PROBLEM SET 3 - SOLUTIONS

Comments on Problem Set 4

PROBLEM 1:

- Be careful with the directions of the "other forces" when you apply momentum conservation in a control volume. Typically, you draw and calculate the forces exerted by the surroundings on the control volume of fluid. At the end, you need calculate the forces exerted by the fluid on the surroundings (in this case on the pipe), which have the same magnitude as the forces on the fluid but opposite direction. Many groups calculated "F_x" and "F_y" without specifying if those were the forces on the fluid or the forces on the pipe, and some groups got confused because of this.

- Onto the same issue, the best way to specify the direction of a force is to show it in a sketch.

PROBLEM 2:

- Remember that we have seen two ways of applying conservation of energy. First, you can apply Bernoulli between two points along a streamline. This is our good old expression:

$$z_1 + p_1/(\rho g) + v_1^2/(2g) = z_2 + p_2/(\rho g) + v_2^2/(2g) + \text{(losses between 1 and 2)}$$

where z_1, p_1, and v_1 are the elevation, pressure, and velocity at point 1 (same for point 2). We used this many times in the old times (a.k.a. before test 1) when we assumed inviscid flow and didn't have losses. Now, we are dealing with real flows, in which there are usually losses. Furthermore, these real flows are not usually uniform in the cross-section, and we are not usually interested in determining velocities at specific points of the cross-section. Rather, we are interested in determining the "cross-sectional averaged" properties of the flow, and thus we work with the cross-sectional average velocity, V. For this reason, we have developed the control volume analysis, which deals with cross-sectional averaged quantities. The conservation of energy between the inflow and the outflow of a Control Volume reads

$$z_{1,CG} + p_{1,CG}/(\rho g) + V_1^2/(2g) = z_{2,CG} + p_{2,CG}/(\rho g) + V_2^2/(2g) + \text{(losses between 1 and 2)}$$

It looks similar to our old good Bernoulli, but it is conceptually different! Now we apply conservation of energy between two sections (not two points), which are the inflow and the outflow of our Control Volume. $z_{1,CG}$ is the elevation of the center of gravity of section 1, $p_{1,CG}$ the pressure at the center of gravity of section 1, and V_1 the average velocity in section 1 (not the velocity at a particular point). So, in the future, be explicit about which version of Bernoulli you are applying, and between which and which section (or which and which point).
NOTE: To apply conservation principles, the control volume must always be chosen so that the flow is well-behaved both in the inflow and the outflow. For this reason, pressure is hydrostatic at these sections. Therefore, the piezometric head, $z_{1,CG} + p_{1,CG}/(\rho g)$, is constant in all the points on the inflow section (and same for the outflow), and you can evaluate the sum of these two terms at any point, not necessarily at the center of gravity.

PROBLEM 3:

- Notice that the reason why we can neglect the headloss in problems 1 and 3 is because in both cases we have a short transition of a converging flow. Since the transition is short, the friction loss is very small. Since the flow is always converging, there is no minor loss.

Problem #4:

A few groups didn't have the formula for head loss quite right – we must also take into account the elevation difference from one side of the hydraulic jump to the other. The formula can easily be derived from the equation: H_1 = H_2 + HL_(1-->2). See solutions for details. A handful of groups also answered the last part incorrectly. Conceptually, if we lose energy going from 1 --> 2, we'd have to gain energy if we went from 2 --> 1 (by conservation of energy). This would yield a negative head loss and negative energy dissipation (which implies a head gain and energy gain). This situation is physically impossible, so the flow only goes from 1 to 2. See solution for mathematical solutions.

Problem #5:

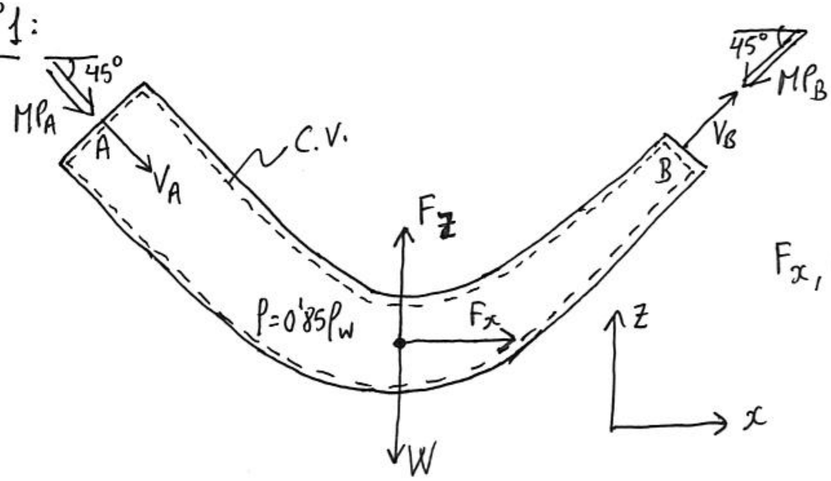
Don't ever assume that two graphs are the same simply by looking at a single plot! They may look the same with the scale you've used, but there still may be large errors. Such is the case in this problem for values of y near the bottom (y = 0) where errors reach 25%. Everyone lost a point or two on this. Also, use Excel or another graphing program to make your plots. Handwritten plots are not acceptable (especially when units and other things are missing), especially for something like this where you have to be very accurate. If you don't know how to make graphs in Excel, ask the TAs. It's an important skill that you'll be able to use throughout your MIT life and beyond.

