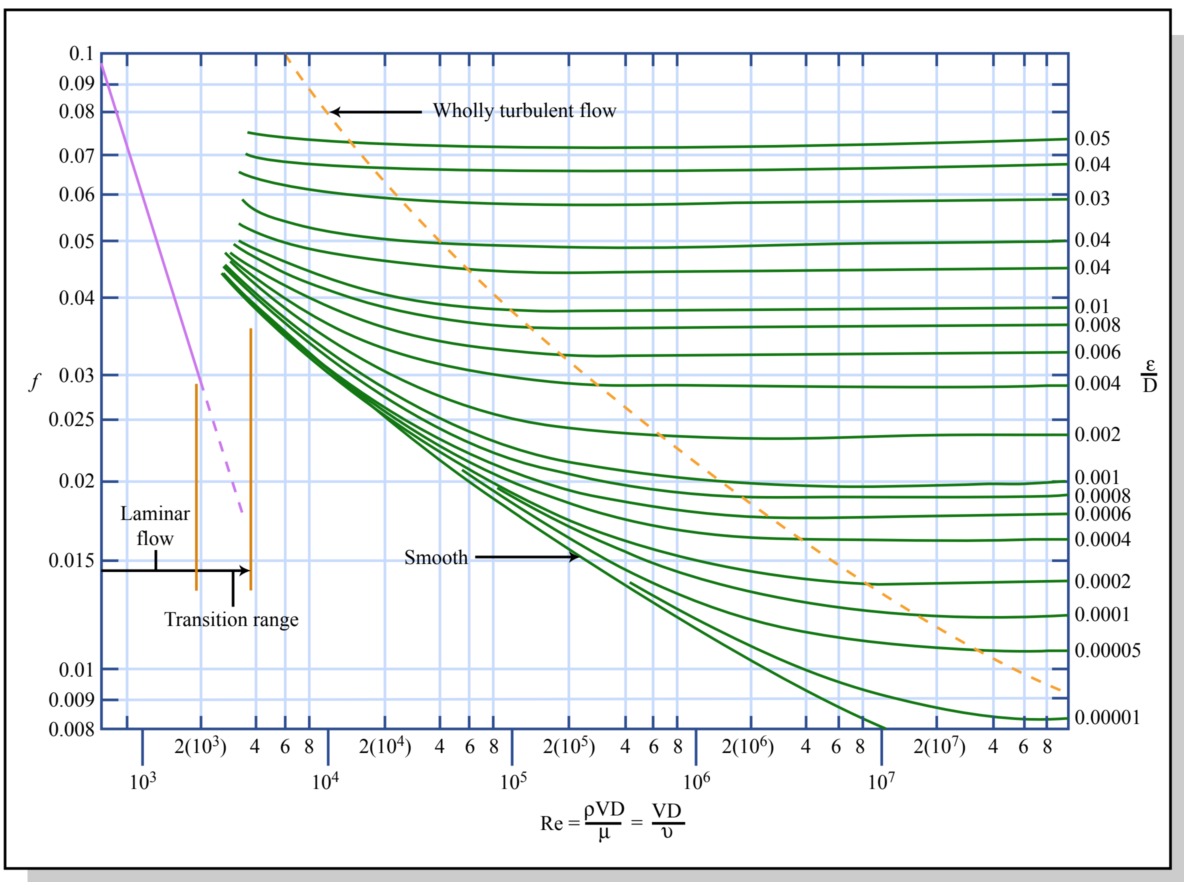
1.060 Engineering Mechanics II

In-Class Examination 14 April, 2006

General Comment

The test is concerned with various aspects of a single problem, and consists of questions requiring you to have solved previous questions. For this reason "default" answers are given so that you can proceed. The "default" answers are not necessarily the correct solutions (but they may be close) so continue to use your own solutions unless they differ from the default values by more than 10%. In answering some of the questions you may find the figure below helpful.

GOOD LUCK!

