

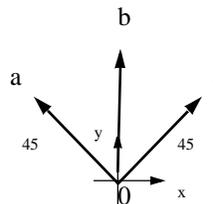
Problem Set #7 Solution 1.050 Solid Mechanics Fall 2004

Problem 7.1

Three strain gages measure the extensional strain in the three directions $0a$, $0b$ and $0c$ at "the point O ". Using the relationship we derived in class

$$\epsilon_{PQ} = \epsilon_x \cos^2 \phi + \gamma_{xy} \cos \phi \sin \phi + \epsilon_y \sin^2 \phi$$

find the three components of strain with respect to the xy axis in terms of ϵ_a , ϵ_b and ϵ_c



We seek the components of (2D) strain at the point referenced to the xy axis.

Let the extensional strains in the directions a , b , and c be ϵ_a , ϵ_b , and ϵ_c . Then, using the above, three times:

$$\epsilon_a = \epsilon_x \cos^2(180 - 45) + \gamma_{xy} \cos(180 - 45) \sin(180 - 45) + \epsilon_y \sin^2(180 - 45)$$

$$\epsilon_b = \epsilon_x \cos^2 90 + \gamma_{xy} \cos 90 \sin 90 + \epsilon_y \sin^2 90$$

$$\epsilon_c = \epsilon_x \cos^2 45 + \gamma_{xy} \cos 45 \sin 45 + \epsilon_y \sin^2 45$$

or, evaluating the trig. functions:

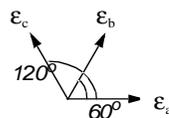
$$\epsilon_a = \epsilon_x \left(\frac{1}{\sqrt{2}}\right)^2 + \gamma_{xy} \left(\frac{-1}{\sqrt{2}}\right) \frac{1}{\sqrt{2}} + \epsilon_y \left(\frac{1}{\sqrt{2}}\right)^2 \quad \text{This which gives, in turn}$$

$$\epsilon_b = \epsilon_y$$

$$\epsilon_c = \epsilon_x \left(\frac{1}{\sqrt{2}}\right)^2 + \gamma_{xy} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + \epsilon_y \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\begin{aligned} \epsilon_x &= \epsilon_a + \epsilon_c - \epsilon_b \\ \epsilon_b &= \epsilon_y \\ \gamma_{xy} &= \epsilon_c - \epsilon_a \end{aligned}$$

Problem 7.2



1.1 A strain gage rosette, fixed to a flat, thin plate, measures the following extensional strains

$$\epsilon_a = 1. \text{ E-}04$$

$$\epsilon_b = 1. \text{ E-}04$$

$$\epsilon_c = 2. \text{ E-}04$$

Determine the state of strain at the point, expressed in terms of components relative to the xy coordinate frame shown.

Using Mohr's circle, determine the strain components relative to an axis oriented at 45° rotation (ccw).

Here we have a problem very much like the first problem, except that the orientation of the three gages is different. We again use the same relationship and express ϵ_a , ϵ_b , and ϵ_c in terms of the components of strain relative to the xy axis:

$$\epsilon_a = \epsilon_x$$

$$\epsilon_b = \epsilon_x \cos^2 60 + \gamma_{xy} \cos 60 \sin 60 + \epsilon_y \sin^2 60$$

$$\epsilon_c = \epsilon_x \cos^2(120) + \gamma_{xy} \cos(120) \sin(120) + \epsilon_y \sin^2(120)$$

or, evaluating the trig. functions:

$$\epsilon_a = \epsilon_x$$

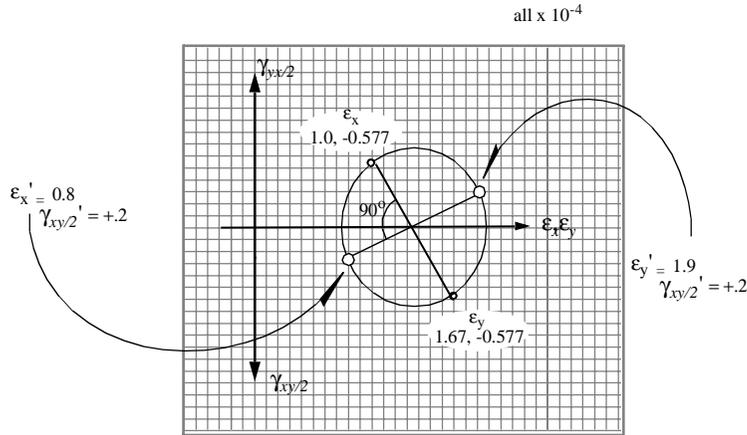
$$\epsilon_b = \frac{\epsilon_x}{4} + \gamma_{xy} \frac{\sqrt{3}}{2} \frac{1}{2} + \epsilon_y \frac{3}{4}$$