Problem #6:

Don't forget to explicitly say which way the force is acting.

PROBLEM SET 4 - SOLUTIONS

- PROBLEM N°1:



F_x, F_z: Forces from the elbow on the fluid.

Conservation of volume between A and B:

$$Q_A = Q_B \Rightarrow V_A \cdot A_A = V_B \cdot A_B \Rightarrow V_B = \frac{A_A}{A_B} V_A$$

$$A_A = \frac{\pi d_A^2}{4} = \frac{\pi \cdot 0'4^2}{4} = 0'1257\ m^2$$

$$A_B = \frac{\pi d_B^2}{4} = \frac{\pi \cdot 0'2^2}{4} = 0'03142\ m^2$$

$$V_B = \frac{0'1257}{0'03142} V_A = 4 V_A$$

Bernoulli between A and B (We neglect losses, which makes sense if the distance between A and B is small, because then we have a short transition of a converging flow).

$$p_A + \rho g z_A + \frac{1}{2} \rho V_A^2 = p_B + \rho g z_B + \frac{1}{2} \rho V_B^2$$

$$z_A = z_B, \quad V_B = 4 V_A$$

$$\frac{1}{2} \rho (4^2 - 1) V_A^2 = p_A - p_B$$

$$\frac{1}{2} 850 \cdot 15 \cdot V_A^2 = 1'5 \cdot 10^5 - 1'3 \cdot 10^5 \Rightarrow V_A = 1'771\ m/s$$

$$V_B = 4 V_A = 7'085\ m/s$$

Thrust on A:

$$MP_A = (\rho V_A^2 + p_{CG,A}) \cdot A_A = (850 \cdot 1'771^2 + 1'5 \cdot 10^5) \cdot 0'1257 = 19190 \text{ N}$$

Thrust on B:

$$MP_B = (\rho V_B^2 + p_{CG,B}) \cdot A_B = (850 \cdot 7'085^2 + 1'3 \cdot 10^5) \cdot 0'03142 = 5425 \text{ N}$$

Weight:

$$W = \rho g \forall = 850 \cdot 9'8 \cdot 0'15 = 1250 \text{ N}$$

Conservation of momentum in x-direction:

$$\sum \text{Forces in } x = MP_A \cos 45^\circ - MP_B \cos 45^\circ + F_x = 0 \Rightarrow$$

$$\Rightarrow \underline{\underline{F_x}} = -19190 \frac{\sqrt{2}}{2} + 5425 \frac{\sqrt{2}}{2} = \underline{\underline{-9733 \text{ N}}} \text{ (to the left)}$$

→ The horizontal force from the fluid on the elbow is of the same magnitude acting to the right.

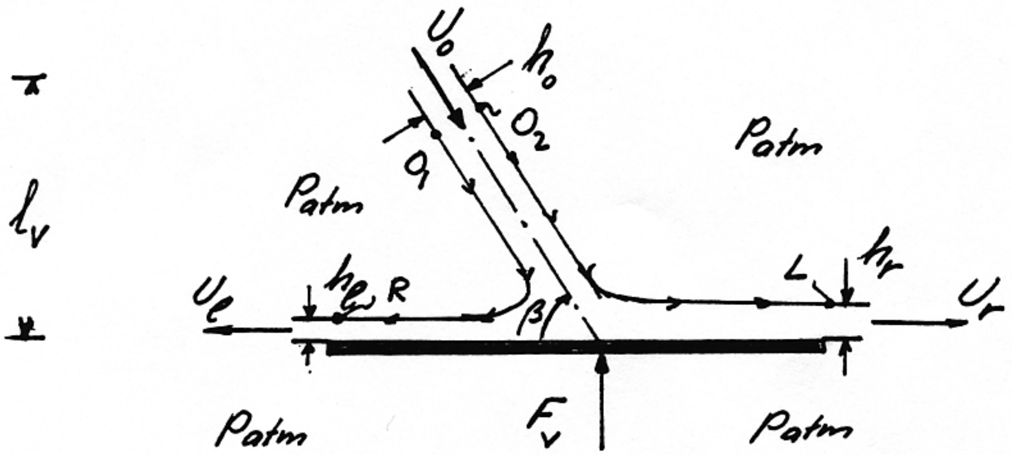
Conservation of momentum in z-direction:

$$\sum \text{Forces in } z = -MP_A \sin 45^\circ - W - MP_B \sin 45^\circ + F_z = 0 \Rightarrow$$

$$\Rightarrow \underline{\underline{F_z}} = 19190 \frac{\sqrt{2}}{2} + 1250 + 5425 \frac{\sqrt{2}}{2} = \underline{\underline{18655 \text{ N}}} \text{ (upwards)}$$

→ The vertical force from the fluid on the elbow is of the same magnitude acting downwards.

