

Problem Set 6

Due on Monday, April 10th

Important note: Please start a new sheet of paper for each problem in the problem set. Write the names of the group members who contributed to the solution at the beginning of each problem.

Problem 1

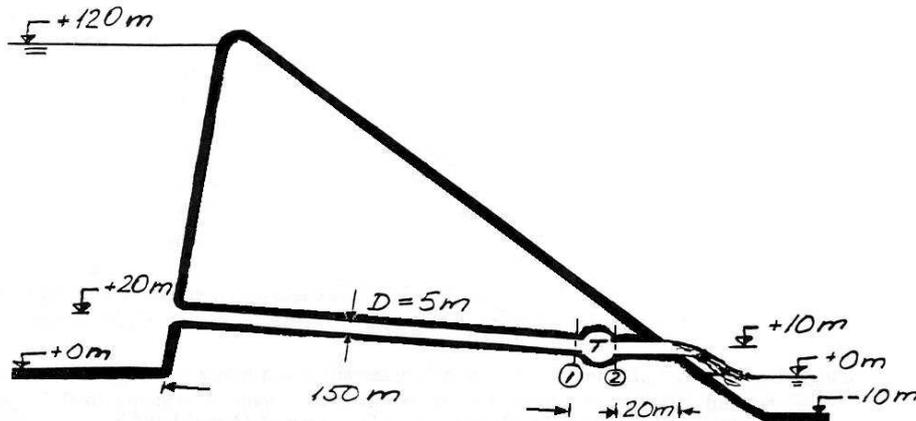


Figure 1: Hydroelectric power plant in Problem 1.

Figure 1 shows a hydroelectric power plant that operates at an efficiency of $\eta = 0.90$ and produces $1.2 \cdot 10^8$ Watts of electric power for the conditions shown, i.e., water level at $+120$ m in the reservoir and at $+0$ m in the tailwater lake. The turbines are located at elevation $+10$ m and the conduits leading to and from the turbines have a diameter of $D = 5.0$ m and lengths of 150 m and 20 m respectively. The conduit leading to the turbines starts at elevation $+20$ m at the reservoir and the other conduit is horizontal. Assume the conduits to have a friction factor of $f = 0.02$ and neglect the minor loss at the inlet from reservoir.

- Determine the head, H_1 , at the entrance of the turbines as a function of the discharge Q .
- Determine the head, H_2 , at the exit from the turbines as a function of the discharge Q .
- With the values of H_1 and H_2 from (a) and (b), determine the discharge, Q , through the power plant in order to achieve the specified power output from the turbines.
- With internal energy per unit mass of water given by $\hat{u} = C_p T = C_v T$, where $C_p = C_v = 4210$ m²/(s²K) and T is temperature in K, estimate the temperature of the water exiting the downstream conduit when the water temperature is 12°C in the reservoir. Be sure to state all assumptions you make in arriving at your estimate.

Problem 2

The design of the city water supply in the last problem set (Problem 6) needs to be completed. A water flowrate of $Q = 0.5 \text{ m}^3/\text{s}$ is pumped from the river, A , to the large reservoir, B , where the water surface is 100 m above the river surface, as shown in Figure 2. The pipe from A to B has length $L = 5000 \text{ m}$, diameter $D = 80 \text{ cm}$, and roughness $\epsilon = 2 \text{ mm}$. The maximum pressure allowed in the pipe is 100 mwc (meters of water column). Account for minor losses only at the reentrant inflow and at the outflow.

- a) Design a system of serial pumps to lift the water from A to B , specifying number of pumps, pump head, and position of the pumps in the pipe (model the pipe as having constant slope from A to B). If you need more than one pump, these should be identical, i.e., all pumps must have the same pump head. The required NPSH is 6 mwc . The design criteria, in order of decreasing priority, are: (i) Satisfy all the pressure constraints; (ii) Minimize the number of pumps; (iii) Minimize the pump head.
- b) Draw the energy grade line, EGL, specifying the relevant values.
- c) Determine the cost of operating the system if the pump efficiency is 0.80 and the cost of electricity is 10 cents per kWh (kilowatthour).

