MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Civil and Environmental Engineering

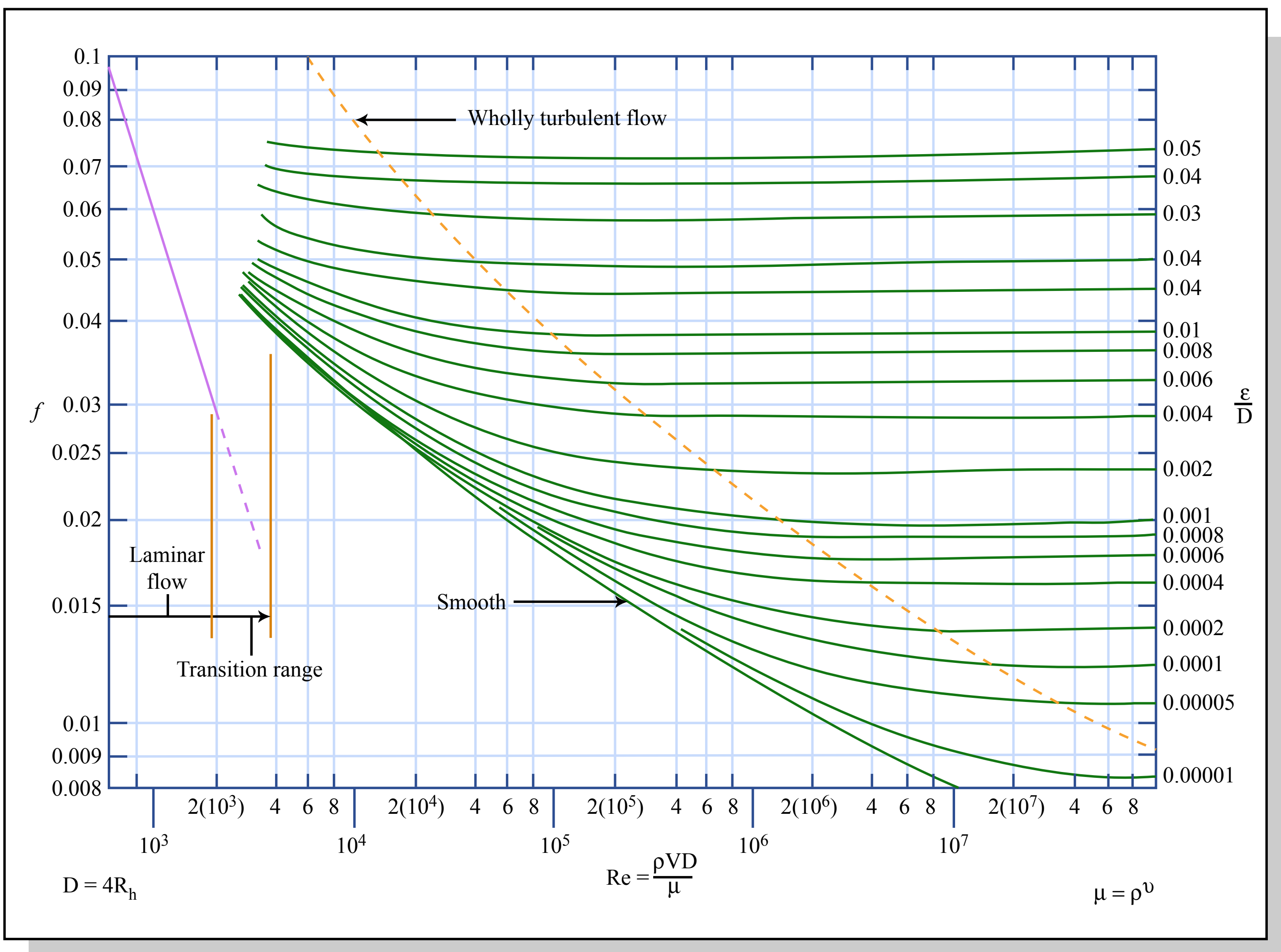
1.060/1.995 Fluid Mechanics

Scheduled Final Examination

Tuesday, May 19, 2005
9 - 12 noon

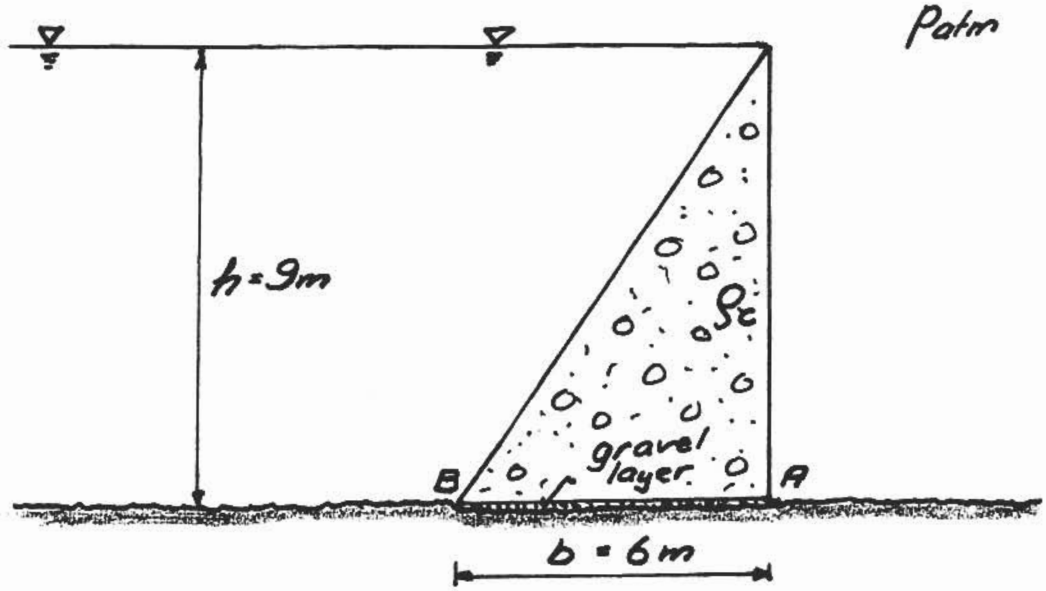
Prof. Ole S. Madsen

1. There are six problems of equal weight. Be sure to allocate an appropriate amount of time for each.
2. Solutions should be expressed in terms of the problem notation and then the numerical results should be obtained.
3. Please indicate clearly, using sketches when necessary, the assumptions and definitions you are introducing in carrying out your analyses. Do not hesitate to make reasonable assumptions, but state the reason why you make them.
4. Please be as neat as possible and clearly indicate what and where your answer is (only one answer!).
5. Unless noted otherwise, the fluid is water: ($\rho = 1{,}000\ kg/m^3, \upsilon = 10^{-6}\ m^2/s$)
6. Cheat sheets #1, 2, and 3 are provided along with the Moody Diagram below.