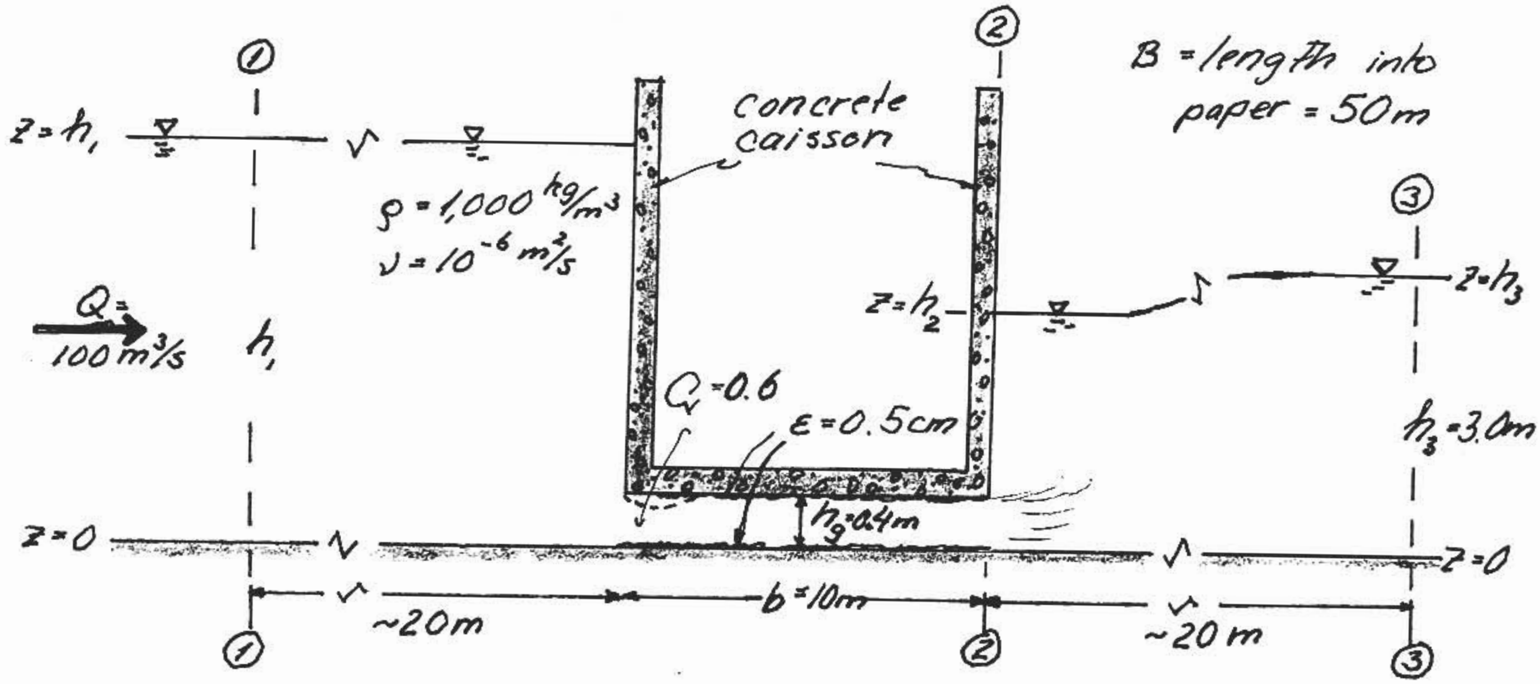


Graph by MIT OCW.

Generalized Moody Diagram: $D = 4R_h = 4(A/P)$

General Problem Description

A 10-m-wide concrete caisson is being transported down a river by tug boats when the mooring lines break. The caisson floats freely down the river and comes to rest against the bank abutments of a bridge thereby creating an obstruction to the natural flow in the river. After a transitional period, during which the river flow adjusts to the presence of the caisson, steady flow conditions are achieved. The sketch below (not to scale) shows the steady state flow scenario in the vicinity of the flow obstruction formed by the caisson.



