1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 7

Shear Failures, Shear Transfer, and Shear Design

- Structural behavior
  - Structural members are subjected to shear forces, generally, in combination with flexure, axial force, and sometimes with torsion.
  - Shear failures are brittle failures primarily because shear resistance in R/C relies on tensile as well as the compressive strength of concrete. Although cracking introduces complications it is still convenient to use classical concepts in analyzing concrete beams under shear failure. Such concepts indicate that shear failure is related to diagonal tensile behavior in concrete. R/C beam must be safe against premature failure due to diagonal tension.

- Failure modes due to shear in beams
  - Diagonal tension failure – sudden
  - Shear-compression failure – gradual
  - Shear-bond failure

  → In general, the design for shear is based on consideration of diagonal (inclined) tension failure.

- Failure of R/C by inclined cracking
  - Inclined cracking load

    $V_c = V_{cc} + V_{ci} + V_d$

    where V_{cc} = shear transferred through the uncracked portion of the concrete,

V_{ci} = vertical component of the aggregate interlocking force in the cracked portion of the concrete, and

V_d = shear force carried through the dowel action of the longitudinal steel.

- Shear strength of the beam without transverse reinforcement is based on the interactive effect of shear stress $v = K_1\left(\frac{V}{bd}\right)$ and flexural stress $f_x = K_2\left(\frac{M}{bd^2}\right)$ leading to dependence on the ratio of $\frac{v}{f} = K \cdot \frac{Vd}{M}$.

Basis of design

- Total ultimate shear force V_u

  $V_u \le \phi V_n$

  where ϕ = the strength reduction factor for shear, and

  V_n = nominal shear strength.

- Nominal shear strength

  $V_n = V_c + V_s$

  where V_c = inclined cracking load of concrete,

  V_s = shear carried by transverse reinforcement.

- Shear strength expression of concrete given by ACI:

$$V_c = \left(1.9\sqrt{f_c^{'}} + 2500\frac{\rho_w V_u d}{M_u}\right)b_w d \le 3.5\sqrt{f_c^{'}}b_w d$$

  where $f_c^{'}$ = compressive strength of the concrete, in psi,

  $\rho_w = \frac{A_s}{b_w d}$ = longitudinal tensile steel ratio,

  b_w = the effective width of the beam, in inches

M_u = total bending moment of the beam, $\frac{V_u d}{M_u} \le 1$, in lbs-in,

d = the effective depth of the beam, in inches, and

s = spacing of stirrups, in inches.

An alternative simpler equation is

$$V_c = 2\sqrt{f_c'} b_w d$$

- The required shear strength to be provided by the steel (vertical web reinforcement):

$$V_s = \frac{\left(V_u - \phi V_c\right)}{\phi} = \frac{V_u}{\phi} - V_c = \frac{A_v f_y d}{s}$$

where f_y = yield strength of the steel and

s = spacing of stirrups.

When stirrups with inclination θ are used, the contribution of steel to shear strength becomes:

$$V_s = \frac{A_v f_y d}{s}\left(\sin\theta + \cos\theta\right)$$

- Contribution of axial forces
- Minimum web reinforcement
- Shear transfer
  - Shear in concrete can cause inclined cracking across a member. It is also possible that shear stresses may cause a sliding type of failure along a well-defined plane. Because of previous load history, external tension, shrinkage, etc., a crack may have formed along such a plane even before shear is applied. Upon application of shear forces we have the problem of quantifying shear stress transferred across the cracked sections.
  - General shear transfer mechanisms are

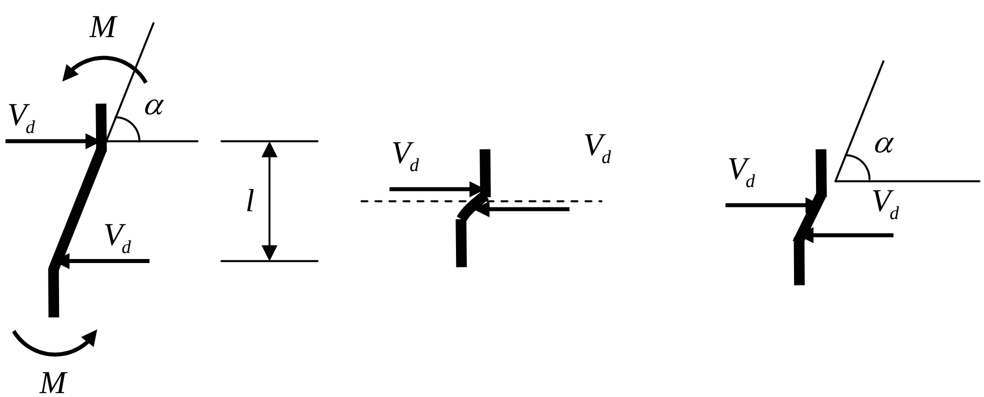
1. Through still uncracked concrete

2. Direct thrust
3. Dowel action
4. Aggregate interlock

→ Reinforcement provides clamping action.