Graph by MIT OCW.

Problem No. 1



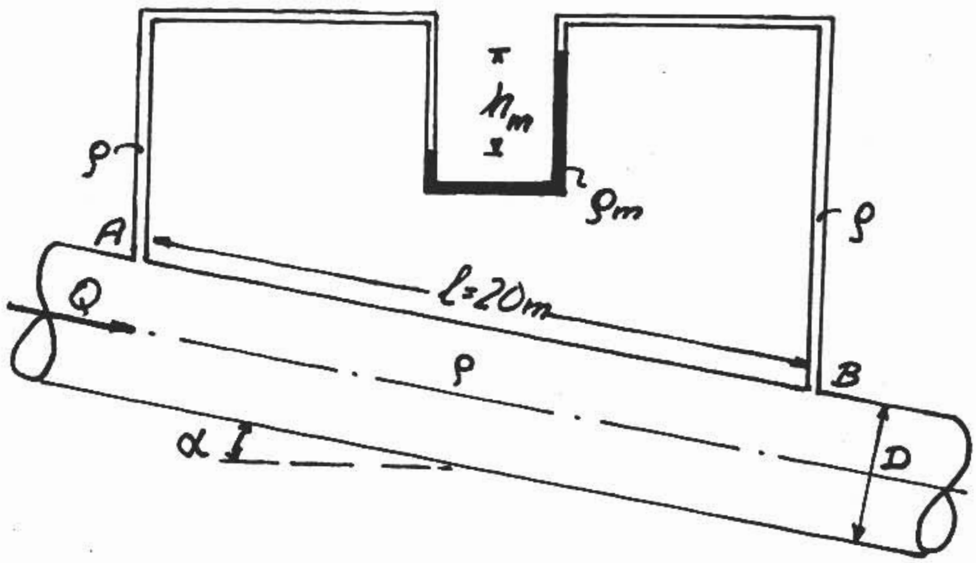
The sketch shows the cross-section of a very long (in direction into the paper) concrete dam. The dam rests on a thin layer of gravel allowing an insignificant amount of water to leak below the bottom of the dam, but causing the water pressure to vary linearly along the bottom of the dam (from B to A). The specific weight of the concrete is 23.6 kN/m^3.

a) Determine the pressure at points A and B.

b) Determine the minimum coefficient of friction (ratio of shear force to normal force) to prevent the dam from sliding.

c) Determine the factor of safety against overturning of the dam (defined as the ratio of stabilizing to overturning moments around the pivot point).

Problem No. 2



The sketch shows a circular pipe of diameter $D = 0.2$m carrying a discharge of water $Q = .053$ m^3/s. The pipe is inclined at angle $\alpha = 1.0°$ (or 0.0175 radians) to the horizontal and is connected to a mercury manometer ($\rho_m = 13.6\rho$) at A and B, with $l = 20$m being the distance from A to B. The manometer reading is given as $h_m = 2.8$cm.

a) Determine the velocity in the pipe.

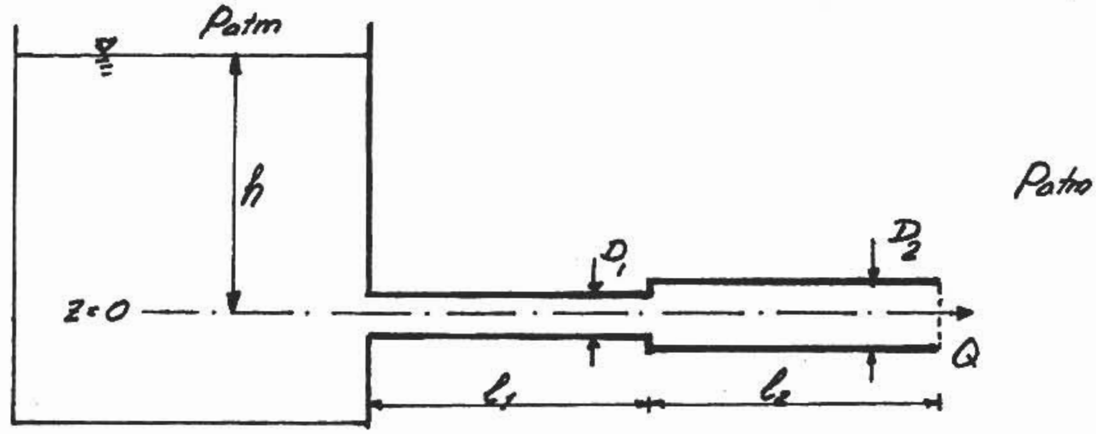
b) Determine the pressure difference between points A and B.

c) Find the magnitude of the shear stress acting between the pipe wall and the fluid (Default value 9Pa).

d) Determine the value of the Darcy-Weisbach friction factor, f.

e) Estimate the pipe roughness, ε.

Problem No. 3



A very large container is filled with water to a level of $z = h$. The container is connected to a horizontal pipe-system consisting of two circular pipes, both of length $l_1 = l_2 = 4.5$m and both of roughness $\varepsilon =$ 0.3mm, one is of diameter $D_1 = 10$cm, the other of diameter $D_2 = 15$ cm. All transitions are sharp-edged (see accompanying sketch) and the discharge is $Q = 3.53 \cdot 10^{-2}$ m^3/s.