$$\epsilon_c = \frac{\epsilon_x}{4} - \gamma_{xy} \frac{\sqrt{3}}{2} \frac{1}{2} + \epsilon_y \frac{3}{4}$$

Which gives

$$\begin{aligned} \epsilon_x &= \epsilon_a = 1. \times 10^{-4} \\ \epsilon_y &= \frac{2}{3} \cdot \left(\epsilon_b + \epsilon_c - \frac{\epsilon_a}{2} \right) = 1.667 \times 10^{-4} \\ \gamma_{xy} &= \frac{2}{\sqrt{3}} (\epsilon_b - \epsilon_c) = -0.577 \times 10^{-4} \end{aligned}$$

For Mohr's circle, we must remember to plot $\gamma_{xy}/2$ in the vertical direction. Only then do our strain transformation equations have the same form as the stress transformation equations.



If we use the equations for the transformation of components of strain in 2D, we obtain:

$$\epsilon'_x = \left[\frac{(\epsilon_x + \epsilon_y)}{2} \right] + \left[\frac{(\epsilon_x - \epsilon_y)}{2} \right] \cdot \cos 2\phi + (\gamma_{xy}/2) \sin 2\phi = 0.776$$

$$\epsilon'_y = \left[\frac{(\epsilon_x + \epsilon_y)}{2} \right] - \left[\frac{(\epsilon_x - \epsilon_y)}{2} \right] \cdot \cos 2\phi - (\gamma_{xy}/2) \sin 2\phi = 1.91$$

$$(\gamma'_{xy}/2) = - \left[\frac{(\epsilon_x - \epsilon_y)}{2} \right] \cdot \sin 2\phi + (\gamma_{xy}/2) \cos 2\phi = 0.17$$

Problem 7.2

A two dimensional displacement field is defined by

$$u(x, y) = \frac{-\alpha}{2} \cdot y \quad \text{and} \quad v(x, y) = \frac{\alpha}{2} \cdot x$$

Sketch the position of the points originally lying along the x axis, the line $y = 0$, due to this displacement field. Assume α is very much less than 1.0.

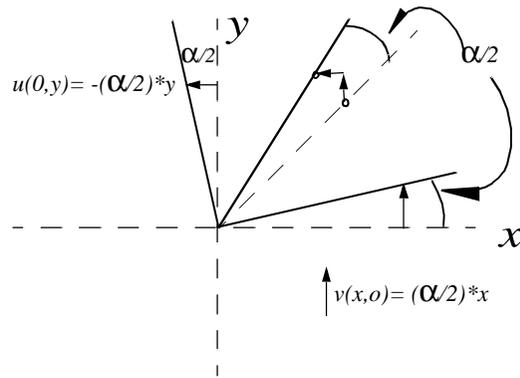
Likewise, on the same sketch, show the position of the points originally lying along the y axis, the line $x = 0$, due to this displacement field.

Likewise, on the same sketch, show the position of the points originally lying along the line $y=x$, due to this displacement field.

Calculate the state of strain at the origin; at the point, x, y .

Respond again but now with $u(x, y) = \frac{\alpha}{2} \cdot y$ and $v(x, y) = \frac{\alpha}{2} \cdot x$

The plot of displacement of points on the $x=0$ and $y=0$ and $x=y$ line elements is shown at the right for the first case. In this case, all the strains are zero. We say this displacement field defines a "rigid body rotation" of $(\alpha/2$ ccw) at the point.



Reversing the sign on the u component of displacement reflects the line $u(0, y)$ line shown about the y axis. We then have a change in the right angle formed by the x - y axes. So there is a shear strain, $\gamma_{xy} = \alpha$. But the extensional strain components remain zero.