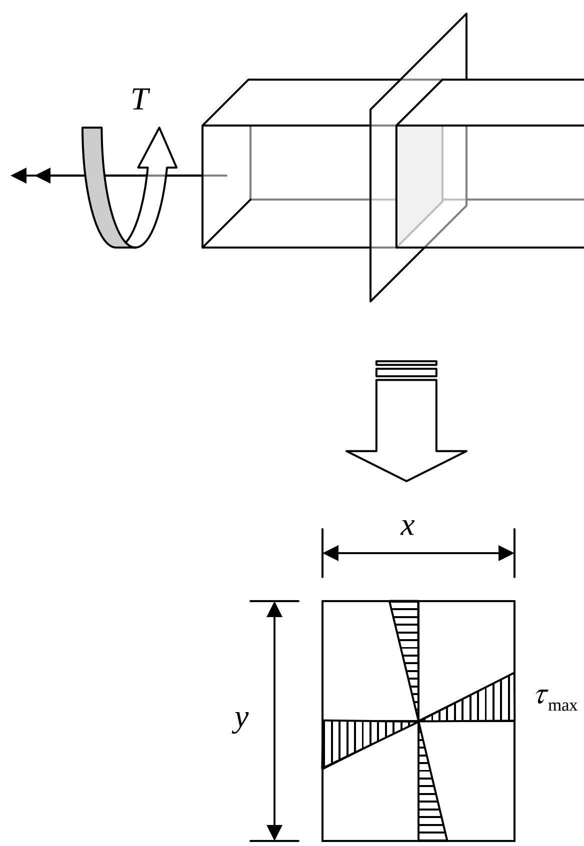
1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 10

Torsion, Shear, and Flexure

- Torsion
  - Stress distribution on a cross section subject to torsion



x: narrow side

y: wide side

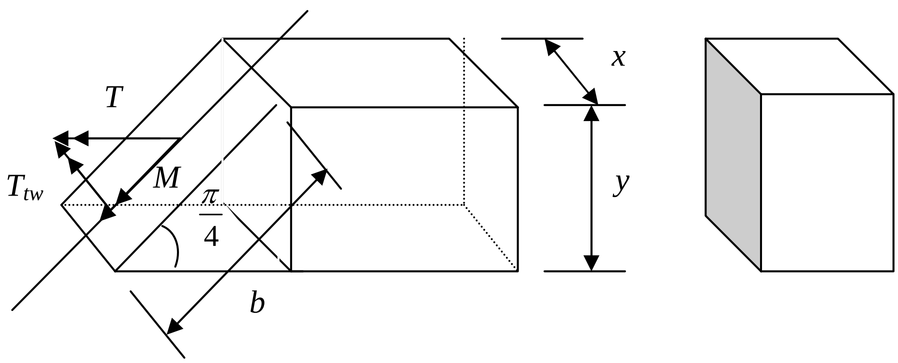
  - Maximum shear stress, τ_{max}

$$\tau_{max} = \eta \frac{T}{x^2 y}$$

where η = shape factor, T = torque, x, y = dimensions of the cross section. The shape factor is different for linear and nonlinear cases.

- Failure mode
  - Torsion failure of plain concrete occurs suddenly with an inclined tension crack in one of the wider faces, then extending into the narrow faces. Concrete crushing occurs in the opposite wider face.

- Torsional strength, T_{up}, of plain concrete
  - Several theories have been presented for computing torsional strength of plain concrete including elastic, plastic, and skew bending theories.
  - Skew bending:



→ T is the applied torque and M, T_{tw}, are the bending and twisting moments, respectively, on the $\frac{\pi}{4}$ plane.

→ $M = \frac{T}{\sqrt{2}}$, $b = \sqrt{2} \cdot y$,

$$S = \frac{\left(\sqrt{2}y\right)x^2}{6} = \frac{x^2 y}{3\sqrt{2}}$$

$$\sigma_t = \frac{M}{S} = \frac{T_{up}/\sqrt{2}}{x^2 y / 3\sqrt{2}} = \frac{3T_{up}}{x^2 y},$$

where T_{up} = ultimate torsion for plain concrete when σ reaches σ_t.