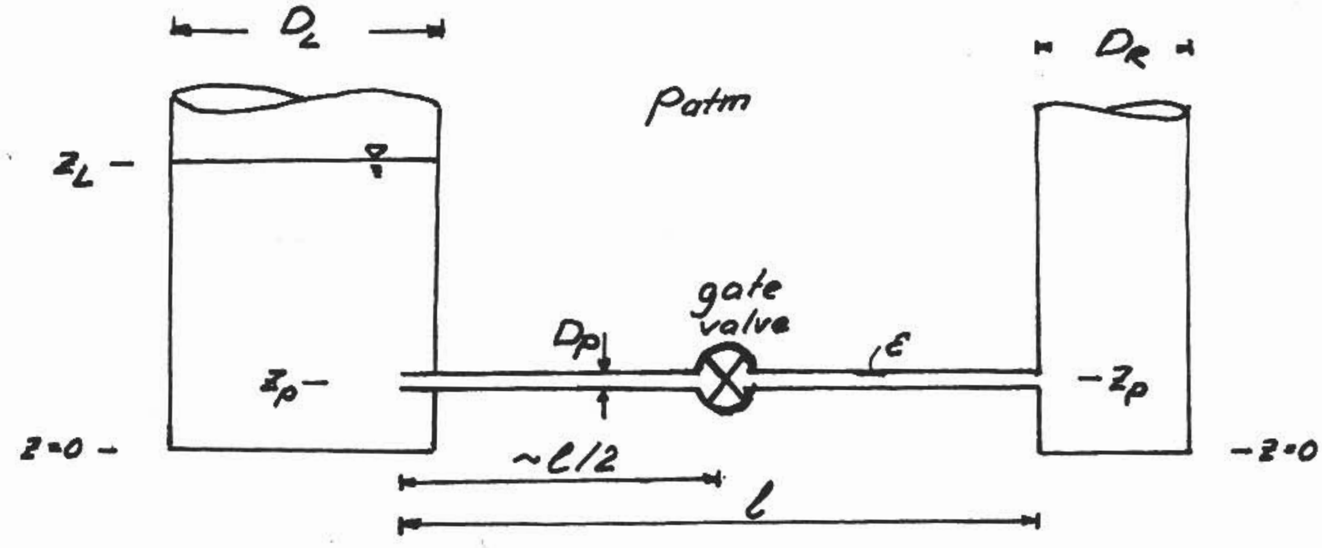
a) Determine the friction factors f_1 and f_2 for the two pipes. (Default values: $f_1 = f_2 = 0.025$)

b) Determine the level h in the container necessary to generate the specified discharge. (Default value: $h = 2.5$m)

c) If you cut off the pipes and threw them away, so that the container was discharging into the air through a 10-cm-diameter sharp-edged orifice what would be the required level h to get the same discharge?

d) Can you explain why a larger head is needed to get the same discharge when the pipes are removed?

Problem No. 4



A straight cast iron pipe (diameter) D_p = 10cm, roughness ε = 0.26mm) of length l = 10m connects two circular cylindrical containers (container L has a diameter D_L = 4m, container R has a diameter D_R = 2m). The L-container holds water initially to an elevation of z_L = 4m whereas the R-container is initially empty. The reentrant pipe inlet (in the L-container) is at elevation z_P = 1m; the pipe is horizontal and has a gate valve located at its midpoint.

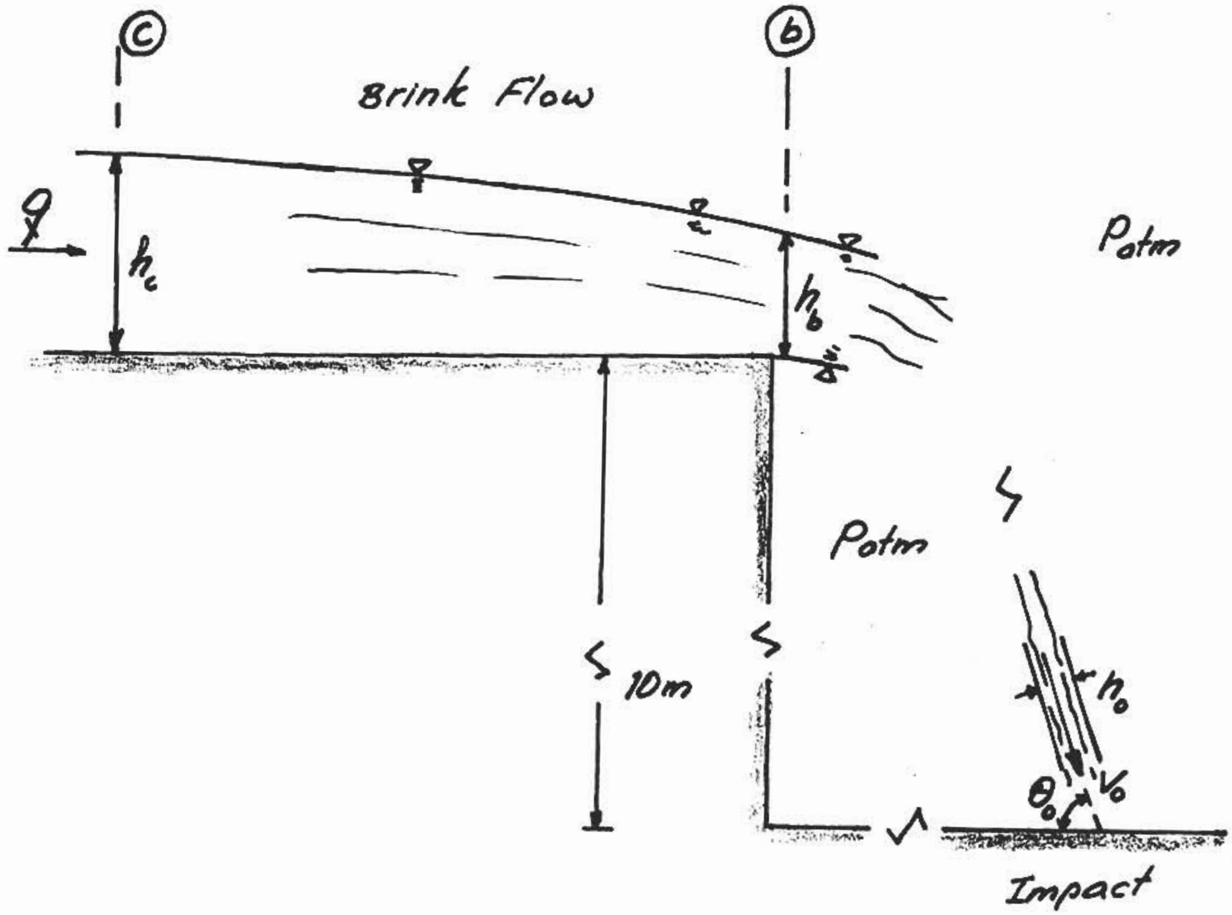
a) Shortly after opening the gate valve ($K_{L,valve}$ = 0.2 when fully open), i.e, before water levels change appreciably in the two containers, determine the initial flow rate, Q_i, from container L to R. (Default value Q_i = 0.025 m^3/s)

b) Corresponding to the initial flow condition *sketch* the Energy Grade Line and the Hydraulic Grade line for the flow in the pipe (sketch but identify important values and discuss briefly reasons for the behavior of the lines you draw).

c) Estimate the time required following opening of the valve for the water level in the R-container to reach z_P.

d) What will be the final level in the R-container?