- **PROBLEM No. 2:**



a) U_e and U_r are horizontal, and therefore contribute no momentum force in vertical direction. The only vertical momentum force is from the incident jet

$$(\rho U_0^2 h_0 + p_0 h_0) \sin\beta = F_v$$

but $p_0 = 0$ [atmospheric pressure on all sides], so

<u>Vertical jet force is downward $= \rho U_0^2 h_0 \sin\beta$</u>

b) Horizontal momentum balance gives, with all pressures being zero since jets are free

$$\rho U_e^2 h_e + \rho U_0^2 h_0 \cos\beta = \rho U_r^2 h_r$$

or since $U_e = U_0 = U_r$

$$h_r - h_\ell = h_o \cos\beta \qquad (1)$$

Conservation of volume (continuity) gives

$$U_o h_o = Q_{in} = U_r h_r + U_\ell h_\ell = \Sigma Q_{out}$$

or

$$h_r + h_\ell = h_o \qquad (2)$$

Combining (1) and (2) gives

$$\underline{h_r = h_o(1+\cos\beta)/2 \; ; \; h_\ell = h_o(1-\cos\beta)/2}$$

c) Application of Bernoulli along streamline from O_1 to R gives

$$U_o^2/2g + p_o/\rho g + z_o = U_r^2/2g + p_r/\rho g + z_r$$

but $p_o - p_r = 0$ and $z_o - z_r$ is negligible, so

$$U_o = U_r$$

Similarly, Bernoulli from O_2 to L gives

$\underline{U_o = U_\ell \text{ (} = U_r \text{ from above)}}$ q.e.d.

d) From (c) we have along streamline from O_1 to R

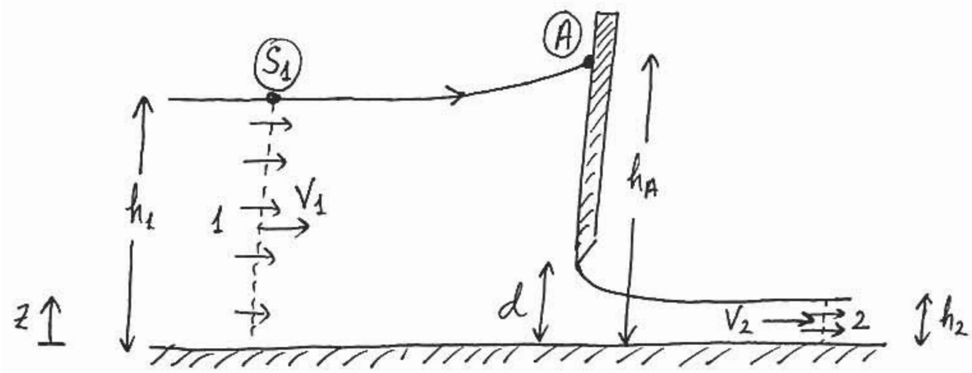
$$U_o^2/2g + (z_o - z_r) = U_r^2/2g$$

Magnitude of $U_o^2/2g = 14^2/(2\cdot 10) \simeq 10\,m$

If $(z_o - z_r) = \ell_v \simeq 1\,m$ $\quad U_r^2 = 14^2 + 2g \Rightarrow U_r \simeq 14.7\,m/s$ with gravity, $14\,m/s$ without. Difference $\simeq 5\%$

<u>I would start to worry if $\ell_v > 1$ to $2m$</u>

- PROBLEM N° 3:



a) Continuity between inflow (1) and outflow (2):

$$Q = h_1 V_1 = h_2 V_2 \Rightarrow V_1 = \frac{h_2}{h_1} V_2$$

This is a short transition of a converging flow, so $\Delta H \approx 0$ and Bernoulli between sections 1 and 2 yields

$$p_1 + \rho g z_1 + \rho \frac{V_1^2}{2} = p_2 + \rho g z_2 + \rho \frac{V_2^2}{2}$$

Since flow in sections 1 and 2 is well-behaved (straight parallel streamlines), $p + \rho g z$ is constant along the depth, i.e.,

$$p_1 + \rho g z_1 = \rho g h_1 \quad \text{(anywhere along section 1)}$$

$$p_2 + \rho g z_2 = \rho g h_2 \quad (\;''\quad ''\quad ''\quad 2)$$

Thus,

$$g h_1 + \frac{1}{2}\left(\frac{h_2}{h_1}\right)^2 V_2^2 = g\, h_2 + \frac{V_2^2}{2}$$

$$V_2 = \sqrt{\frac{2g(h_1 - h_2)}{1-(h_2/h_1)^2}} \overset{h_2 = 0'61 d = 0'61\,m}{=} \sqrt{\frac{2 \cdot 9'8 \cdot (3 - 0'61)}{1-(0'61/3)^2}} = 6'990 \text{ m/s}$$

$$\underline{\underline{Q}} = V_2\, h_2 = 6'990 \cdot 0'61 = \underline{\underline{4'26 \ \frac{m^3/s}{m}}}$$

$$V_1 = \frac{Q}{h_1} = \frac{4'264}{3} = 1'421 \text{ m/s}$$

b)

