

## Problem Set 3

*Due on Monday, March 6th*

**Important note:** Please start a new sheet of paper for each problem in the problem set. Write the names of the group members who contributed to the solution at the beginning of each problem.

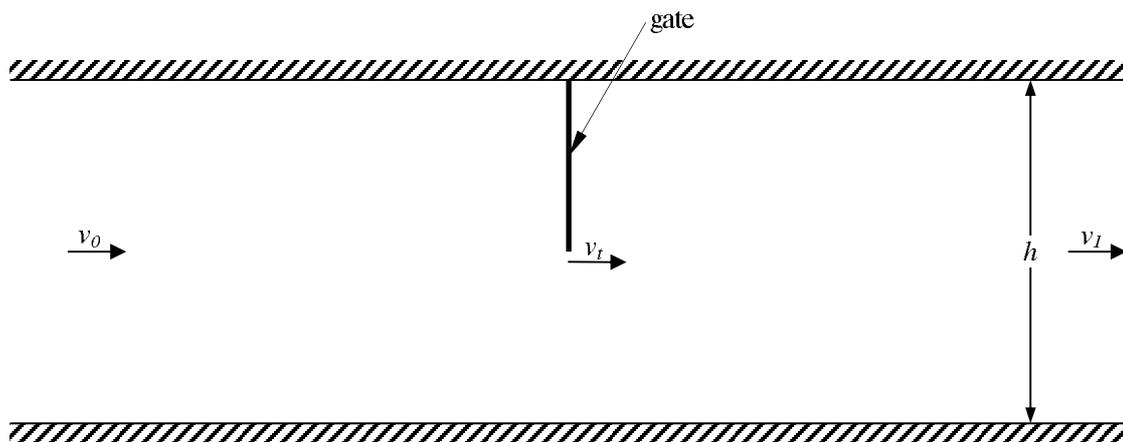
**Problem 1**

Figure 1: Rectangular duct in Problem 1.

A gate of length  $h/2$  is present in a horizontal wide rectangular duct, as shown in Figure 1.

- Sketch a flow net using 4 stream tubes.
- In the figure,  $v_0$  and  $v_1$  are the velocities in the center of the duct far upstream and far downstream the gate. Is  $v_0 = v_1$ ?
- Estimate from your flow net the mean velocity slightly below the tip of the gate,  $v_t$ , relative to  $v_0$ .
- Estimate the pressure slightly below the tip of the gate, relative to the pressure at the same level far upstream.

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Please see:

Problem 3.56 in Munson, Bruce R., Donald F. Young, and Theodore H. Okiishi. *Fundamentals of Fluid Mechanics*. Instructor's Manual. 2nd ed. New York, NY: John Wiley & Sons, Inc., 1994, pp. 3-50.

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Please see:

Problem 3.22 in Munson, Bruce R., Donald F. Young, and Theodore H. Okiishi. *Fundamentals of Fluid Mechanics*. Instructor's Manual. 2nd ed. New York, NY: John Wiley & Sons, Inc., 1994, pp. 3-20.