Problem No. 5



A steady flow of q = discharge per unit width = $Vh = 3.13$ [m^3/s per m] proceeds in a very long, wide, and mildly sloping rectangular channel towards the brink of a drop-structure. As the flow approaches the brink, it passes through critical depth, h_c, a short distance, ~3-4 h_c, upstream of the brink. At the brink the depth of flow is h_b ($<h_c$). The near-brink conditions are shown in the accompanying sketch.

a) Why must the flow pass through critical depth near the brink?

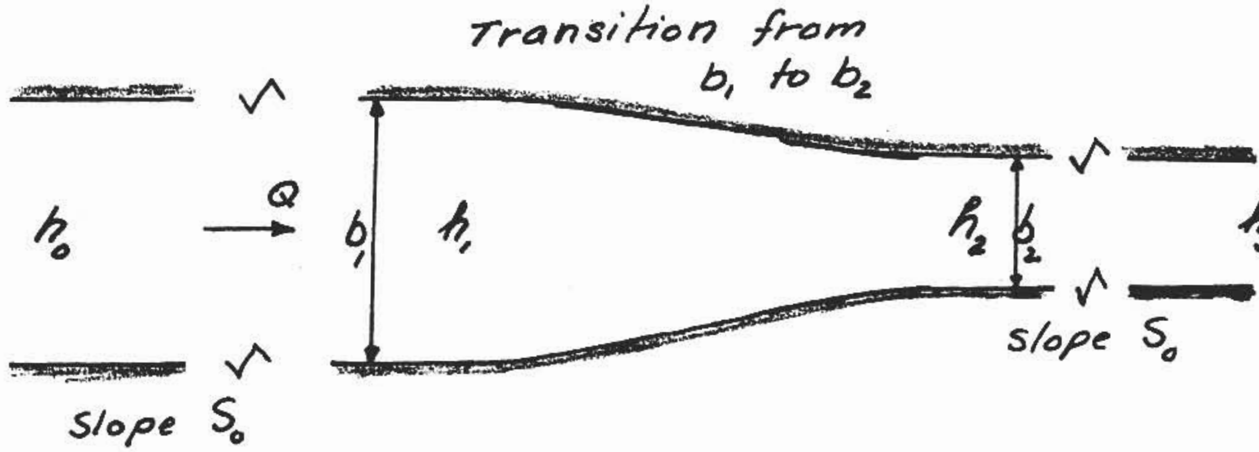
b) At the critical flow section ("c" in the sketch), determine h_c and the specific head, E_c.

c) Using the momentum principle between sections "c" and "b", where "b" (see sketch) is located right after the flow passes over the brink as a free jet, estimate the depth of flow and the velocity at the brink, h_{bm} and V_{bm}.

d) Using the Bernoulli principle between sections "c" and "b", estimate the depth and velocity at the brink, h_{bB} and V_{bB}.

e) With your best estimate of V_b the brink velocity (if you do not like either value you obtained use V_b = 4.5m/s), and assuming V_b to be horizontal, estimate the velocity, V_0, thickness of the jet, h_0, and angle of incidence, θ_0, when the jet hits the drop-structure's horizontal bottom located 10m below the brink [neglect friction from the air surrounding the freely falling jet].

Problem No. 6



An extremely long channel of rectangular crossection carries a discharge of $Q = 100\text{m}^3/\text{s}$, has a uniform slope of $S_0 = 0.001$ and a roughness corresponding to a Manning's $n = 0.015$ [SI-units]. Near the middle of the channel, its width changes from $b_1 = 40\text{m}$ to $b_2 = 20\text{m}$ through a relatively short, smooth transition (see sketch).

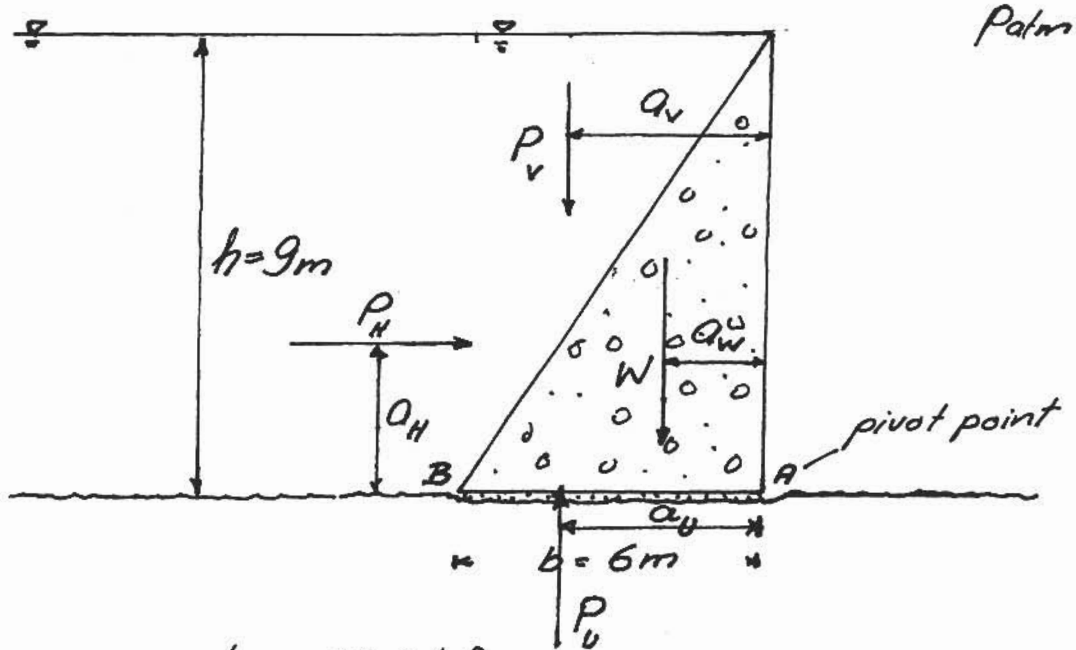
a) Determine the depths h_o (far upstream of the transition), h_1 (immediately upstream of the transition), h_2 (immediately downstream of the transition), and h_3 (far downstream of the transition).

b) Identify the gradually varied flow profiles connecting h_0 and h_1; and h_2 and h_3.

c) Would the depth approximately be h_0 at a distance of (i) 100m; (ii) 1,000m or (iii) 10,000m upstream of the transition? Justify your answer.

