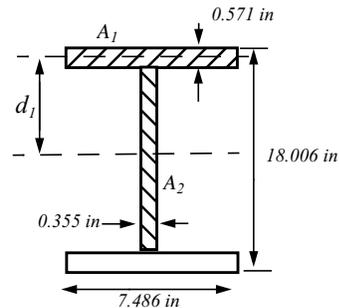


Problem Set #10 Solution 1.050 Solid Mechanics Fall 2004

Problem 10.1

Let A_1 and A_2 be the cross-sectional areas of the flange and web respectively. Then we can compute, with the dimensions give, the total area, the weight per unit length (given the weight density of steel) and the moment(s) of inertia - using the parallel axis theorem.



$$A_1 = (7.486)(0.571) = 4.275 \text{ in}^2$$

$$A_2 = [18.006 - (2)(0.571)](0.355) = 5.987 \text{ in}^2$$

$$\text{total } A = A_1 + A_2 + A_1 = 14.537 \text{ in}^2$$

$$\rho_{\text{steel}} = 7850 \frac{\text{kg}}{\text{m}^3} = 0.2835 \frac{\text{lbs}}{\text{in}^3}$$

$$W/L = (14.537)(0.2835) = 4.1212 \frac{\text{lbs}}{\text{in}} = 49.45 \frac{\text{lbs}}{\text{ft}}$$

$$I_{xx} = \frac{(0.355)(18.006 - (2)(0.571))^3}{12} + (2) \left[\frac{(7.486)(0.571)^3}{12} + (4.275)(d_1)^2 \right]$$

$$d_1 = \frac{18.006 - 0.571}{2} = 8.7175 \text{ in}$$

$$I_{xx} = 791.87 \text{ in}^4$$

$$I_{yy} = \frac{[18.006 - (2)(0.571)](0.355)^3}{12} + (2) \left[\frac{(0.571)(7.486)^3}{12} \right]$$

$$I_{yy} = 39.99 \text{ in}^4$$

Problem 10.2

From $\sigma_y = M_b y / I$ and $M_b = EI / \rho$, we get that $\sigma_y = E y / \rho$.

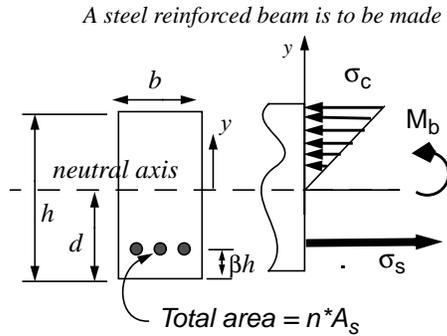
So we have

$$E_{\text{steel}} = 200 \times 10^9 \frac{\text{N}}{\text{m}^2} = 29 \times 10^6 \frac{\text{lbs}}{\text{in}^2}$$

$$\sigma_y = \frac{(29 \times 10^6)(0.0625)}{12} = 151 \times 10^3 \text{ psi} > 120 \times 10^3 \text{ psi}$$

As you can see, if the radius of cylinder is reduced to 12 in, the stress in the steel wire will exceed the yield strength and thus make the steel wire fail (or undergone permanent plastic deformation). This is clearly not good and thus you should tell your boss about this and inform him that the minimum radius of the cylinder that can be used for this steel wire storage is 16 in (which will give $\sigma_y = 113 \times 10^3$ psi). But for safety reason and if it is stored for a long time period, you might want to keep the stress in the steel wire well below the yield stress. In this case, the original 20 in radius is a good choice.

Problem 10.3



A steel reinforced beam is to be made such that the steel and the concrete fail simultaneously.
 If $E_s = 30 \text{ e}06 \text{ psi}$ steel
 $E_c = 3.6 \text{ e}06 \text{ psi}$ concrete
 and taking
 $\sigma_{\text{failure steel}} = 40,000 \text{ psi}$
 $\sigma_{\text{failure concrete}} = 4,000 \text{ psi}$ (compression)

how must β be related to d/h for this to be the case?