1. Transfer of shear through intact concrete such as the compression region in a beam
2. Direct thrust
   - Models of shear transfer:
     1. Arch analogy: Beam example
     2. Truss analogy (strut-and-tie action): Corbel example

     → Failure mechanisms of corbels:
       - Flexural-tension failure
       - Diagonal splitting failure
       - Diagonal cracks and shear force failure
       - Splitting along flexural reinforcement failure
       - Local cracking at support
       - Local splitting due to cracking
3. Dowel action
   - Three mechanisms of dowel action:



Flexural
$$V_d = \frac{2M}{l}$$

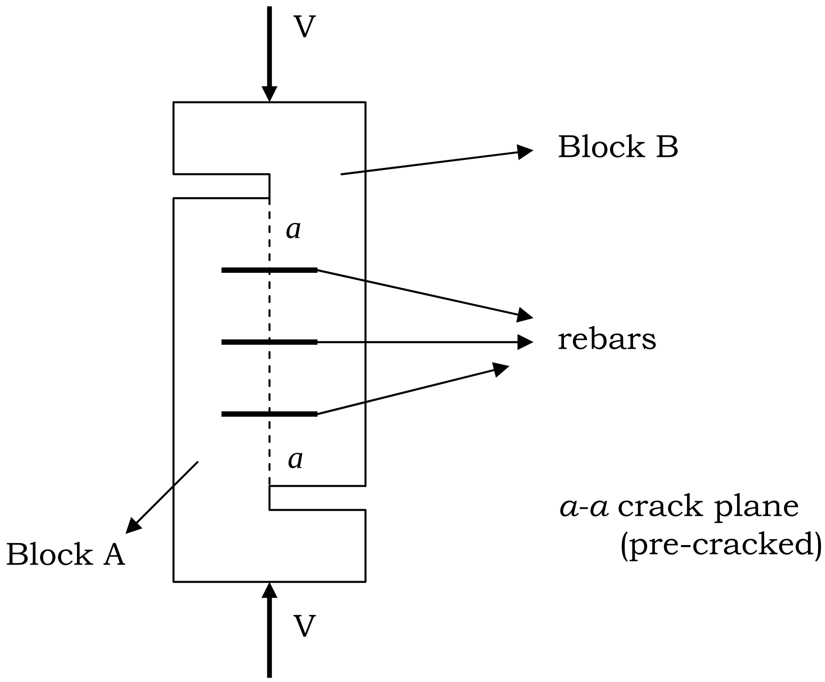
V_d shear
$$V_d = \frac{A_s f_y}{\sqrt{2}}$$

Kinking
$$V_d = A_s f_y \cos\alpha$$

- Shear transfer through dowel action is approximately 25~30% of the shear resisted by the interface shear mechanism.
- Note that the shear yield stress may be determined from von Mises yield function:

$$\tau_y = \frac{A_s f_y}{\sqrt{3}}$$

4. Interface shear transfer: Aggregate interlock + Dowel action
   - o Simple friction behavior



   - o Assume that the movement of Block A is restraint. Upon application of V Block B moves downward and tends to go to right-opening of crack. Crack plane is in compression. Dowel is in tension.
   - o Shear force due to simple friction and dowel

     $V_f = \mu A_s f_y$

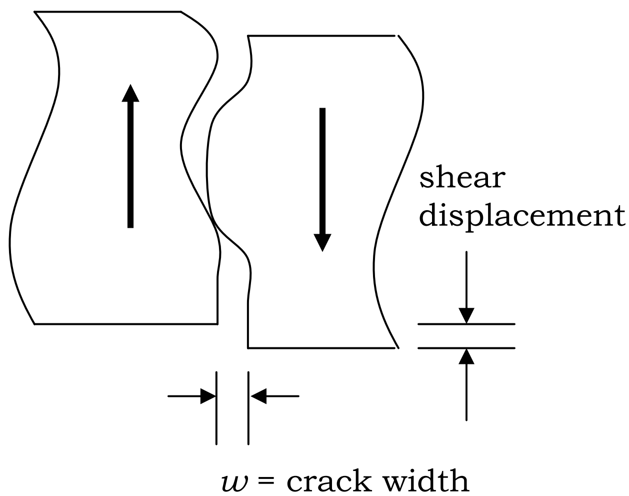
     where μ = friction coefficient.

- Total shear capacity

$$V = V_d + V_f = \frac{A_s f_y}{\sqrt{3}} + \mu A_s f_y = A_s f_y \left(\frac{1}{\sqrt{3}} + \mu \right)$$

→ $A_s = \dfrac{V}{f_y \left(\frac{1}{\sqrt{3}} + \mu \right)}$ = required area of the steel

- Modeling of aggregate interlock and shear modulus of cracked concrete in R/C elements