HAVE A FANTASTIC SUMMER

Problem No: 1



a)

$$p_B = \rho g h = 88.2\,kPa$$

$$p_A = P_{atm} = 0$$

Forces & Arms around A

Horizontal pressure force from water $= \frac{1}{2}\rho g h^2 = 397\,kN/m$

$$P_H = 397\,kN/m \;;\; a_H = h/3 = 3\,m$$

Vertical pressure force from water $= \frac{1}{2}\rho g h b = 265\,kN/m$

$$P_v = 265\,kN/m \;;\; a_v = 2b/3 = 4m$$

Weight of dam $= \frac{1}{2}\rho_c g\, hb = \frac{1}{2}\, 23.6\cdot 6\cdot 9 = 637\,kN/m$

$$W = 637\,kN/m \;;\; a_w = b/3 = 2\,m$$

Uplift force on base $= \frac{1}{2} p_B\, b = \frac{1}{2}\rho g h b = P_v$

$$P_U = P_v = 265\,kN/m \;;\; a_U = 2b/3 = 4m = a_v$$

b)

$$\mu_{min} = \text{coef. of friction} = \frac{P_H}{P_v + W - P_U} = \frac{P_H}{W} = \underline{0.62}$$

c)

$$F_s = \left(\frac{P_H a_H + P_U a_U}{W a_w + P_v a_v}\right)^{-1} = \frac{2{,}334\ kNm/m}{2{,}251\ kNm/m} = \underline{1.04}$$

Problem No: 2

a)

$$Q = VA \Rightarrow \underline{V} = Q/A = 0.053 / \left[\tfrac{\pi}{4}(0.2)^2\right] = \underline{1.69\ m/s}$$

b)

$$p_A + \rho g z_A - (\rho_m - \rho) g h_m = p_B + \rho g z_B$$

$$p_A - p_B = (\rho_m - \rho) g h_m - \rho g (z_A - z_B) = (\rho_m - \rho) g h_m - \rho g\, l \sin\beta$$

$$\underline{p_A - p_B} = 12.6 \rho g h_m - \rho g l \sin 1^\circ = 3{,}457 - 3{,}420 = \underline{37 Pa}$$

c) Forces in x : Pressure + Gravity = Friction

$$(p_{A'} - p_{B'}) A + \rho A l g \sin\beta = \tau (Pl) \ ; \ p_{A'} - p_{B'} = p_A - p_B$$

$$[(p_A - p_B) + \rho g l \sin\beta] A = 3{,}457 \cdot \tfrac{\pi}{4}(0.2)^2 = \tau (\pi D l)$$

$$\underline{\tau = 8.64\ Pa}$$

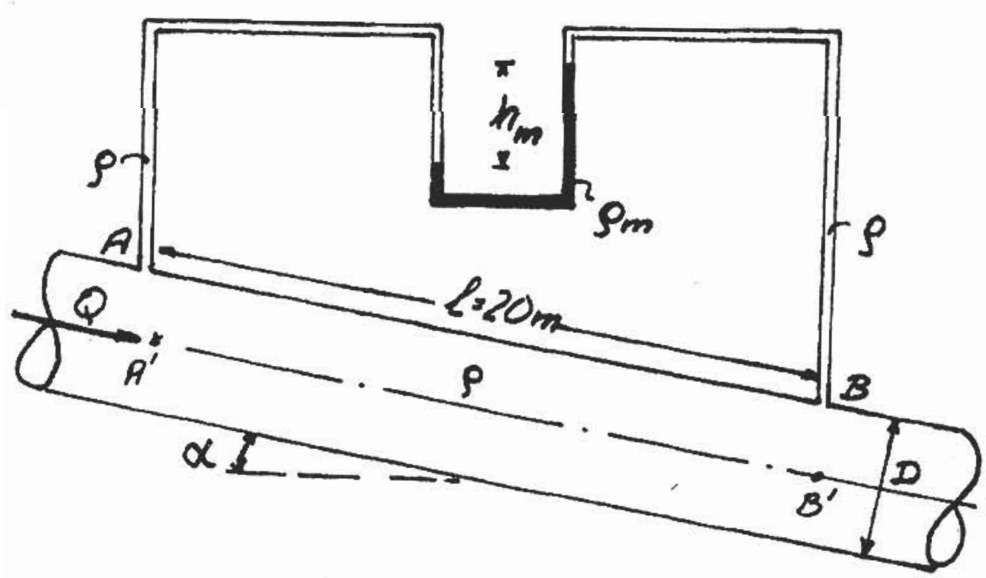
d)

$$\tau = \frac{f}{8} \rho V^2 \Rightarrow \underline{f} = 8\tau/(\rho V^2) = \underline{0.024(2)}$$

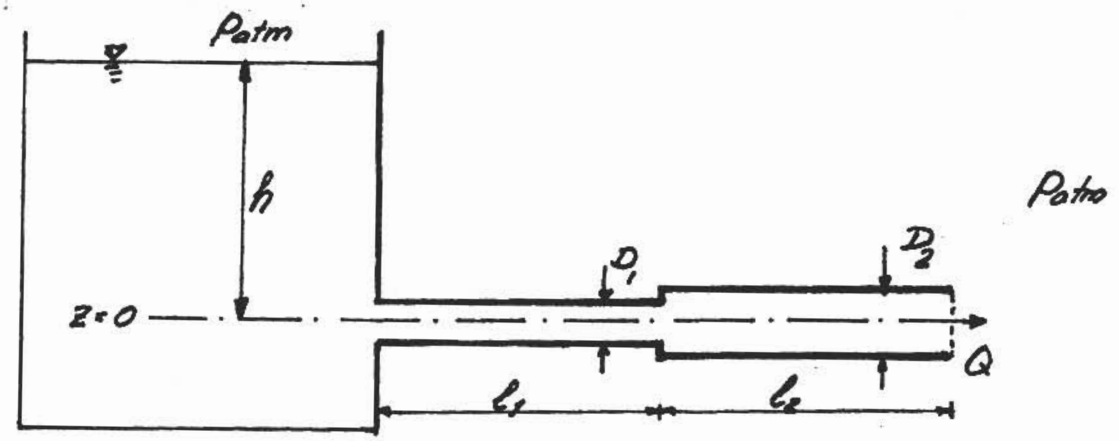
e)

