

## Problem Set 4

*Due on Monday, March 20th*

**Important note:** Please start a new sheet of paper for each problem in the problem set. Write the names of the group members who contributed to the solution at the beginning of each problem.

### Problem 1

A converging elbow turns water through an angle of 90 degrees in a vertical plane, as shown in Figure 1. The flow cross-sectional diameter is 40 cm at the elbow inlet (section A), and 20 cm at the elbow outlet (section B). The elbow flow passage volume is  $0.15 \text{ m}^3$ . An oil of specific gravity 0.85 flows through the elbow. The elbow inlet pressure (section A) is  $150 \text{ kPa}$  and the elbow outlet pressure is  $130 \text{ kPa}$ . Sections A and B have the same elevation.

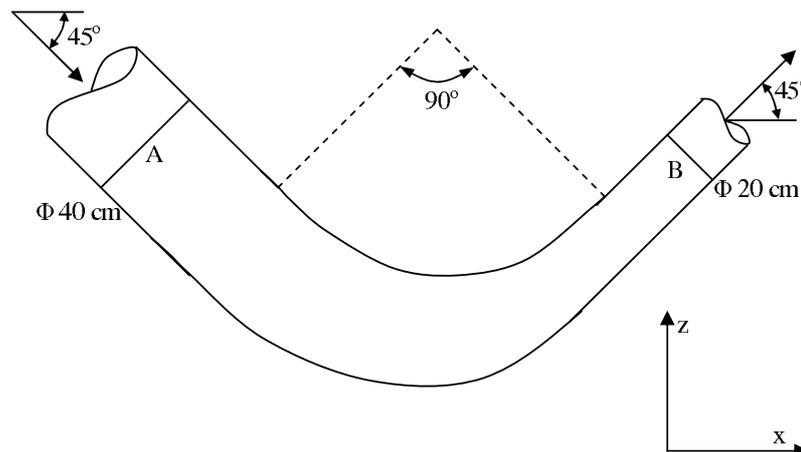


Figure 1: Elbow junction in Problem 1.

Calculate the horizontal ( $x$ -direction) and vertical ( $z$ -direction) forces exerted by the fluid on the elbow. Neglect losses.

NOTE: Gravity is acting in the  $-z$  direction.

## Problem 2

In a hydraulic structure, known as a drop structure, a free surface flow in a rectangular channel flows over a brink, drops in a free fall, and impacts the horizontal bottom of the drop at an angle  $\beta$ . The plane jet's impact with the bottom causes it to split up into two separate jets –one to the left, the other to the right. The impact of a plane jet at an angle  $\beta$  to a surface forms an important part of the hydraulic analysis of a drop structure and is shown idealized in Figure 2. This is the problem you are asked to solve.

