1.051 Structural Engineering Design

Recitation 2

Basic Flexure Design Method for Rectangular Singly Reinforced Beams in Accordance with the ACI-318

Reference: Chapter 8, 9, 10, Building Code Requirements for Reinforced Concrete ACI-318 and Commentary (ACI-318R), American Concrete Institute, Detroit, MI

Design Principle:

$$M_u \leq \Phi \cdot M_n \qquad \text{Equation (1)}$$

Major Design Equations:

Design Moment

$$M_u \leq \Phi \cdot \rho \cdot f_y \cdot b \cdot d^2 \cdot \left(1 - \frac{\rho \cdot f_y}{1.7 \cdot f_c'}\right) \qquad \text{Equation (2)}$$

where
M_u = factored applied moment
M_n = nominal flexural strength of a section
Φ = strength reduction factor (0.9 for flexural design)
ρ = ratio of tension reinforcement, see Equation (4)
= $\frac{A_s}{b \cdot d}$ Equation (3)
f_y = specified yield strength of reinforcement, psi
b = width of compression face of member, in.
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
f'_c = specified compressive strength of concrete, psi

Ratio of Steel Reinforcement

$$\rho_{min} \leq \rho \leq \frac{3}{4}\rho_b \qquad \text{Equation (4)}$$

where ρ_{min} = minimum ratio of tension reinforcement
= $\frac{200}{f_y}$ Equation (5)

ρ_b = reinforcement ratio producing balanced strain conditions

$$\rho_b = \frac{0.85 \cdot \beta_1 \cdot f'_c}{f_y} \cdot \frac{87{,}000}{87{,}000 + f_y} \qquad \text{Equation (6)}$$

β_1 = 0.85 if $f'_c \le 4$ ksi
= $0.85 - 0.05\,(f'_c - 4)$ if $f'_c > 4$ ksi
= not less than 0.65 in any case

Crack Width

$$w = 0.000091 \cdot f_s \cdot \sqrt[3]{d_c \cdot A} \qquad \text{Equation (7)}$$

where w = crack width
= 0.016 in. for an interior exposure condition
= 0.013 in. for an exterior exposure condition

f_s = $0.6\, f_y$, kips

d_c = distance from tension face to center of the row of reinforcing bars closest to the outside surface

A = effective tension area of concrete divided by the number of reinforcing bars

$$= \frac{A_{eff}}{N} \qquad \text{Equation (8)}$$

A_{eff} = product of the web width and a height of web equal to twice the distance between the centroid of the steel and tension surface

$$N = \frac{\text{Total area of steel } A_s}{\text{Area of largest bar}} \qquad \text{Equation (9)}$$

Required Design parameters:

- Dead Load (DL)
- Live Load (LL)
- Other loading types: Snow, wind, etc.
- Concrete material properties: f'_c, f'_r, E_c, γ_c
- Steel material properties: f_y, E_s

Basic Design Procedure:

In real design problems, loading conditions are obtained from structural analysis and real loading estimates. Frame structures are by far the most commonly encountered structural systems in design. Beam loadings mostly rely on loads transferred from slabs being supported and such loads are dependent on the type of occupancy. For instance, the service load of an office floor would be much less than that of a machine room. Also, weights of construction materials also contribute significantly in most cases. Self-weight of the beam under consideration is normally required.

Section design is an iterative process as (1) the section size is, in most cases, not yet determined, self-weight (DL) of the beam need to be updated with the section size, and (2) the initially assumed section size may not meet the required capacity and need to be redesigned. The following presents a general design procedure:

1. Determine the service loads
2. Assume h and estimate b by the rule of thumb of effective section (dimensions should satisfy architectural requirements, if any)
3. Check minimum thickness of beam according to Table 9.5(a) in the code
4. Estimate self weight
5. Determine initial dead loads
6. Perform preliminary elastic analysis {Plot bending moment diagram (BMD) + shear force diagram (SFD)}; Choose the largest moment and shear values from the BMD and SFD for design
7. Compute ρ_{min} and ρ_b
8. Choose a ρ that satisfies Equation (4)
9. Compute bd^2 from Equation (2) and compare to the assumed bd^2 used for self weight estimation
10. If the assumed bd^2 is larger than the computed (required) bd^2, and that the values are not too different, go to Step 11 for steel provision design. Otherwise, go to Step 2 using the computed bd^2 value as the assumed value and reiterate the design procedure until the difference becomes small.
11. With the chosen ρ, b, d, and Equation (3), determine the total A_s required.
12. Design the steel reinforcement arrangement with appropriate concrete covers and spacing stipulated in the code. Bar size and the corresponding number of bars for the determined A_s can be found in typical design aids or computed accurately based on the knowledge that bar size #n = n/8 inch diameter for n<10. Spacing between bars should be greater than 1 in. or one bar diameter, whichever is larger
13. Sketch the section with the designed rebar arrangements
14. Check crack widths by Equation (7)
15. Calculate deflections and check with Table 9.5(b) in the code.
16. Design Completed

The above design guideline is mostly applicable to rectangular singly reinforced R.C. beams only. If the design involves other complicated section shapes and/or doubly reinforcement provisions, this procedure will need slight modifications.

Some Useful Rules of Thumb:

- $\frac{d}{b} = 1.5 \sim 2.0$ for beam spans of 15 ~ 25 feet
- $\frac{d}{b} = 3.0 \sim 4.0$ for beam spans > 25 feet
- Larger the $\frac{d}{b}$, the more efficient is the section due to less deflection
- For initial estimation, h – d = 2.5”
- h should be rounded to the nearest whole number
- b is taken as an even number

These rules of thumb are very commonly used in practice. But they need not be strictly followed if the real situation proves them impractical.