The caisson spans the entire width of the river and leaves a gap of uniform height $h_g = 0.4$ m, length = b = width of caisson = 10 m, and width (in direction into the paper) of B = width of the river = 50 m between the caisson and the river bottom. Over the distance covered by the sketch the river bottom may be assumed horizontal at $z = 0$. Thus, the discharge in the river, $Q = 100$ m^3/s, must pass under the caisson, i.e. through a closed conduit of "very" rectangular cross-section, $A_g = h_g B$, and length = b = 10 m.

Due to the flow obstruction created by the caisson the water ($\rho = 1{,}000$ kg/m^3 and $\nu = 10^{-6}$ m^2/s) is backed up to a depth h_1 a <u>short</u> distance (~20 m) upstream of the caisson. The flow enters the gap between the caisson and the river bottom through a contraction (caisson has sharp corners and the contraction coefficient is $C_v = 0.6$), and shoots out from under the caisson at section 2-2, where the depth, measured along the downstream sidewall of the caisson, is h_2. Following exit from under the caisson the flow expands and forms a uniform flow of depth $h_3 = 3.0$ m, a relatively <u>short</u> distance (~20 m) downstream of the caisson.

Question No: 1 (10%)

Determine the velocity in the gap below the caisson, $V_{g,}$, and show that the velocity heads of the flows upstream and downstream of the caisson where the depths are $h_1 > h_3 = 3.0$ m, are negligibly small compared to the velocity head of the flow in the gap between caisson and river bottom (Default values $V_g = 5$ m/s, $V_3 = 0.7$ m/s $> V_1$)

Question No: 2 (22%)

Consider the flow expansion that takes place after the flow exits from under the caisson (at 2-2) to the uniform river flow achieved a short distance (~20 m) downstream of the caisson, where the depth is $h_3 = 3.0$ m, and determine the depth h_2 along the downstream sidewall of the caisson. (Default value: $h_2 = 2.7$ m).

Question No: 3 (33%)

Determine the depth h_l a short distance (~20 m) upstream of the caisson required to drive the discharge Q under the caisson. Assume (i) the depth at the outflow from under the caisson to be $h_2 = 2.7$ m (regardless of the value you obtained in Q #2); (ii) the caisson and the river bottoms to have the same roughness, $\varepsilon = 0.5$ cm; (iii) the caisson to have sharp corners so that $C_v = 0.6$; and (iv) the velocity head at 1-1 is negligibly small, as was shown in Q #1). (Default value $h_1 = 5$ m)

Question No: 4) (10%)

Determine the total headloss, $\Delta H_{1\text{-}3}$, caused by the flow obstruction created by the caisson, and the portion of this headloss contributed by the flow expansion from 2-2 to 3-3, $\Delta H_{2\text{-}3}$. (Default values $\Delta H_{1\text{-}3} = 2$ m, $\Delta H_{2\text{-}3} = 1$ m)

Question No: 5 (15%)

Sketch the EGL and HGL for the flow in the closed conduit of length = b = 10m formed by the gap between the caisson and the river bottom.

Question No: 6 (10%)

Determine the total horizontal force acting on the fluid from the surrounding boundaries between 1-1 and 3-3. Is this horizontal force equal to the horizontal force acting from the fluid on the caisson?

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SOLUTIONS

Question No: 1

Steady flow ⇒ Conservation of volume ⇒ $Q = VA = \text{Const.}$

$Q = 100\ m^3/s$; $A_g = h_g \cdot B = 0.4 \cdot 50 = 20 m^2$: $\underline{V_g = Q/A_g = 5.0 \frac{m}{s}}$

$V_3 = Q/A_3 = Q/(h_3 \cdot B) = 100/(3 \cdot 50) = 0.67 m/s$

$V_1 = Q/A_1 < V_3$ since $A_1 = h_1 B > A_3 = h_3 B$.

$V_3^2/2g$ = vel. head @ 3 $= (0.67)^2/2 \cdot 9.8 = 0.02 m \ll V_g^2/2g = 1.28\ m$.