From Moody using $f = 0.024$ and $Re = \frac{VD}{\nu} = 3.4 \cdot 10^5$

$\varepsilon / D = 0.002 \quad \Rightarrow \underline{\varepsilon} = 2 \cdot D \cdot 10^{-3} = 0.4 \cdot 10^{-3} m = \underline{0.4 mm}$



Problem No: 3



a) $V_1 = Q/A_1 = 4.5\, m/s \Rightarrow Re_1 = \frac{V_1 D_1}{\nu} = 4.5 \cdot 10^5$; $\varepsilon/D_1 = 0.003 \Rightarrow \underline{f_1 = 0.026}$

$V_2 = V_1 \cdot (D_1/D_2)^2 = 2.0\, m/s \Rightarrow Re_2 = \frac{V_2 D_2}{\nu} = 3 \cdot 10^5$; $\varepsilon/D_2 = 0.002 \Rightarrow \underline{f_2 = 0.024}$

b) $H_1 = h = (K_{L,ent} + f_1 l_1/D_1) V_1^2/2g + (K_{L,exp} + f_2 l_2/D_2) V_2^2/2g + H_2$; $K_{L,ent} = (1/0.6 - 1)^2 = 0.44$; $K_{L,exp} = ((D_2/D_1)^2 - 1)^2 = 1.56$; $H_2 = \frac{V_2^2}{2g}$

$$\underline{h} = \left(0.44 + 0.026 \frac{4.5}{0.1}\right)\frac{4.5^2}{2 \cdot 9.8} + \left(1.56 + 0.024 \frac{4.5}{0.15}\right)\frac{2^2}{2 \cdot 9.8} + \frac{2^2}{2 \cdot 9.8} = \underline{2.33\, m}$$

c)

$$Q = 0.0353 = \left(\frac{\pi}{4} 0.1^2\right) C_c \cdot \sqrt{2 \cdot 9.8\, h} \Rightarrow \text{with } C_c = 0.6$$

$$\underline{h = 2.86\, m}$$

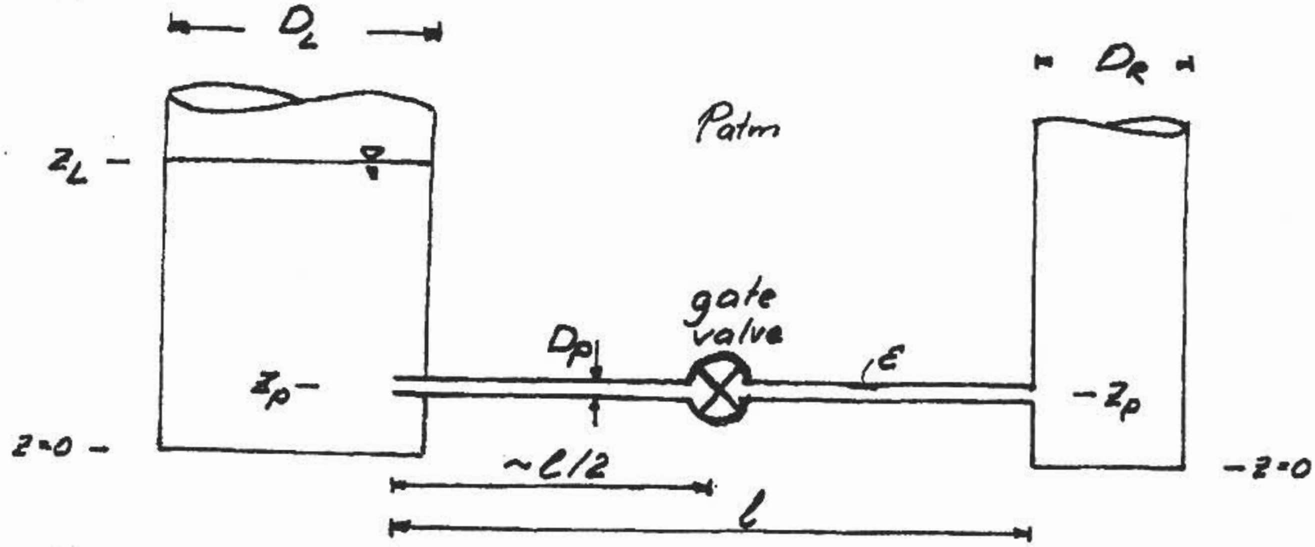
d)

Explanation is that pressure pipe at vena contracta after inflow from container is negative, i.e. with pipe installed "suction" is created at outflow from contained.

$$P_{v.c.} = \left(H_1 - \frac{V_{v.c.}^2}{2g}\right)\rho g = \left(h_{\text{from } b} - \frac{(V_1/0.6)^2}{2g}\right)\rho g = -0.54\, \rho g < 0$$

[note: $\Delta h_{b-c} = -0.53 \approx P_{v.c.}/\rho g$]

Problem No: 4



a)

$$H_L = z_L = H_R + \Delta H = V_p^2/2g + z_p + \underbrace{\frac{p_R}{\rho g}}_{0 \text{ since free jet}} + \left(K_{L,ent} + f(\ell/D_p) + K_{L,valve}\right)\frac{V_p^2}{2g}$$

$$V_p = \left\{\frac{2g(z_L - z_p)}{1 + K_{L,ent} + K_{L,valve} + f\cdot\frac{10}{0.1}}\right\}^{1/2} = \left\{\frac{58.8}{2.2 + 100f}\right\}^{1/2}$$

$(1/C_c - 1)^2 = 1$ since $C_c = 0.5$.