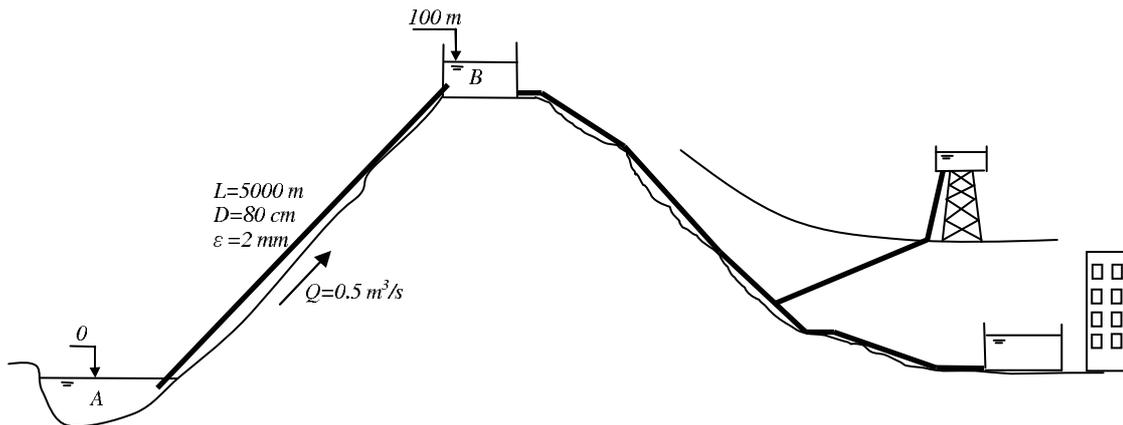


Figure 2: Water supply in Problem 2.

Problem 3

The outflow and the inflow for a centrifugal pump are perpendicular to each other. For this reason, if we want to keep the same direction of the pipe before and after the pump, we need to introduce an elbow (elbow at C in Figure 3). The pump is located at elevation $z = 0$ and supplies water from a large tank A , whose free surface is kept at constant elevation $z = 5\text{ m}$, to a large tank G , whose free surface is kept at constant elevation $z = 25\text{ m}$. The pipe is horizontal from A to E and from F to G , and vertical from E to F . The pipe from the lower tank to the pump has a length $L_{AB} = 100\text{ m}$. The pipe from the pump to the upper tank has a length $L_{CG} \approx L_{BG} = 600\text{ m}$. The distance between B and C is short and friction losses between B and C are negligible. All pipes have diameter $D = 0.3\text{ m}$ and roughness $\epsilon = 0.3\text{ mm}$. All elbows are regular 90° threaded elbows and both the inflow from A and the outflow to G are reentrant.

- Determine the pump head to provide a flowrate $Q = 0.1\text{ m}^3/\text{s}$.
- Would you be worried about cavitation in the pump?
- Calculate the magnitude and direction of the total horizontal force exerted **on** the elbow at C .
- Keeping the same pipe alignment and the same flowrate, consider the possibility of interchanging the elbow and the pump, so that the elbow would be located at B **before** the pump. Give a reasoned answer to the following questions:
 - Would the new pump head be larger or smaller than the result obtained in (a)?
 - Would the pump be more or less susceptible to experience cavitation?
 - Would the magnitude of the total horizontal force on the new 90° elbow at B be larger or smaller than in (c)?
 - Overall, which configuration is better?

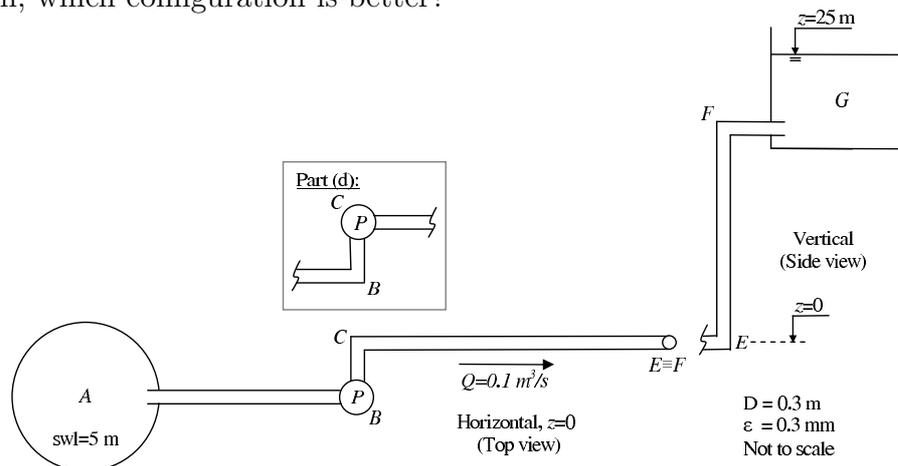


Figure 3: Pump setup in Problem 3.