$\underline{V_1^2/2g} < \underline{V_3^2/2g} \approx 1.5\%$ of $V_g^2/2g$: Negligible

Question No: 2

Steady flow – short transition (no friction force/loss) – expanding flow (expansion loss ΔH) ⇒ Momentum & Volume conservation must be used(!) between 2-2 and 3-3 :

$$\overbrace{\tfrac{1}{2}\rho g h_2^2 + \underbrace{\rho V_g^2 h_g}_{= \rho Q \cdot V_g / B}}^{MP_2} = \overbrace{\tfrac{1}{2}\rho g h_3^2 + \underbrace{\overbrace{\rho V_3^2 h_3}^{= \rho Q V_3/B}}_{V_3^2 \ll V_g^2 \text{ from Q\#1 so could neglect this}}}^{MP_3}$$

$$\underline{h_2} = \sqrt{h_3^2 - 2(Q/B)(V_g - V_3)/g} =$$

$$\sqrt{3^2 - \frac{1}{9.8} 2\left(\frac{100}{50}\right)(5.0 - 0.67)} = \sqrt{9 - (2.04 - \underset{\text{from } V_3^2}{0.27})} = \underline{2.69 m}$$

If ρV_3^2 (M_3) is neglected : $\underline{h_2 = 2.64 m}$

Note: Approximate same answer if

$$H_2 = h_2 + V_g^2/2g \cong H_3 + \Delta H_{23} = h_3 + V_3^2/2g + \approx \frac{(V_g - V_3)^2}{2g} \left[\text{See comment Prob.\#4}\right]$$

Question NO:3

There will be pressure forces on fluid between 1-1 and 2-2 from the upstream sidewall of the caisson ⇒ we can not assume hydrostatic pressure ⇒ we don't know forces acting on fluid ⇒ Bernoulli and volume conservation must be used between 1-1 & 2-2.

$$H_1 = H_2 + \sum \Delta H_{1\to2} = H_2 + \Delta H_f + \Delta H_m$$

$$H_1 = \frac{V_1^2}{2g} + \frac{p_{CG,1}}{\rho g} + z_{CG,1} = \frac{V_1^2}{2g} + \frac{p_{CG,1} + \rho g z_{CG,1}}{\rho g}$$

but $p + \rho g z$ = constant at 1-1 (well behaved flow) = $\rho g h_1$, so

$\underline{H_1 = h_1 + V_1^2/2g}$ ($V_1^2/2g$ negligible from Q #1, so it can be dropped)

$$\underline{H_2} = \frac{V_g^2}{2g} + \frac{p_{CG,2}}{\rho g} + z_{CG,2} = \underline{\frac{V_g^2}{2g} + h_2}$$

$z_{CG,2} = h_g/2 = 0.2\,m$; $p_{CG,2}$ = pressure in receiving fluid = $\rho g (h_2 - z_{CG,2})$

$$\Delta H_f = f\left(\frac{l_p}{4R_h}\right)\frac{V_g^2}{2g} = f\frac{b}{4R_h}\frac{V_g^2}{2g} \qquad (b = 10m)$$

$$R_h = \frac{A_g}{P_g} = \underbrace{\frac{h_g \cdot B}{2B + 2h_g}}_{\text{both top \& bottom of conduit carries friction}} \simeq \frac{0.4 \cdot 50}{2 \cdot 50 + \underbrace{2 \cdot 0.4}_{\approx 0}} = 0.2\,m \;\left(= \frac{h_g}{2}\right)$$

$$\underline{4R_h} = \underline{\text{"}D\text{"}} = 2h_g = \underline{0.8m}$$

$$Re_g = \frac{V_g(4R_h)}{\nu} = \frac{5.0 \cdot 0.8}{10^{-6}} = 4 \cdot 10^{+6}; \quad \frac{\varepsilon}{4R_h} = \frac{0.5 \cdot 10^{-2}\,m}{0.8\,m} = 6.2 \cdot 10^{-3}$$

$\underline{\text{MOODY DIAGRAM}}$: $f = 0.032$ [Note: Not always 0.02!]