$$f = f\left(Re = \frac{V_p D}{\nu} = V_p \cdot 10^5,\ \varepsilon/D = 0.0026\right) \Rightarrow f^{(0)} = 0.02 \Rightarrow V_p^{(0)} = 3.74\,\frac{m}{s}$$

$$f^{(1)} = f(V_p^{(0)} 10^5 = 3.7\cdot 10^5;\ \varepsilon/D = 0.0026) = 0.026 \Rightarrow V_p^{(1)} = 3.50\ m/s$$

$$f^{(2)} = f(3.5\cdot 10^5,\ 0.0026) = 0.026 \quad \text{(can't see a difference)}$$

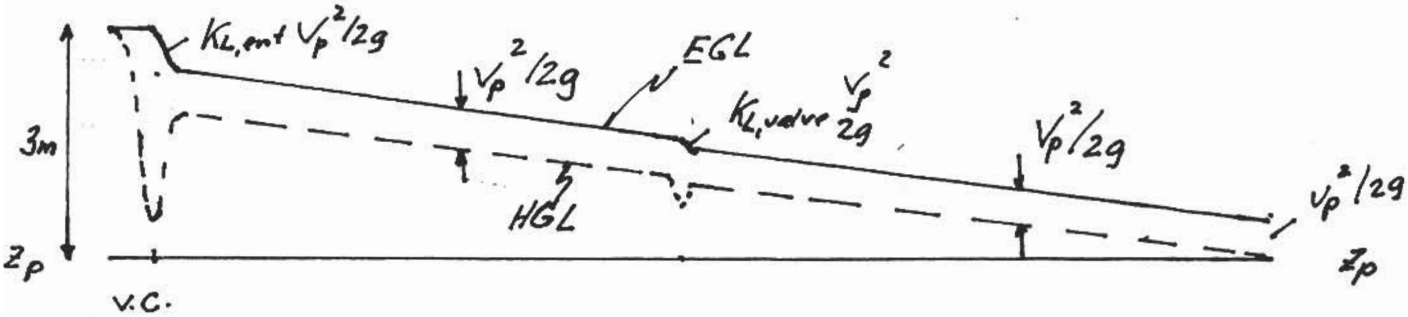
$\underline{V_p = 3.5\ m/s}$; $\underline{Q} = V_p A_p = V_p \frac{\pi}{4} D_p^2 = \underline{0.0275\ m^3/s}$

b)

EGL $= V^2/2g + p/\rho g$ above z_p; HGL is $V^2/2g$ below EGL

$V_p^2/2g = 0.625\,m$; at vena contracta $V_{vc} = 2V_p \Rightarrow V_{vc}^2/2g = 2.5\,m$

$\Delta H_f = 50\cdot 0.026\cdot V_p^2/2g \approx 0.8\,m$ over $5\,m$.



c)

When $z = z_p$ in the R-container a volume of $(\pi/4)\, D_R^2\, z_p = 3.14 m^3$ has drained from the L-container. Due to this, the L-container has dropped $\Delta z_L = 3.14/(\frac{\pi}{4} D_L^2) = 0.25 m$. From (a) we have that $V_p \propto \sqrt{z_L - z_p}$ So, V_p starts at $3.50 m/s$ and ends at $3.50 \cdot \sqrt{2.75/3.0} = 3.35 m/s$. Average $\simeq \overline{V_p} = 3.43 m/s \Rightarrow Q_{ave} = 0.0269 m^3/s$

To get $3.14 m^3$ would take

$$\underline{t = 3.14 / Q_{ave} \simeq 117 s \approx 2 \text{ minutes}}$$

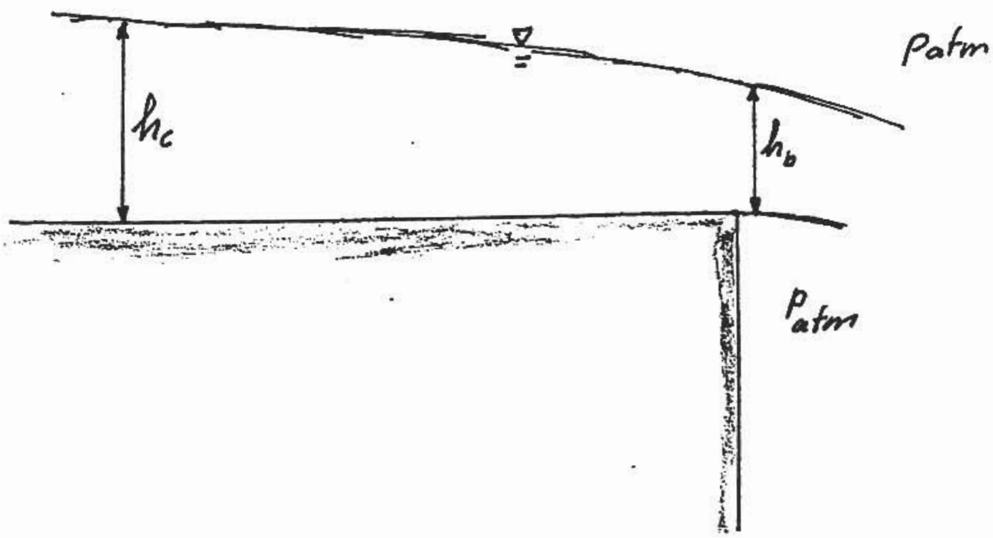
d)

$$\frac{\pi}{4} D_R^2 z_{R\infty} + \frac{\pi}{4} D_L^2 z_{L\infty} = \frac{\pi}{4}(D_R^2 + D_L^2)\, z_\infty = \frac{\pi}{4} D_L^2 z_L .$$

$z_{R\infty} = z_{L\infty}$ when flow stops

$$\underline{z_\infty} = z_L \frac{D_L^2}{D_R^2 + D_L^2} = \frac{4}{5} z_L = \underline{3.2 m}$$