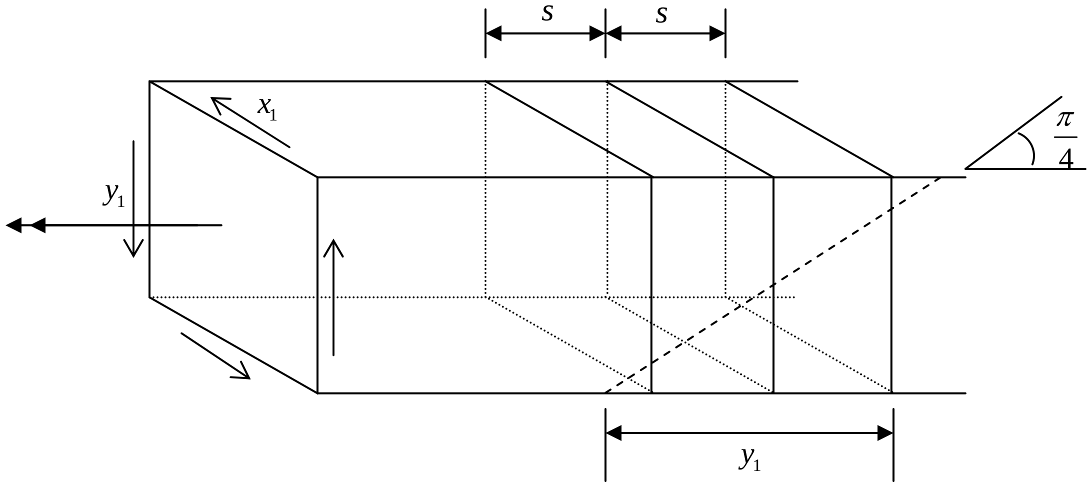
→ $T_{up} = \frac{x^2 y}{3}\sigma_t$

$$\sigma_t = 0.85 f_r \approx 0.85\left(7.5\sqrt{f_c^{'}}\right) = 6\sqrt{f_c^{'}}$$

$$\rightarrow T_{cr} = \frac{x^2 y}{3} \cdot 6\sqrt{f_c^{'}} = 2x^2 y \sqrt{f_c^{'}} = T_{up}$$

- Torsional strength contributed by steel
  - Consider the system consisting of longitudinal and transverse (stirrups) steel:

    (x_1, y_1 are the dimensions of steel frame as shown.)



  - Torsional moment with respect to axis of the vertical stirrups

$$T_{s1} = \left(A_t \alpha_1 f_s\right) \frac{y_1}{s} x_1$$

where A_t = area of one stirrup leg,

f_s = stirrup stress, and

s = stirrup spacing.

  - Torsional moment with respect to axis of the horizontal stirrups

$$T_{s2} = \left(A_t \alpha_2 f_s\right) \frac{x_1}{s} y_1$$

  - Total torsional moment

$$T_s = \frac{A_t f_s}{s} x_1 y_1 \left(\alpha_1 + \alpha_2\right)$$

$$T_s = \frac{A_t f_s}{s} x_1 y_1 \alpha_t$$ (α_t is determined from experiment.)

- Design concept
  - Total ultimate torsion capacity, T_u

    $T_u = T_c + T_s$

    where T_c = torsional capacity contributed by concrete, and

    T_s = torsional capacity contributed by reinforcement.

    $T_c = \beta T_{up} \quad (\beta \approx 0.4)$

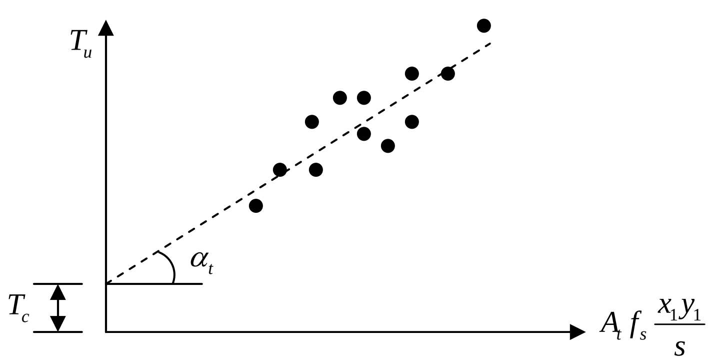
    Thus,

    $T_c = 0.8\sqrt{f_c^{'}}x^2 y$

    The coefficient β represents reduction in torsional strength provided by concrete after cracking. Upon cracking of concrete stress and strain are partially transferred to steel. Stiffness and strength of the system will depend on the amount of transverse and longitudinal reinforcements.

  - The final failure may be in one of the following ways:
    1. Under reinforced → Both transverse and longitudinal steel yield before failure.
    2. Over reinforced → Concrete crushes before yielding of steel.
    3. Partially over (under) reinforced

  - For under reinforced elements, α_t is independent of the steel ratio.



- Code suggestion:

$$\alpha_t = 0.66 + 0.33\frac{y_1}{x_1} \le 1.50$$

Role of longitudinal steel

1. It anchors the stirrups, particularly at corners.
2. It provides dowel resistance.
3. It controls crack widening.

Condition of under reinforcement

$$A_l \le 2A_t \frac{x_1 + y_1}{s}$$

where A_l = volume per length of longitudinal steel.

→ Steel yields first.