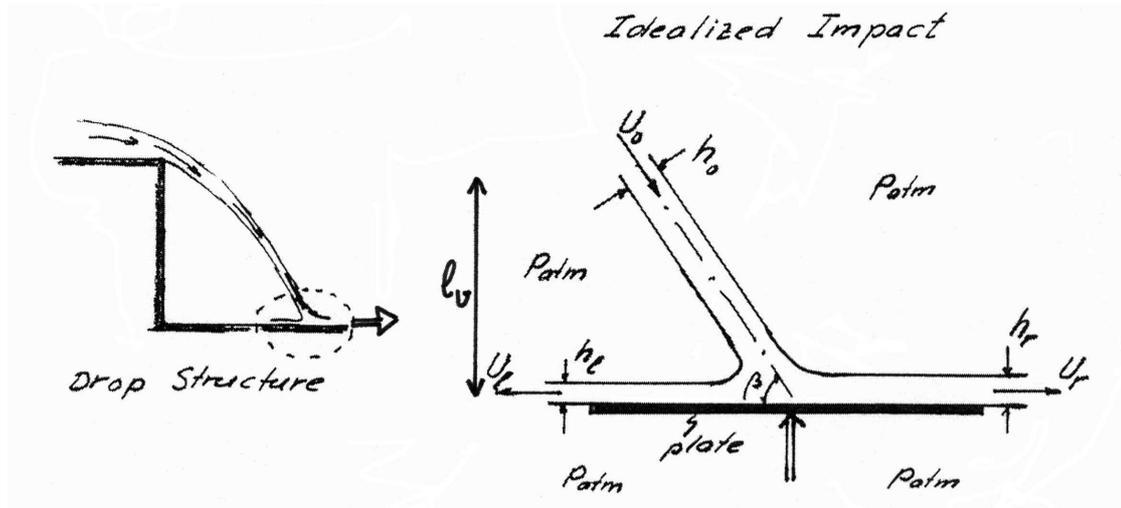


Figure 2: Idealized impact in drop structure (Problem 2).

The velocity of the plane jet incident on the plate at angle  $\beta$  is  $U_0$  and its thickness is  $h_0$ . The jet splits into two with thicknesses and velocities  $h_l, U_l$  and  $h_r, U_r$ , respectively.

- Neglecting gravity determine the vertical force (per unit length into paper) exerted by the jet impact on the plate.
- Assuming that  $U_0 = U_l = U_r$  and neglecting friction along the plate, determine the thicknesses of the split jets,  $h_l$  and  $h_r$ .
- If gravity (i.e., elevation head differences) and losses are neglected, show that application of Bernoulli's equation along streamlines leads to the assumption of  $U_0 = U_l = U_r$  made in part **b**.
- If  $U_0 = 14 \text{ m/s}$ , how large could the vertical scale,  $l_v$  (shown in the figure), be before you would start worrying about having neglected gravity (elevation differences) in part **c** above?

### Problem 3

Water flow in open channels can be controlled and measured with a sluice gate. Water flows under the gate in a wide channel as shown in Figure 3. Sections 1 (depth  $h_1$ ) and 2 (depth  $h_2$ ) are respectively located at moderate distances upstream and downstream of the gate. At these sections, the flow is uniform and well behaved. The depth directly under the gate,  $d$ , is 1.00 m. The coefficient of contraction for the outlet of the gate is 0.61. Bottom friction can be neglected.

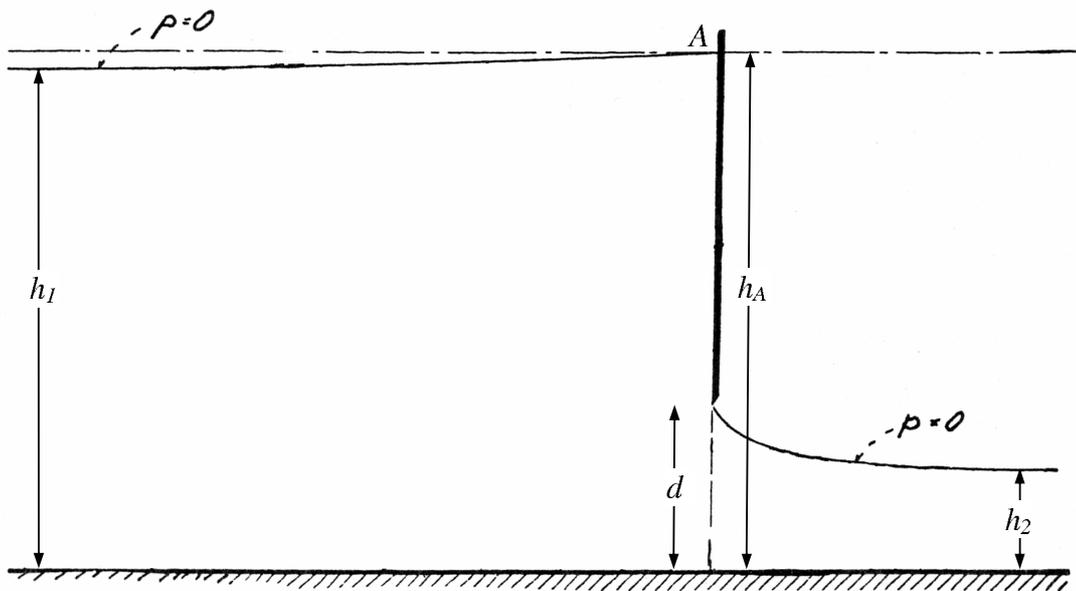


Figure 3: Sluice gate in Problem 3.