## Problem 4

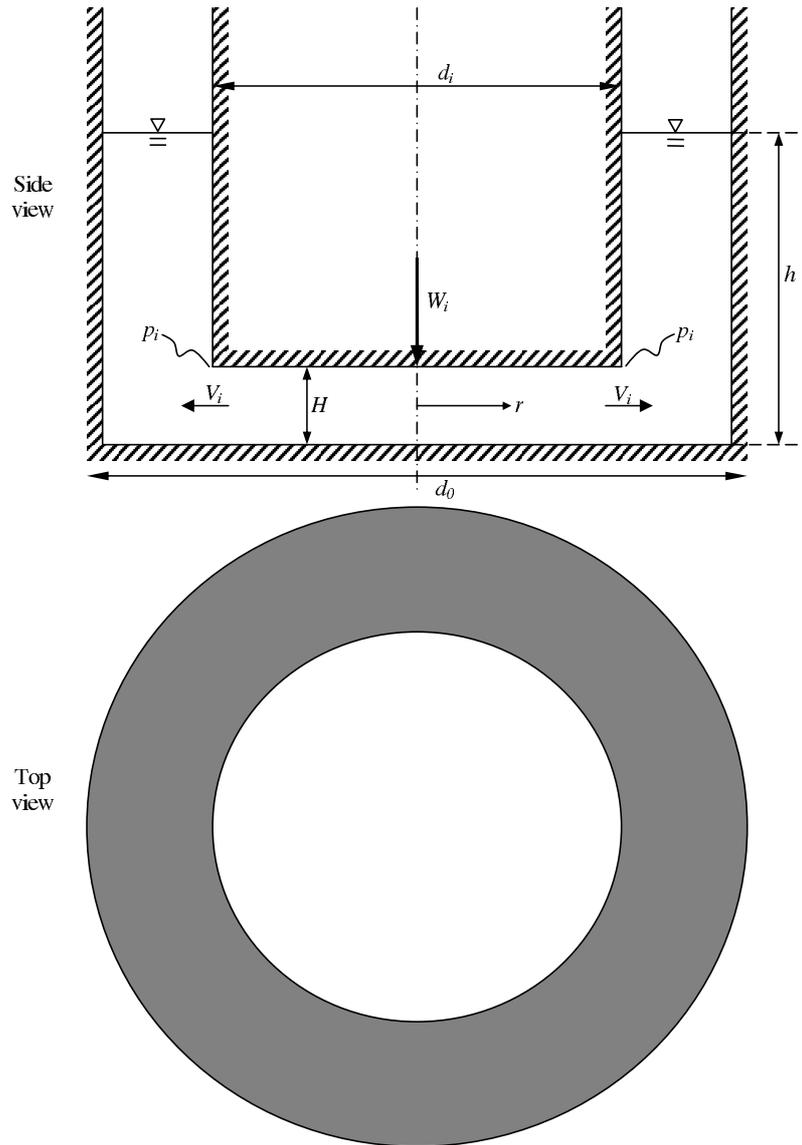


Figure 4: Concentric cylinders in Problem 4.

Figure 4 shows an inner circular cylinder of diameter  $d_i$  placed concentrically inside an outer cylinder of diameter  $d_0$ . The inner cylinder is being pushed vertically into the outer cylinder at a constant downward velocity of  $W_i$ . The outer cylinder was originally filled with a fluid of density  $\rho$  to a level of  $h_0$  above its bottom.

a) Sketch the streamlines of the flow under the inner cylinder as seen from the top (i.e., the streamlines in the white circle in the "Top view" of Figure 4).

- b)** Denoting the height of the gap between the two cylinder bottoms by  $H$ , derive an expression for the depth of water in the outer cylinder,  $h$ , for  $H \leq h_0$  (assume the free surface to be horizontal).
- c)** Derive an expression for  $V_i$ , the horizontal velocity in the radial direction averaged over the gap height,  $H$ , at the outflow from under the inner cylinder, i.e.,  $V_i$  at  $r = d_i/2$ .
- d)** How does the gap-averaged radial velocity,  $V(r)$ , vary with  $r$  for  $0 \leq r \leq d_i/2$ ?
- e)** Assume that  $V_i$  and  $V(r)$  are known and that the pressure at the outflow corner of the inner cylinder,  $r = d_i/2$ , is  $p_i = \rho g(h - H)$ . Obtain an expression for  $p(r)$  as a function of the known values. For this part, you can neglect the pressure variation with  $z$  in the gap between the two cylinder bottoms and any effects of unsteadiness.
- f)** With what vertical force is the inner cylinder being pushed downwards? Neglect the inner cylinder's weight.

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Please see:

Problem 4.57 in Munson, Bruce R., Donald F. Young, and Theodore H. Okiishi. *Fundamentals of Fluid Mechanics*. Instructor's Manual. 2nd ed. New York, NY: John Wiley & Sons, Inc., 1994, pp. 4-51.