Crack surface

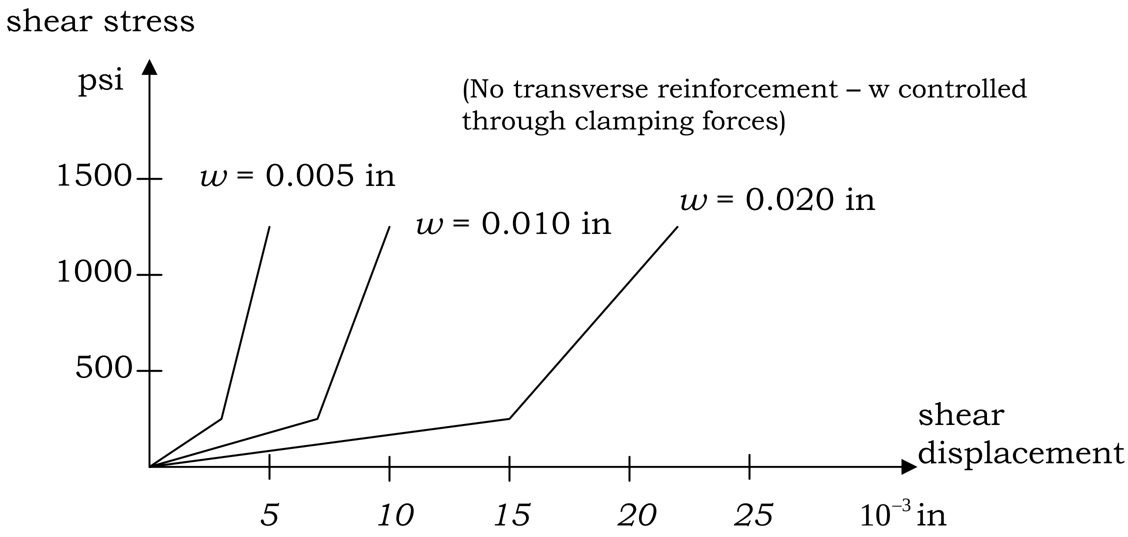
→ Sufficient shear displacement should take place before interlock occurs.

→ Crack width will increase with increased shear displacement.

- A fundamental theory was developed at M.I.T.
- At contact points:

  Frictional resistance due to general roughness of a crack in concrete,

  Additional frictional resistance due to local roughness. (also involves cyclic shear effects)



- Calculation of the deflection due to shear-slip
- Equilibrium + Compatibility + Deformation

$$\delta = \frac{\alpha}{1 + K_D \frac{\beta}{K_N}} w_0 + \frac{1}{\frac{K_N}{\beta} + K_D} V$$

where δ = shear-slip deflection,

α = a coefficient representing gaps produced between asperities,

w_0 = initial crack displacement,

K_D , K_N = coefficients relating to dowel and normal stiffnesses,

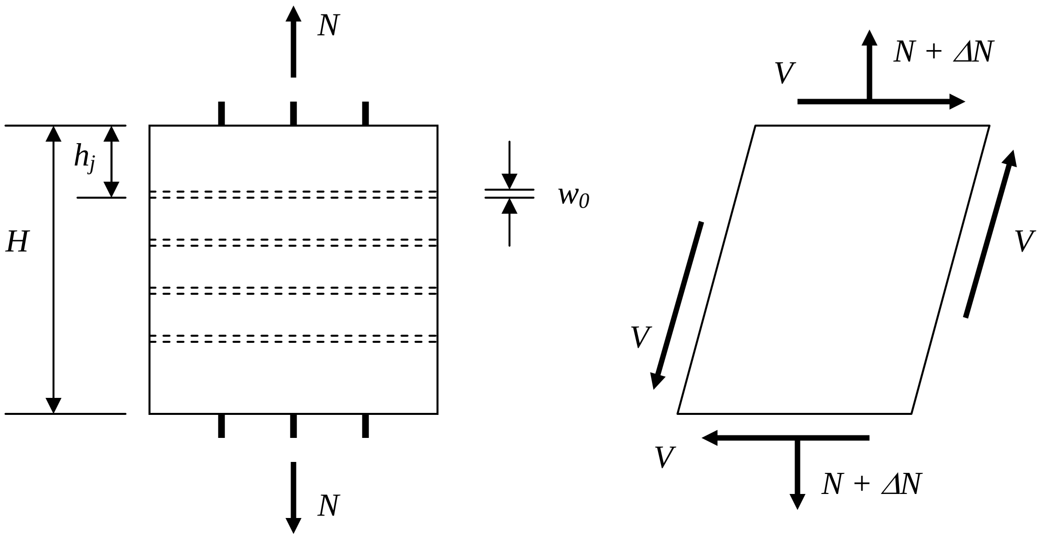
β = a coefficient representing frictional effects at contact points, and

V = applied shear load.

→ α increases with the number of loading cycles.

→ β decreases with the number of loading cycles.

- Overall cracked panel shear modulus G_{cr}



  - Overall effective shear modulus is calculated by

$$G_{cr} = \left[\frac{1}{h\left(\frac{K_N}{\hat{\beta}} + K_D\right)} + \frac{1}{G_e} \right]^{-1}$$

where h = distance between two cracks,

$\hat{\beta}$ = a coefficient obtained from regression analysis, and

G_e = elastic shear modulus.

- Examples of structural applications where inclusion of shear transfer mechanism is important to reduce the required transverse reinforcement for constructability and efficiency
  - Nuclear containment structures (R/C, P/C, hybrid systems)
  - Offshore concrete gravity structures
  - Shear walls

- Design Example – Failure investigation of a prestressed concrete bridge girder