- If  $h_1$  is measured and found to be 3.00 m, what is the discharge per unit width,  $Q$ ?
- Find the depth along upstream face of gate,  $h_A$ .
- For these flow conditions, what is the horizontal force on the gate due to the fluid?
- If the pressure variation on the gate were hydrostatic, what would be the total horizontal force on the gate?
- Why is the force obtained for **d** different from the force obtained for **c**?

## Problem 4

Figure 4 shows the phenomenon of hydraulic jump in a rectangular channel, where a rather abrupt change occurs in the depth of the flow.

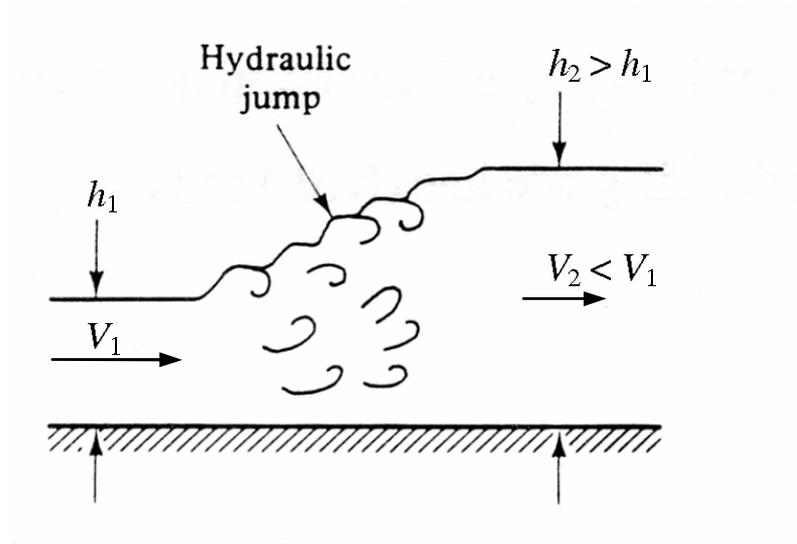


Figure 4: Hydraulic jump in Problem 4.

- The upstream depth and velocity are  $h_1 = 1 \text{ m}$  and  $V_1 = 10 \text{ m/s}$ . Use continuity and momentum conservation (with no friction) to calculate the downstream depth and velocity,  $h_2$  and  $V_2$ .
- For these flow conditions, calculate the headloss and the rate of energy dissipation in the hydraulic jump.
- If flow direction was changed, i.e., if flow was from right to left, and you knew  $h_2$  and  $V_2$  to have the values obtained in **a**, show that  $h_1$  and  $V_1$  would also have the same values as in **a**.
- Now, for the reversed flow (flow from right to left), what is the difference in total head between inflow and outflow? What is the rate of energy dissipation? What is wrong with this picture?

## Problem 5

The vertical velocity distribution for water flow in a wide rectangular open channel of depth  $h$  can be approximately described by a power-law profile

$$u = u_s \left( \frac{y}{h} \right)^{1/n} \quad (1)$$

with  $n \geq 1$ , where  $u_s$  is the maximum velocity, which happens at the surface, and  $0 \leq y \leq h$  is the vertical coordinate ( $y = 0$  at channel bottom).