## Problem 6

To throw a curveball, a pitcher has to put a spin on the ball. This spin causes the ball to deflect from a straight trajectory, through a physical mechanism called the Magnus effect.

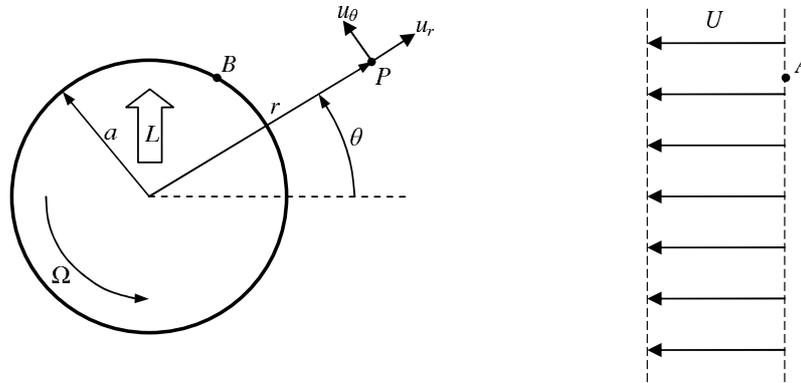


Figure 6: Cylinder in Problem 6.

We will study a simplified 2-D version of this phenomenon, in which the ball will be modeled by a cylinder of the same radius. Figure 6 shows a cylinder of radius  $a$  spinning in counter-clockwise direction at an angular speed  $\Omega$ . In the far field, there is a uniform wind velocity  $U$  towards the left. For simplicity, we will adopt a 2-D polar coordinate system  $(r, \theta)$  with origin at the center on the ball, as represented in the figure. In this coordinate system, the velocity vector has components  $(u_r, u_\theta)$ . Ignore the arrow marked with an  $L$  until part **e**. Gravity is neglected in this problem.

For this flow, the velocity potential is given by

$$\phi = -U \left( r + \frac{a^2}{r} \right) \cos \theta + a^2 \Omega \theta.$$

In 2-D polar coordinates, the velocity field is related to the velocity potential through

$$u_r = \frac{\partial \phi}{\partial r}, \quad u_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}.$$

- Deduce the expression for the velocity field,  $(u_r, u_\theta)$ .
- The radius of a baseball ball is  $a = 3.75 \text{ cm}$ . In a curveball, the ball moves at a speed of about  $U = 30 \text{ m/s}$  and rotates at an angular speed of  $\Omega = 25 \text{ rps}$ . Deduce the location of the stagnation points in the flow for these values of the parameters.
- Based on your knowledge of the stagnation points and the far field velocity, sketch the streamlines for the values of  $a$ ,  $U$ , and  $\Omega$  given in **b**.

**d)** By applying Bernoulli's theorem between a point in the far field,  $A$ , and a point on the cylinder surface,  $B$ , show that the pressure on the cylinder surface ( $r = a$ ) is given by

$$p(r = a, \theta) = \frac{1}{2}\rho \left( U^2(1 - 4 \sin^2 \theta) - (a\Omega)^2 - 4a\Omega U \sin \theta \right)$$

where  $\rho$  is the air density.

**e)** Calculate  $L$ , the lift force per unit length (perpendicular to the plane of the paper) exerted by the flow on the cylinder, by integrating the vertical contribution of the pressure force on the cylinder surface.

**f)** Estimate the lift force on a baseball ball by assuming it equivalent to a cylinder of length  $l = 2a$  and taking the values of  $a$ ,  $U$ , and  $\Omega$  given in **b**. If the weight of a baseball ball is  $1.4 \text{ N}$ , estimate the ratio between the lift force and the weight. Measurements show that, for the given parameter values, the lift force in a real baseball ball is about  $1/3$  of its weight. How well does our model work?