$$\underline{\Delta H_f} = f \cdot \frac{b}{4R_h} \frac{V_g^2}{2g} = 0.032 \cdot \frac{10}{0.8} \frac{V_g^2}{2g} = \underline{0.4 \frac{V_g^2}{2g}}$$

$$\underline{\Delta H_m} = K_{L,ent} \cdot \frac{V_g^2}{2g} = \left(\frac{1}{C_v} - 1\right)^2 \frac{V_g^2}{2g} = \underline{0.44 \frac{V_g^2}{2g}}$$

$$h_1 = h_2 + (\underset{\substack{\uparrow \\ \text{vel head} \\ \text{at 2}}}{1} + \underset{\substack{\uparrow \\ \text{friction}}}{0.4} + \underset{\substack{\uparrow \\ \text{expansion}}}{0.44}) \frac{V_g^2}{2g} - \frac{V_1^2}{2g} = h_2 + 1.84 \frac{V_g^2}{2g} - \frac{V_1^2}{2g}$$

$V_1^2/2g \ll 1.84\, V_g^2/2g$ (from Q #1) so drop it

$$\underline{h_1} = 2.7 + 1.84 \frac{5^2}{2 \cdot 9.8} = \underline{\underline{5.05\ m}}$$

(Check: $V_1^2/2g = (Q/Bh_1)^2/2g = \left(\frac{100}{50 \cdot 5.05}\right)^2/2 \cdot 9.8 < 0.01\,m \approx \text{"}0\text{"}$)

Question No: 4

Headloss: Obviously Bernoulli is to be used

$$H_1 = h_1 + \frac{V_1^2}{2g} = H_3 + \Delta H = h_3 + \frac{V_3^2}{2g} + \Delta H_{1-3}$$

$$\underline{\Delta H_{1-3}} = h_1 - h_3 - \frac{V_3^2 - V_1^2}{2g} = 5.05 - 3.0 - \underbrace{(0.02 - 0.01)}_{\approx 0} = \underline{2.05 m}$$