Problem 4

Water is to be pumped from one large, open tank (1) to a second large, open tank (2) as shown in Figure 4. The pipe diameter is $D = 0.15 \text{ m}$ and the total length of the pipe between the pipe entrance and exit is 50 m . Before the pump, there is regular 90° , flanged elbow. Assume rough turbulent flow throughout the problem.

a) When the pipe is new, its roughness is $\epsilon_{new} = 0.15 \text{ mm}$. Under these conditions, and independent of the performance characteristics of the pump, obtain a relationship between the pump head, H_p , and the flowrate, Q , of the form $H_p = A + BQ^2$. Here A and B are constants for you to determine. This relationship is called the system curve.

b) Consider the centrifugal pump whose performance characteristics are shown in the graph inserted in the figure. Represent the system curve obtained in (a) in the pump-performance graph. Using your graph, determine the pump head, H_p , and the flowrate, Q , when this centrifugal pump is installed in our system. (NOTE: A copy of the pump-performance graph for you to draw on is provided at the end of the problem set).

c) Calculate the BHP (break horse power) for the pump when operating as specified in (b).

d) When the pipe gets old, roughness increases due to corrosion. Assume $\epsilon_{old} = 1.5 \text{ mm}$. Under these conditions, calculate the flowrate, Q , pump head, H_p , and BHP, and show the point at the pump-performance graph that corresponds to your answer.

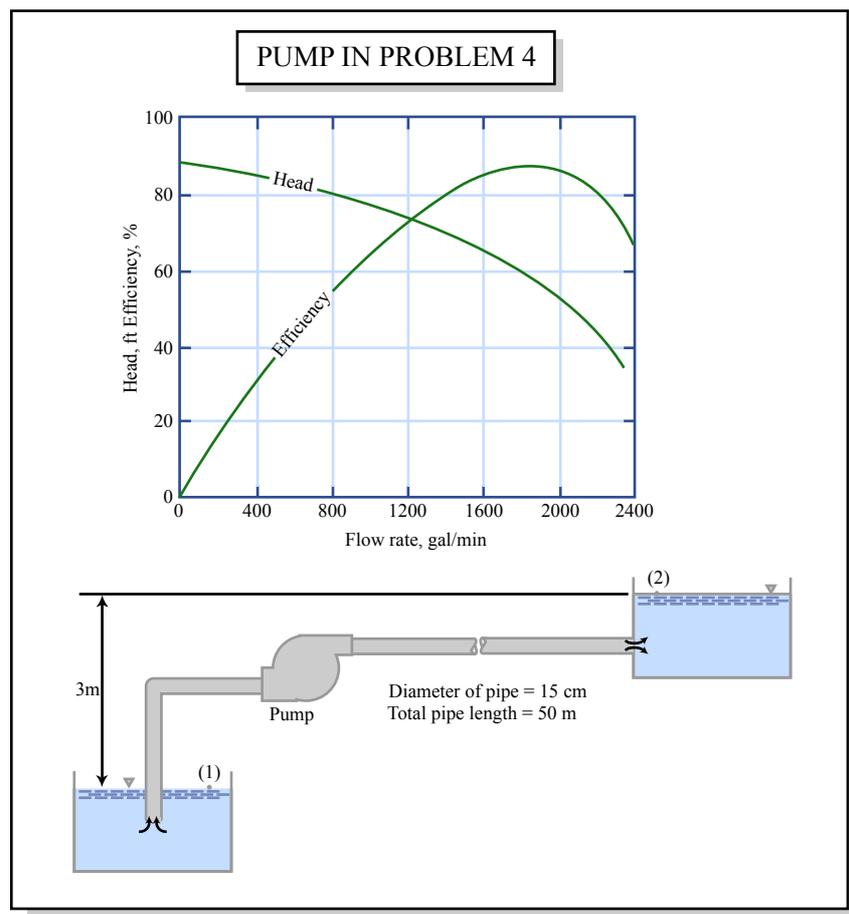


Figure by MIT OCW.

Adapted from:

Figure E11.3 in Young, Donald F., Bruce R. Munson, and Theodore H. Okiishi.

A Brief Introduction to Fluid Mechanics. 2nd ed. New York, NY: John Wiley & Sons, Inc., 2001, pp. 461.

Problem 5

Water at 20°C flows at an average velocity of 80 cm/s through a horizontal **rectangular** duct as shown in Figure 5. The duct is 1 m high and 2 m wide. A circular cylinder 40 cm in diameter is placed in the duct as shown, so that it spans the entire width of the duct. A differential manometer is set up so that the pressure difference between A and B can be measured. The distance between locations A and B is 3 m , and the specific gravity of the manometer fluid is 1.30 . The manometer reading is $d = 6.8\text{ cm}$. The velocity profile is uniform at sections 1 and 2, as shown in the figure. The equivalent roughness of the duct walls is $\epsilon = 5.3\text{ mm}$.