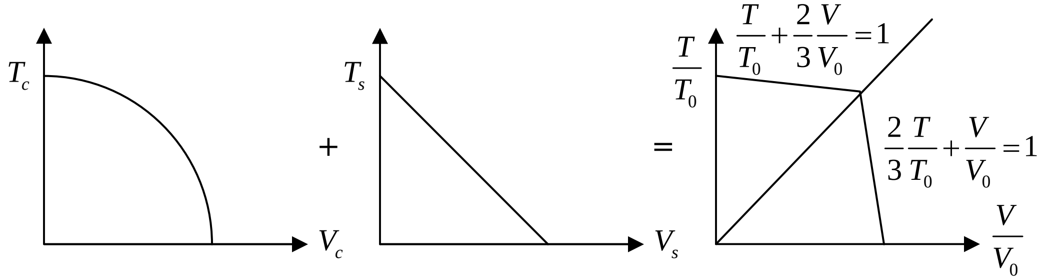
Torsion combined with flexure

Torsion combined with shear

- Generally shear exists simultaneously with bending.

  → The existence of shear will reduce the resisting ability in torsion. Thus, it is necessary to consider the case of torsion combined with shear.

- For RC beams with transverse reinforcement:
  - Pure torsion: $T_u = T_c + T_s$
  - Pure shear: $V_u = V_c + V_s$



- ACI Code
  - Design for torsion

    → Same interaction as in members without transverse reinforcement.

  - Excess torque

    → Over and above that resisted by concrete, the same amount of reinforcement is provided in members subject to torsion plus shear as would be required for purely torsional members.

    → This torsional reinforcement is added to that required for carrying bending moments and flexural shears.

  - $T_u \le \phi T_n = \phi\left(T_c + T_s\right)$

    where T_u = factored torque,

    ϕ = capacity reduction factor for torsion = 0.75,

    T_n = nominal strength for torsion,

    T_c = torsional moment carried by concrete, and

    T_s = torsional moment carried by steel.

  - $T_c = \dfrac{T_0}{\sqrt{1+\left(\dfrac{T_0}{V_0}\right)^2\left(\dfrac{V_c}{T_c}\right)^2}}$

    where $T_0 = 0.8\sqrt{f_c^{'}}x^2y$ = pure torsion and $V_0 = 2\sqrt{f_c^{'}}bd$ = pure shear.

    → $\dfrac{T_0}{V_0} = 0.4\dfrac{x^2y}{bd} = \dfrac{0.4}{C_T}$ and $C_T = \dfrac{bd}{x^2y}$

  - Assume $\dfrac{V_c}{T_c} = \dfrac{V_u}{T_u}$, such that

→ $T_c = \dfrac{0.8\sqrt{f_c^{'}}x^2 y}{\sqrt{1+\left(\dfrac{0.4}{C_T}\dfrac{V_u}{T_u}\right)^2}}$, $V_c = \dfrac{2\sqrt{f_c^{'}}bd}{\sqrt{1+\left(2.5C_T\dfrac{T_u}{V_u}\right)^2}}$

$T_s = \dfrac{\alpha_t A_t f_y}{s} x_1 y_1$, $V_s = \dfrac{A_v f_y d}{s}$

→ $T_u = \phi\left(T_c + T_s\right) = \phi T_n$

$T_s = \dfrac{T_u - \phi T_c}{\phi}$

→ $A_t = \dfrac{sT_s}{\alpha_t f_y x_1 y_1} = \dfrac{\left(T_u - \phi T_c\right)s}{\alpha_t \phi f_y x_1 y_1}$

- $T_s \le 4T_c$ is required to assure yielding of steel first.
- Minimum spacing of torsional stirrups → $4\left(x_1 + y_1\right)$ or 12 in.

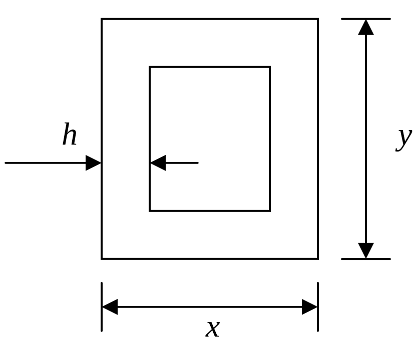
Condition of neglecting torsional effects

- Torsional effects may be neglected if

$T_u < 0.5\phi\sqrt{f_c^{'}}\sum_{i=1}^{n}\left(x_i^2 y_i\right)$

where $\sum_{i=1}^{n}\left(x_i^2 y_i\right)$ = sum of the small rectangles for irregular shapes.

Hollow sections



- When $h > \frac{x}{4}$, consider the cross section as solid.
- When $\frac{x}{10} \le h \le \frac{x}{4}$, assume it as solid but multiply $\sum\left(x^2 y\right)$ by $\left(4\frac{h}{x}\right)$.
- When $h < \frac{x}{10}$, consider it as a thin-walled section. → Check for instability (local buckling).

- General formulation of post-cracking behavior of flexure, shear, and tension interaction in R/C beams

- Discussion of applications: Concrete guideway systems from monorail and maglev transportation infrastructure.

- Design Example – Shear and torsion