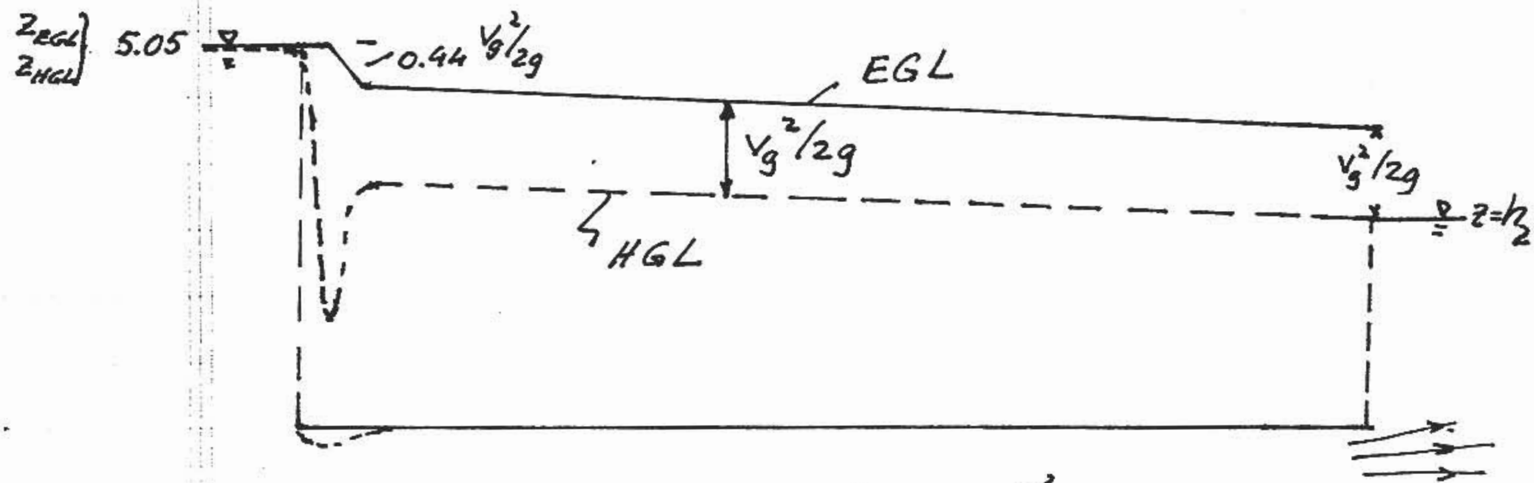
Bernoulli from 2-2 to 3-3 gives

$$H_2 = h_2 + \frac{V_g^2}{2g} = H_3 + \Delta H_{2-3} = h_3 + \frac{V_3^2}{2g} + \Delta H_{2-3}$$

$$\underline{\Delta H_{2-3}} = h_2 - h_3 + \frac{V_g^2}{2g} - \underbrace{\frac{V_1^2}{2g}}_{0 \text{ from Q\#1}} = 2.7 - 3.0 + \frac{5^2}{2 \cdot 9.8} = \underline{0.98 m}$$

[Note: If using simple formula for expansion loss $\Delta H_{exp} = (V_g - V_3)^2/2g = (5 - 0.67)^2/2g = 0.96\ m$ it would work here, BUT THIS IS NOT ALWAY SO!]

Question No: 5



$V_1^2/2g = 0.008\,m \approx 0$; $V_g^2/2g = \frac{5^2}{2\cdot 9.8} = 1.28\,m$

$\Delta H_m = 0.44\, V_g^2/2g$; $V_{v.c.}/V_g = 1/0.6 = 1.67$

$V_{v.c.}^2/2g = 2.8\, V_g^2/2g$

Start

Both start at $z = h_1$ (since $V_1^2/2g \approx 0$)

z_{EGL} unchanged up to Vena Contracta (short transition of converging flow

z_{HGL} drops to a distance of $V_{v.c.}^2/2g = 2.8\, V_g^2/2g$ below z_{EGL} at Vena Contracta (V increases, p decreases)

Expansion after Vena Contracta:

z_{EGL} drops by $\Delta H_m = 0.44\, V_g^2/2g$

z_{HGL} rises from low point to be $V_g^2/2g$ below z_{EGL}

Rest of the way

z_{EGL} varies linearly until end, where it is $V_g^2/2g$ above $z = h_2$. Decrease is due to ΔH_f

z_{HGL} varies linearly - is parallel to EGL since $V = V_g$ = constant - and meets the free surface at exit from under caisson.

Question No: 6

Momentum between 1-1 and 3-3

$MP_1 = MP_3 + F_H \Rightarrow F_H$ = horizontal force on fluid

F_H (positive in upstream) $= MP_1 - MP_3 =$

$$\left(\tfrac{1}{2}\rho g h_1^2 + \rho V_1^2 h_1\right)B - \left(\tfrac{1}{2}\rho g h_3^2 + \rho V_3^2 h_3\right)B =$$

$$\left(125 \cdot 10^3 + 0.8 \cdot 10^3\right)B - \left(44 \cdot 10^3 + 1.33 \cdot 10^3\right)B$$

Note: Low contributions of ρV^2 terms to MPs
Reason: Smallness of $V_1^2/2g$ & $V_3^2/2g$ shown in Q#1

$$F_H = 4.02 \cdot 10^6\ N$$

but part of this force comes from the shear stress acting on the river bed under the caisson. With $\tau_s = \frac{1}{8} f \rho V_g^2 = \frac{1}{8} \cdot 0.032 \cdot 1000 \cdot 5^2 = 100\ N/m$ this force is $F_{RB} = \tau_s \cdot b \cdot B = 100 \cdot 10 \cdot 50 = 5 \cdot 10^4\ N$, i.e. about ~ 1% of total force – could be neglected. For completeness:

F_c = Force on Caisson $= F_H - \tau_s bB = 3.97 \cdot 10^6\ N \approx 4\ MN$ acting in the downstream direction.