- What is the pressure difference between points A and B ?
- What is the average shear stress on the duct walls, τ_s , from section 1 to section 2, if the presence of the cylinder is neglected?
- Estimate the magnitude and direction of the total force from the fluid on the cylinder.
- What is the corresponding drag coefficient, C_D , of the cylinder?
- Why is it better to have the cylinder at its present location than at point X ?
- Is the value calculated in (d) a lower or an upper bound for the actual value of C_D ?
- Explain how you would calculate the other bound for the actual C_D of the cylinder using the information provided in the problem, i.e., explain (but do not carry out the computations) how you would calculate an upper bound for C_D if you determined that your result in (d) is a lower bound or vice versa.

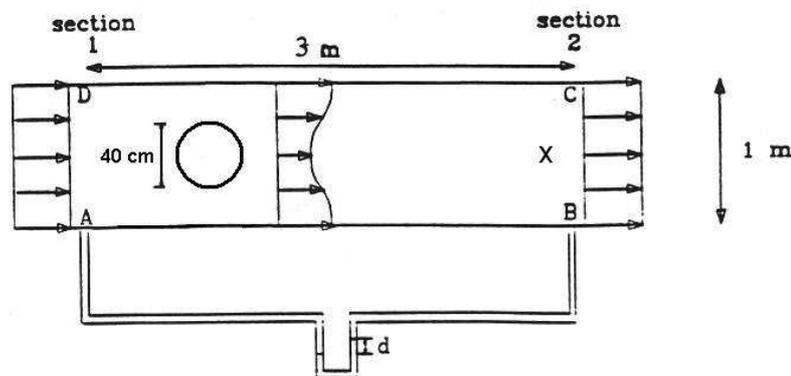


Figure 5: Flow past a cylinder in Problem 5.

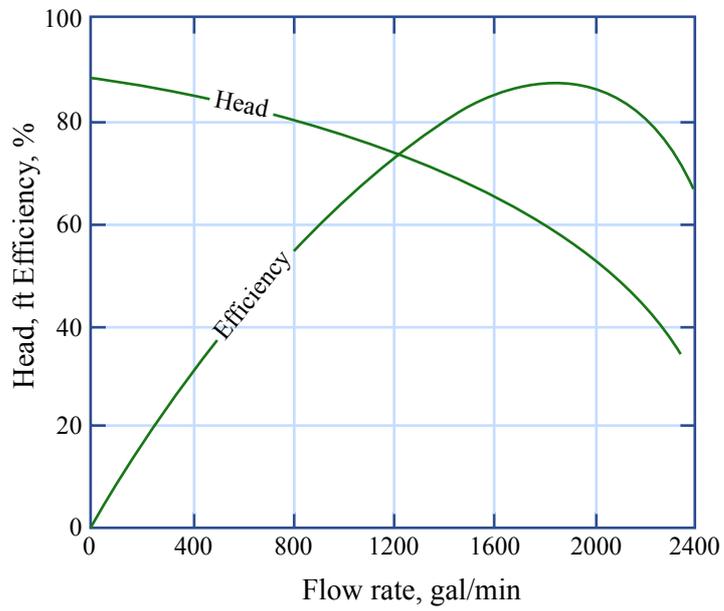
Problem 6

On a perfect Summer day (no wind, temperature = $25^{\circ}C$), a spherical ball of diameter $D = 7.4\text{ cm}$, mass $m = 0.14\text{ kg}$, and a relative surface roughness of $\epsilon/D = 1.5E-3$ (see Figure 9.18 in Young et al.), is thrown from a mound ($x = 0$) a distance of $l = 60\text{ ft}$ to where a person stands and will try to hit the ball as it passes over a plate.

a) If the ball is thrown with an initial horizontal velocity $U_0 = 70\text{ mph}$, **estimate** how long it takes the ball to reach the plate, and the velocity of the ball as it passes the plate, U_p .

b) **Estimate** the trajectory followed by the ball considered in **(a)** between mound and plate. In particular, estimate the vertical drop experienced by the ball from its release point ($z = z_0$) until it passes the plate.

PUMP-PERFORMANCE GRAPH FOR PROBLEM 4



Graph by MIT OCW.

Adapted from:

Figure E11.3 in Young, Donald F., Bruce R. Munson, and Theodore H. Okiishi.

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