- a) Find the depth-averaged velocity  $U$  as a function of  $n$  and  $u_s$ .
- b) Find the momentum coefficient  $K_m$  as a function of  $n$ .
- c) Find the energy coefficient  $K_e$  as a function of  $n$ .

In Problem Set 2, Problem 4, we deduced that the velocity profile in an open channel is given by a logarithmic law. Now we want to approximate this logarithmic law by the simpler power-law profile given by (1). Assume the flow characteristics given in Problem Set 2 ( $u_* = 4 \text{ cm/s}$ ,  $h = 2 \text{ m}$ , and  $y_0 = 0.3 \text{ mm}$ ) for the remainder of the problem.

- d) Taking  $u_s$  in the power-law profile equal to the surface velocity in the logarithmic profile, calculate the value of  $n$  for which both profiles yield the same total discharge. Represent both profiles on the same figure and comment on the accuracy of the approximation.
- e) Use your power-law profile approximation to estimate the values of  $K_m$  and  $K_e$  for the logarithmic profile.

## Problem 6

Figure 5 shows a section of a horizontal pipe in which a nozzle-sleeve is inserted. The pipe has a **square cross-section** with a sidelength of  $a_p = 10 \text{ cm}$ , and the nozzle-sleeve introduces a smooth transition that reduces this sidelength to  $a_N = a_p/2$ . A mercury manometer ( $\rho_m = 13.6\rho$ ) is connected to the pipe as shown and gives a reading of  $h_m = 6.1 \text{ cm}$ . Effects of wall friction are neglected.

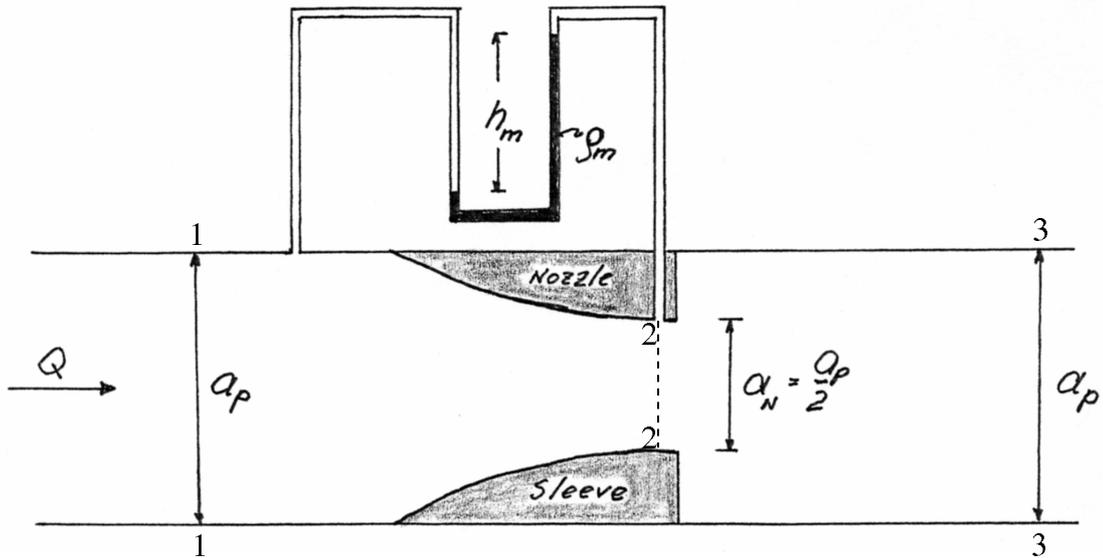


Figure 5: Pipe with nozzle-sleeve in Problem 6.

- Determine the discharge in the pipe,  $Q$ .
- Determine the headloss associated with the nozzle-sleeve inserted in the pipe.
- Determine the force exerted by the flow on the nozzle-sleeve. (Hint: Since you can calculate the pressure difference between sections 1-1 and 3-3, you can obtain the force by analyzing the control volume between these two sections).