Now we apply Bernoulli along the streamline from (S_1) to (A):

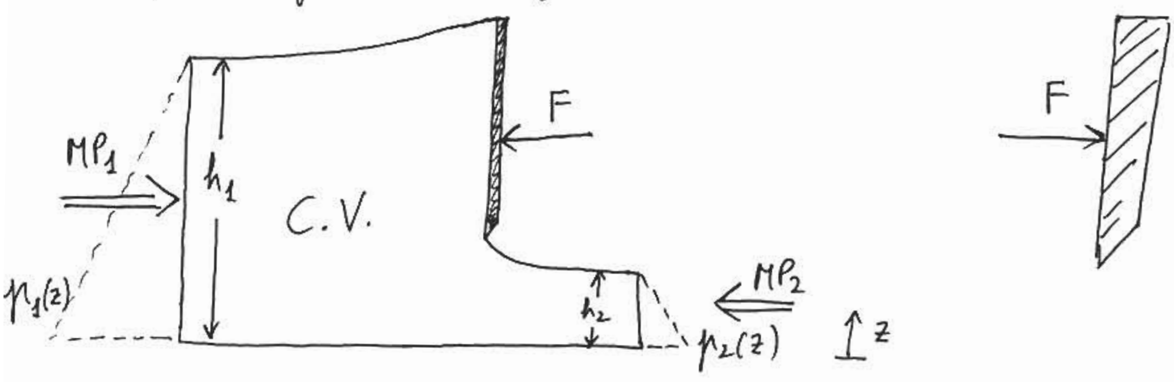
$$\underset{0}{\underset{\parallel}{p_{S1}}} + \rho g h_1 + \frac{1}{2}\rho V_1^2 = \underset{0}{\underset{\parallel}{p_A}} + \rho g h_A + \frac{1}{2}\rho \underset{0}{\underset{\parallel}{v_A^2}}$$

$v_A = 0$ since (A) is a convex corner

$$\underline{\underline{h_A}} = h_1 + \frac{V_1^2}{2g} = 3 + \frac{1'421^2}{2\cdot 9'8} = \underline{\underline{3'10\ m}}$$

c)

To calculate the horizontal force on the gate, we'll apply conservation of linear momentum in x-direction to the control volume between sections 1 and 2:
(Bottom friction is neglected since it is a "short" transition)



$$MP_1 = (p_{CG_1} + \rho V_1^2)\, h_1 = \underbrace{\left(\frac{\rho g h_1}{2} + \rho V_1^2\right)}_{\text{well behaved flow at 1.}} h_1 = \left(\frac{9800\cdot 3}{2} + 1000\cdot 1'421^2\right)\cdot 3 =$$

$$= 50158\ N/m$$

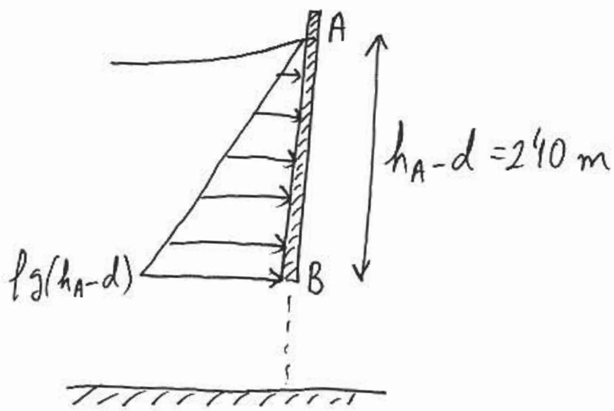
$$MP_2 = \left(\frac{\rho g h_2}{2} + \rho V_2^2\right) h_2 = \left(\frac{9800\cdot 0'61}{2} + 1000\cdot 6'990^2\right)\cdot 0'61 = 31628\ N/m$$

$$\sum F_x = 0 \text{ (steady flow)} \Rightarrow MP_1 - F - MP_2 = 0 \Rightarrow \underline{\underline{F}} = MP_1 - MP_2 = \underline{\underline{18530\ N/m}}$$

(force from fluid on gate acts to the right).

d)

Assuming hydrostatic pressure:

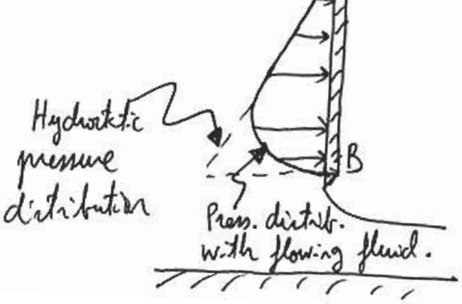


$\underline{\underline{F}}$ = area of the triangle of pressure $= \frac{1}{2}\rho g(h_A-d)^2 =$