Problem No: 5



a)
Since flow approaches the brink in a mildly sloping channel, and the flow after the brink [bottom is "vertical"] is the ultimate in terms of a steep slope, we have a transition from mild to steep slope $\Rightarrow$ Flow passes through critical "at" transition, i.e. near the brink

b)
Since channel is rectangular we have $Fr = V/\sqrt{gh}$.
For critical flow, therefore

$$Fr = 1 = V_c/\sqrt{gh_c} \Rightarrow \underline{V_c = \sqrt{gh_c}}$$

$$q = Vh = V_c h_c = \sqrt{g}\, h_c^{3/2} \quad \underline{h_c} = \sqrt[3]{q^2/g} = \left(\frac{3.13^2}{9.8}\right)^{1/3} = \underline{1.00\,m}$$

$$\underline{E_c} = h_c + \frac{V_c^2}{2g} = h_c + \frac{1}{2}h_c = \underline{1.5\,m}$$

c)
$$MP_c = MP_b \quad \text{(short distance, friction may be set = 0)}$$

$$MP_c = (\rho V_c^2 + p_{cg,c})\, h_c \cdot 1 = (\rho g h_c + \tfrac{1}{2}\rho g h_c)\, h_c = \tfrac{3}{2}\rho g h_c^2$$

$$MP_b = (\rho V_b^2 + p_{cg,b})\, h_b \cdot 1 = \rho V_b^2 h_b \quad \text{(since free jet: } p_{cg,b} = 0)$$

$$\tfrac{3}{2}\rho g h_c^2 = \rho (V_b h_b) V_b = \rho q V_b = \rho(\sqrt{gh_c}\, h_c)\, V_b$$

$$\underline{V_{bm}} = \tfrac{3}{2}\sqrt{gh_c} = 1.5\sqrt{9.8 \cdot 1} = \underline{4.70\ m/s}$$

$$\underline{h_{bm}} = q/V_{bm} = \tfrac{2}{3}h_c = \underline{0.67\,m}$$

d) The "center of gravity" streamline starts at "c" with a total head (measure above bottom) of $H_c = E_c = \frac{3}{2} h_c$

At "b" it has $z_{cg,b} = \frac{1}{2} h_{bB}$, pressure ≈ 0, and velocity V_{bB}, so

$$H_b = \frac{1}{2} h_{bB} + \frac{V_{bB}^2}{2g} = \frac{1}{2} h_{bB} + \frac{(V_{bB} h_{bB})^2}{2g h_{bB}^2} = \frac{1}{2} h_{bB} + \frac{q^2}{2g h_{bB}^2}$$

Short transition ~~+~~ Converging Flow $\Rightarrow H_c = H_b$

or with $q = V_c h_c = \sqrt{g}\, h_c^{3/2}$

$$H_c = \frac{3}{2} h_c = H_b = \frac{1}{2} h_{bB} + h_c^3 / 2h_{bB}^2 \Rightarrow \left(3 - \frac{h_{bB}}{h_c}\right)^{1/2} = \frac{h_c}{h_{bB}}$$

Start iteration with $h_{bB}/h_c = 0.67$ from (c)

$$h_{bB}/h_c = 0.65 \Rightarrow \underline{h_{bB}} = \underline{0.65\,m}$$

$$\underline{V_{bB}} = q/h_{bB} = \underline{4.80\ m/s}$$

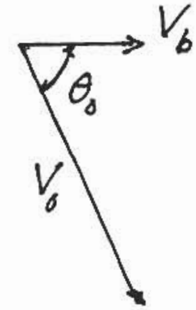
e) Best "estimate" of $V_b = 4.75\ m/s$.

As jet falls it looses no energy since air-resistance may be neglected. Thus, the head at the brink:

$$H_b = H_c = \frac{3}{2} h_c = H_{jet} \text{ everywhere}$$

$$H_{jet} = \frac{V_j^2}{2g} + p_j + z_j = \frac{V_j^2}{2g} + 0 + z_j \quad (p_j \cong p_{atm} = 0)$$

So, $V_j = \sqrt{2g(H_c - z_j)}$



At impact, $z_j \approx -10\,m$, so

$$\underline{V_0} = \sqrt{2g(1.5 - (-10))} = \underline{15.0\ m/s}$$

$$V_0 \cdot h_0 = q \Rightarrow h_0 = 3.13/V_0 = \underline{0.21\,m}$$

Without air resistance the initial horizontal velocity at the brink – V_b – is unchanged during free fall. Thus,

$$\cos\theta_0 = V_b/V_0 = 4.75/15 = 0.317 \Rightarrow \underline{\theta_0 = 71.5^\circ}$$

Problem No: 6

a)

Normal flow: $Q = \frac{1}{n}\frac{(h_n b)^{5/3}}{(b+2h_n)^{2/3}}\sqrt{S_0}$ or

$$h_n = \left(\frac{Qn}{b\sqrt{S_0}}\right)^{3/5}\left(1+2\frac{h_n}{b}\right)^{2/5} = \begin{matrix} 1.108(1+0.05h_{n1})^{2/5} & b=b_1=40m \\ 1.679(1+0.1h_{n2})^{2/5} & b=b_2=20m \end{matrix}$$

Solve by iteration: $h_{n1} = 1.13m$; $h_{n2} = 1.79m$

Far from transition only possibility is normal flow - So

$\underline{h_0 = h_{n1} = 1.13m}$; $\underline{h_3 = h_{n2} = 1.79m}$

$V_{n1} = Q/b_1 h_{n1} = 2.21\, m/s \Rightarrow Fr_{n1} = 0.66 < 1$
$V_{n2} = Q/b_2 h_{n2} = 2.79\, m/s \Rightarrow Fr_{n2} = 0.67 < 1$ } Notice $Fr_n \approx$ const!

Normal Flow is subcritical - So <u>slope is MILD</u>

There is no downstream control after transition ⇒

Normal flow established immediately after transition

$\underline{h_2 = h_{n2} = 1.79m}$

Transition is short and converging, so $\Delta H \sim 0$, or

$E_1 = h_1 + [Q^2/(2g b_1^2)]/h_1^2 = h_2 + Q^2/(2g(h_2 b_2)^2) \Rightarrow$

$$h_1 = h_{n2} + \frac{V_{n2}^2}{2g} - \frac{Q^2}{2g\,b_1^2}\frac{1}{h_1^2} = 2.187 - 0.319/h_1^2$$

Solve by iteration:

$\underline{h_1 = 2.25\, m}$

b)

Slope is MILD $h_0 = h_{n1}$ far upstream is backed up to reach $h_1 = 2.25m$ before transition: <u>M1-Curve</u>

c)

$\Delta h = h_1 - h_{n1} \approx 1.12m$; Surface is larger than horizontal.

If $dh/dx = S_0 = 10^{-3}$ then $\Delta h = 1.12m$ is achieved over ~ 1km

(ii) is the answer - but (iii) may not be way large!