$$= \frac{9800}{2}\cdot 2'40^2 = \underline{\underline{21609\ N/m}}$$

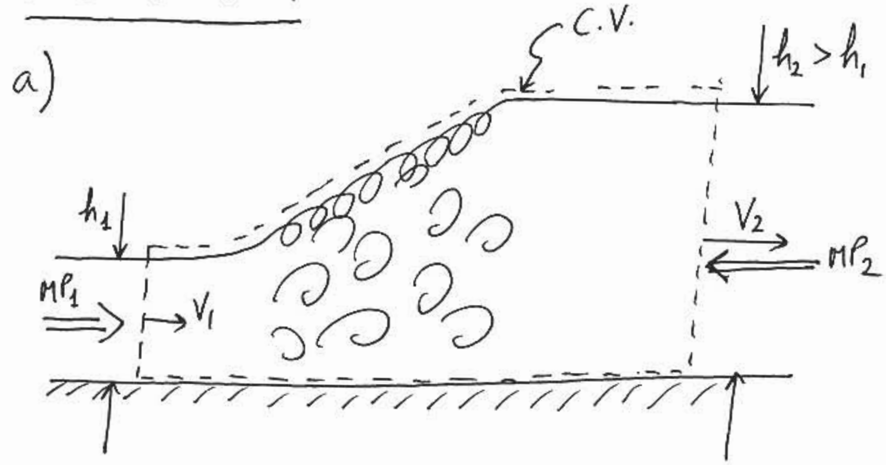
e)

When flow is moving past the gate, there is velocity near the bottom of the gate. Therefore, the pressure will be reduced near the bottom of the gate with respect to hydrostatic pressure (this is suggested by Bernoulli equation). Note also that point B is in contact with the atmosphere, so $p_B = 0$. Thus, the total force on the gate (18530 N/m) is smaller than in the hydrostatic case (21609 N/m).



- PROBLEM Nº 4:

a)



Per unit width into the paper.

Continuity: $V_1 h_1 = V_2 h_2 \Rightarrow V_2 = V_1 \dfrac{h_1}{h_2}$ (1)

Momentum : $MP_1 = MP_2$

NOTE: Short transition $\Rightarrow$ Friction is negligible (But $\Delta H \neq 0$ since flow is expanding).

$$\left(\frac{1}{2}\rho g h_1 + \rho V_1^2\right) h_1 = \left(\frac{1}{2}\rho g h_2 + \rho V_2^2\right) h_2 \quad (2)$$

Substituting (1) into (2):

$$\frac{1}{2}\rho g h_1^2 + \rho V_1^2 h_1 = \frac{1}{2}\rho g h_2^2 + \rho \frac{V_1^2 h_1^2}{h_2^2} h_2$$

$$\frac{g}{2} h_2^3 - \left(\frac{1}{2} g h_1^2 + V_1^2 h_1\right) h_2 + V_1^2 h_1^2 = 0$$

$$h_2^3 - h_1^2 h_2 - \frac{2V_1^2 h_1 h_2}{g} + \frac{2}{g} V_1^2 h_1^2 = 0$$

$$h_2 (h_2^2 - h_1^2) - \frac{2V_1^2}{g} h_1 (h_2 - h_1) = 0$$

$$h_2 (h_2 + h_1)(h_2 - h_1) - \frac{2V_1^2}{g} h_1 (h_2 - h_1) = 0$$

$h_2 > h_1 \Rightarrow h_2 - h_1 \neq 0$, therefore

Take + sign to get $h_2 > 0$

$$h_2^2 + h_1 h_2 - \frac{2V_1^2}{g} h_1 = 0 \Rightarrow h_2 = h_1 \left(\frac{-1 + \sqrt{1 + 8V_1^2/gh_1}}{2}\right)$$

$$\underline{\underline{h_2}} = \frac{-1 + \sqrt{1 + 8 \cdot 10^2/9'8}}{2} = \underline{\underline{4'05\,m}} \; ; \; \underline{\underline{V_2}} = V_1 \frac{h_1}{h_2} = \underline{\underline{2'47\ m/s}}$$

b)

Energy conservation:

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + \underbrace{\Delta H_{1\to 2}}_{\text{HEAD LOSS}}$$

Well-behaved flow in sections 1 and 2 $\Rightarrow$ $\frac{p_1}{\rho g} + z_1 = \text{constant} = h_1$

Thus, $\quad \frac{p_2}{\rho g} + z_2 = \text{constant} = h_2$

$$\underline{\underline{\Delta H_{1\to 2}}} = \frac{V_1^2 - V_2^2}{2g} + (h_1 - h_2) = \frac{10^2 - 2'47^2}{2 \cdot 9'8} + (1 - 4'05) = \underline{\underline{1'75\ m}}$$

Rate of energy dissipation <u>per unit width of the channel</u>:

$$\underline{\underline{\dot{E}_{1\to 2}}} = \rho g Q \Delta H_{1\to 2} = 9800 \cdot (10 \cdot 1) \cdot 1'75 = \underline{\underline{1'72 \cdot 10^5\ \tfrac{J}{ms}}}$$

c) To obtain h_1 and V_1 from h_2 and V_2, we need to apply continuity and momentum, as we did in (a). The two equations remain unchanged when we reverse flow direction. Therefore, if h_2 and V_2 have the same values as in (a), h_1 and V_1 will also have the same values, that is: $\underline{\underline{h_1 = 1\ m}}$ and $\underline{\underline{V_1 = 10\ m/s}}$.

d) Now energy conservation yields:

$$\frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 = \frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + \Delta H_{2\to 1} \Rightarrow \underline{\underline{\Delta H_{2\to 1}}} = -\Delta H_{1\to 2} = \underline{\underline{-1'75\,m < 0!!}}$$

compare with (b)

and $\underline{\underline{\dot{E}_{2\to 1}}} = \rho g Q \Delta H_{2\to 1} = -\underline{\underline{1'72 \cdot 10^5\ J/ms}} < 0!!$

Obviously, energy dissipation cannot be negative, since this would mean creation of energy. Therefore, we conclude that this hypothetical flow from right to left is impossible. This conclusion is true for any values of h_1, V_1 such that $h_1 < h_2$: In a hydraulic jump, flow always goes from smaller to larger depth. This is consistent with the fact that we expect energy dissipation in flow expansions and not in (smooth) flow contractions. Since the hydraulic jump introduces turbulent dissipation, it should correspond to a flow expansion.

- PROBLEM N°5:

a)

$$\underline{\underline{U}} = \frac{1}{h}\int_0^h u(y)\,dy = \frac{u_s}{h^{1+1/n}}\int_0^h y^{1/n}\,dy =$$

$$= \frac{u_s}{h^{\frac{n+1}{n}}}\left[\frac{n}{n+1}\,y^{\frac{n+1}{n}}\right]_0^h = \frac{u_s}{h^{\frac{n+1}{n}}}\,h^{\frac{n+1}{n}}\frac{n}{n+1} = \underline{\underline{\frac{n}{n+1}u_s}}$$

b)

The momentum coefficient is defined as

$$K_m = \frac{\int_A q_\perp^2\,dA}{U^2 A} \;;\quad q_\perp = u = u_s\left(\frac{y}{h}\right)^{1/n} \;;\; U = \text{average velocity}$$

Dividing the numerator and the denominator by the width of the channel,

$$\underline{\underline{K_m}} = \frac{\int_0^h u_s^2\left(\frac{y}{h}\right)^{2/n} dy}{u_s^2\left(\frac{n}{n+1}\right)^2 h} = \left(\frac{n+1}{n}\right)^2 \frac{1}{h^{1+2/n}}\int_0^h y^{2/n}\,dh =$$

$$= \left(\frac{n+1}{n}\right)^2 \frac{1}{h^{1+2/n}}\,\frac{1}{1+2/n}\,h^{1+2/n} = \underline{\underline{\frac{(n+1)^2}{n(n+2)}}}$$

c)

The energy coefficient is defined as

$$\underline{\underline{K_e}} = \frac{\int_A q_\perp^3\,dA}{U^3 A} = \frac{\int_0^h u_s^3\left(\frac{y}{h}\right)^{3/n} dy}{u_s^3\left(\frac{n}{n+1}\right)^3 h} =$$

$$= \left(\frac{n+1}{n}\right)^3 \frac{1}{h^{1+3/n}}\,\frac{1}{1+3/n}\,h^{1+3/n} = \underline{\underline{\frac{(n+1)^3}{n^2(n+3)}}}$$

d) From PROBLEM SET 2, PROBLEM N° 4:

$$u = 0'1 \ln\left(\frac{y + 3\cdot 10^{-4}}{3\cdot 10^{-4}}\right) \qquad 0 \leq y \leq 2 \quad (\text{s.i.})$$

$$u_s = u(y=2) = 0'1 \ln\left(\frac{2 + 3\cdot 10^{-4}}{3\cdot 10^{-4}}\right) = 0'881 \ m/s$$

$$U = 0'780 \ m/s \ (\text{from PS2})$$

$$h = 2 \ m$$

Denote the power-law approximation with a tilde ($\sim$):

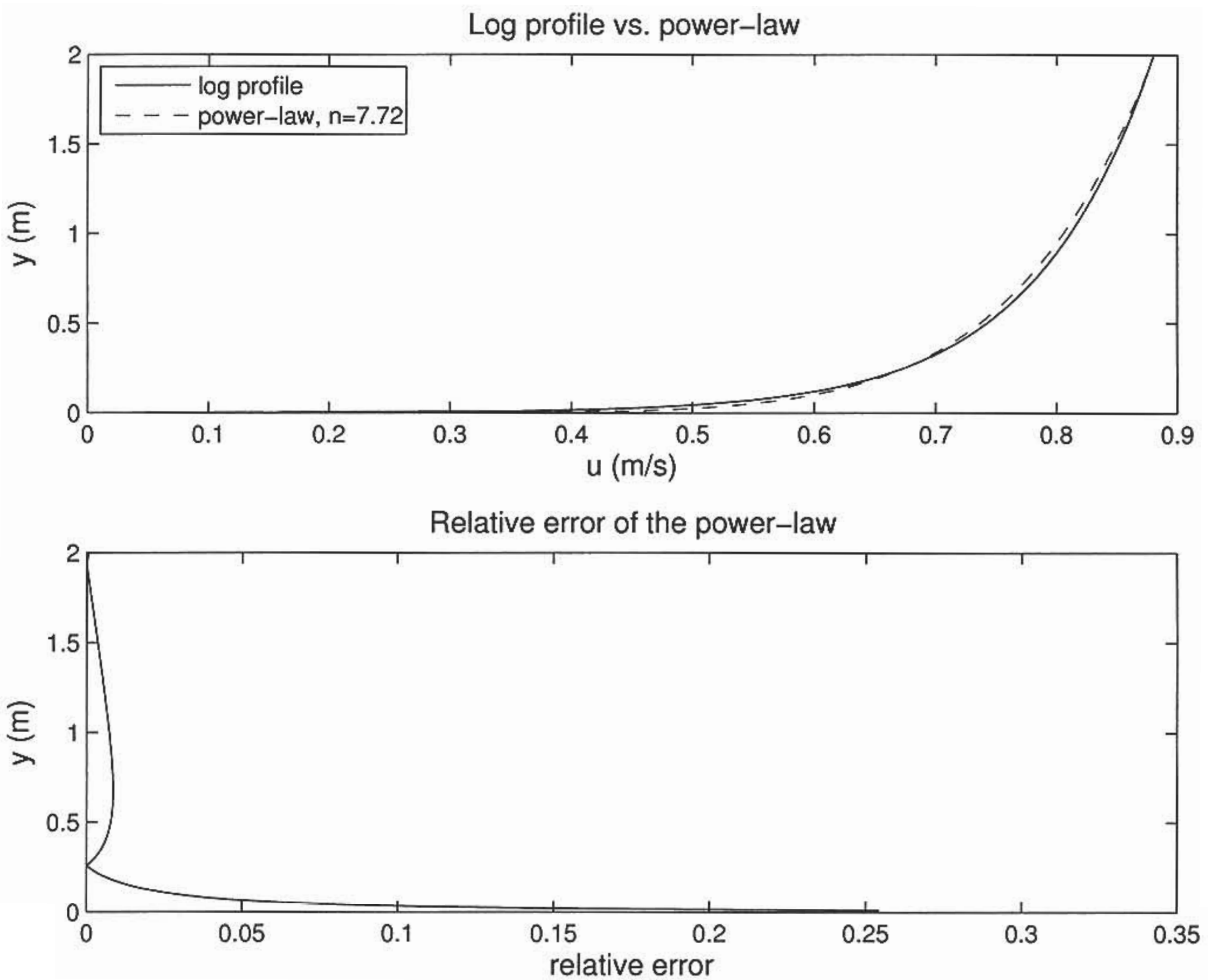
$$\tilde{u}_s = u_s = 0'881 \ m/s$$

$$\tilde{U} = U \Rightarrow \frac{n}{n+1}\tilde{u}_s = \frac{n}{n+1} \cdot 0'881 = 0'780 \Rightarrow \underline{\underline{n = 7'72}}$$

On next page, I have represented the log profile vs. the power-law approximation, as well as the relative error of the power-law (defined as $\left|\frac{\tilde{u}(y) - u(y)}{u(y)}\right|$). As seen in the figure, the agreement is good. The power-law profile slightly underpredicts the log profile (error $\simeq 1\%$) in the upper region, while it yields a 25% overprediction of the near-bottom velocity. Other than in the near-bottom (relevant, e.g., for sediment transport calculations), the agreement is very good.

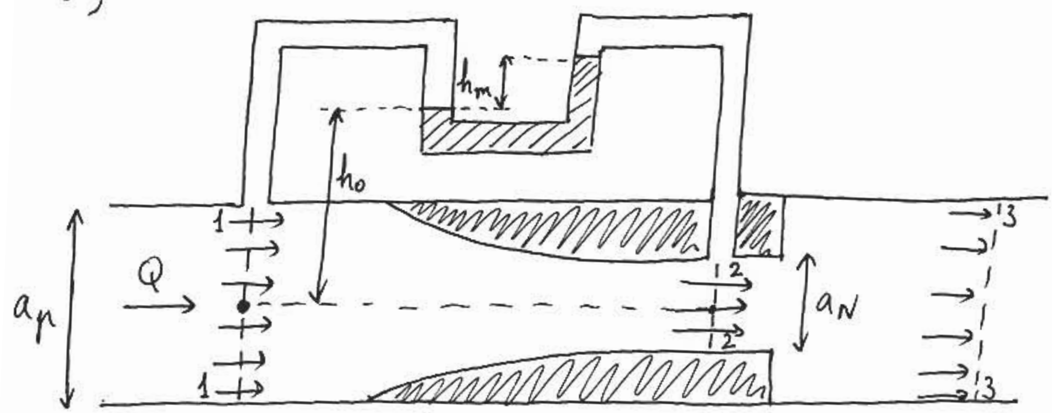
e) $\underline{\underline{K_m}} = \frac{(7'72+1)^2}{7'72\,(7'72+2)} = \underline{\underline{1'013}}$; $\underline{\underline{K_e}} = \frac{(7'72+1)^3}{7'72^2(7'72+3)} = \underline{\underline{1'038}}$

Both have values close to 1, so the momentum and energy of the flow are well represented by the average velocity values. These results are also consistent with what we saw in <u>recitation 5</u>: $\delta^2 \simeq 0'013 \rightarrow K_m = 1 + \delta^2$, $K_e = 1 + 3\delta^2$.


Log profile vs. power−law
log profile
power−law, n=7.72
y (m)
u (m/s)
0
0.5
1
1.5
2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Relative error of the power−law
y (m)
relative error
0
0.5
1
1.5
2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

- PROBLEM Nº 6:

a)



At 1-1 and 2-2 we have well-behaved flow and pressure is hydrostatic. With this, from the manometer reading, we can obtain the pressure difference between 1 and 2:

$$p_{1,CG} - \rho g h_0 - \rho_m g h_m + \rho g (h_m + h_0) = p_{2,CG} \Rightarrow$$

$$\Rightarrow p_{1,CG} - p_{2,CG} = (\rho_m - \rho) g h_m = 12600 \cdot 9'8 \cdot 0'061 = 7532 \text{ Pa}$$

The CG of sections 1 and 2 are on the same streamline. Neglecting wall friction effects (since transition is short), we apply Bernoulli between 1-1 and 2-2 considering the center streamline:

$$p_{1,CG} + \frac{1}{2} \rho V_1^2 = p_{2,CG} + \frac{1}{2} \rho V_2^2 \qquad (1)$$

where $V_1 = Q/A_1$ and $V_2 = Q/A_2$ ($K_m = K_e = 1$ assumed) and we have applied that $z_{1,CG} = z_{2,CG}$.

Continuity dictates:

$$Q = V_1 \, a_p^2 = V_2 \, a_N^2 \Rightarrow V_2 = \frac{a_p^2}{a_N^2} V_1 = 4 V_1 \quad (2)$$

Plugging (2) into (1):

$$p_{1,CG} + \frac{1}{2}\rho V_1^2 = p_{2,CG} + 8\rho V_1^2 \Rightarrow V_1 = \sqrt{\frac{2(p_{1,CG}-p_{2,CG})}{15\rho}} = \sqrt{\frac{2\cdot 7532}{15\cdot 1000}} = 1'00\ m/s$$

$V_2 = 4'00\ m/s$, $\underline{\underline{Q}} = V_1\, a_p^2 = 1\cdot 0'1^2 = \underline{\underline{0'01\ m^3/s}}$

b)

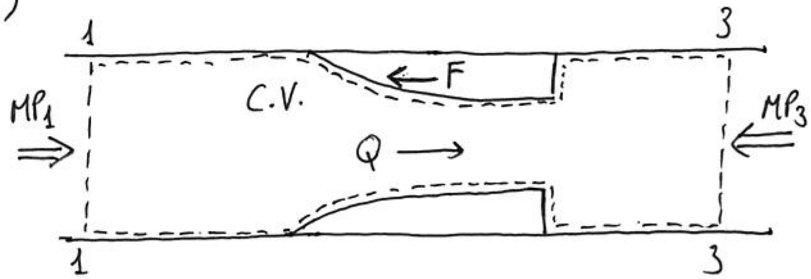
As seen in Lecture 13, the headloss in an abrupt expansion is

$$\Delta H_{exp} = \frac{(V_2 - V_3)^2}{2g}$$

By continuity, $Q = V_1\, a_p^2 = V_3\, a_p^2 \Rightarrow V_3 = V_1$. Therefore

$$\underline{\underline{\Delta H_{exp}}} = \frac{(4-1)^2}{2\cdot 9'8} = \underline{\underline{0'459\ m}}$$

c)



Conservation of energy between 1-1 and 3-3 dictates

$$\frac{p_{1,CG}}{\rho g} + \cancel{z_{1,CG}} + \cancel{\frac{V_1^2}{2g}} = \frac{p_{3,CG}}{\rho g} + \cancel{z_{3,CG}} + \cancel{\frac{V_3^2}{2g}} + \Delta H_{exp} \quad \left(\begin{matrix}K_e \approx 1\\ \text{assumed}\end{matrix}\right)$$

Since $z_{1,CG} = z_{3,CG}$; $V_1 = V_3$; $\Delta H_f = 0$ (wall friction is neglected since transition is short).

$$p_{1,CG} - p_{3,CG} = \rho g\, \Delta H_{exp} = 9800 \cdot 0'459 = 4498\ Pa$$

Momentum conservation in the C.V. represented above yields:

$$0 = MP_1 - MP_3 - F \Rightarrow F = (\rho V_1^2 + p_{1,CG})A_1 - (\rho V_3^2 + p_{3,CG})A_3 =$$

$\left(\begin{matrix}K_m \approx 1\\ \text{assumed}\end{matrix}\right)$

$$= (p_{1,CG} - p_{3,CG})\, a_p^2 = 4498 \cdot 0'1^2 \simeq \underline{\underline{45\ N}} \text{ (to the left)}$$

The force from the fluid on the nozzle is 45 N to the right.