Now, letting $\lambda = \frac{2 \cdot E_s \cdot nA_s}{E_c \cdot bh}$ find d/h and β values for

a range of "realistic" values for the area ratio, (nA_s/bh) , hence for a range of values for λ .

Make a sketch of one possible composite cross-section showing the location of the reinforcing rod. Take the diameter of the rod as 0.5 inches.

The magnitude of the maximum compressive stress in the concrete will be seen at $y = (h-d)$. The maximum tensile stress in the steel will be seen at $y = -(d-\beta h)$. They are¹:

$$|\sigma_c|_{\text{max}} = \frac{M_b \cdot (h-d)E_c}{EI} \quad \text{and} \quad |\sigma_s|_{\text{max}} = \frac{M_b \cdot (d-\beta h)E_s}{EI}$$

If the steel and concrete are to fail at the same bending moment, then the maximum tensile stress in the steel must be 10 times the maximum compressive stress in the concrete. So $(d-\beta h)E_s = 10 \cdot (h-d)E_c$. With the Young's moduli given this can be written,

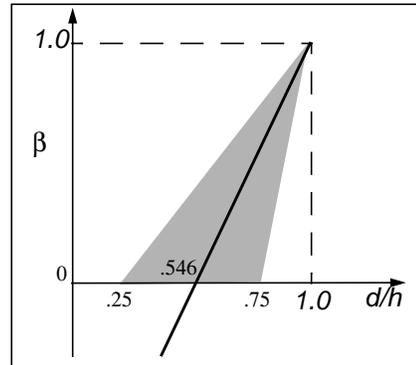
$$\left(\frac{d}{h} - \beta\right) = (1.2)\left(1 - \frac{d}{h}\right) \quad \text{or} \quad \beta = 2.2 \cdot \frac{d}{h} - 1.2$$

We plot this linear relationship at the right. Beta and d/h must be positive and both less than 1.0

We can generalize this relationship if we allow the relationship between the maximum stresses to be arbitrary and also allow for different ratios for the Young's moduli.

Letting $C = \frac{|\sigma_s|_{\text{max}}}{|\sigma_c|_{\text{max}}} \cdot \frac{E_c}{E_s}$

we obtain $\left(\frac{d}{h} - \beta\right) = C\left(1 - \frac{d}{h}\right) \quad \text{or} \quad \beta = (1+C) \cdot \frac{d}{h} - C$



The intercept for $\beta = 0$ is then just $d/h = C/(1+C)$. Furthermore, all lines for any C must pass through the point $(1,1)$. We show a shaded region which would include all plots for $(1/3) < C < 3$. The line is for the particular case, $C = 1.2$.

Now we join with the above relationship, the requirement that follows from force equilibrium in the direction of the axis of the beam. We have.

$$\left(\frac{d}{h}\right)^2 - (2 + \lambda) \cdot \left(\frac{d}{h}\right) + (1 + \beta\lambda) = 0 \quad \text{where} \quad \lambda = \frac{2E_s nA_s}{E_c bh}$$

We can eliminate beta in terms of d/h and obtain:

$$\left(\frac{d}{h}\right)^2 - (2 + \lambda) \cdot \left(\frac{d}{h}\right) + \left(1 + \left((1+C) \cdot \frac{d}{h} - C\right)\lambda\right) = 0$$

This, in time, simplifies considerably:

$$\left(\frac{d}{h}\right)^2 - 2 \cdot \left(\frac{d}{h}\right) + 1 + \lambda \cdot \left(\frac{d}{h} - 1\right)C = 0$$

or, factoring the quadratic in d/h

$$\left(\frac{d}{h} - 1\right)^2 + \lambda \cdot \left(\frac{d}{h} - 1\right)C = 0$$

We can not have $d/h = 1$, so this common term may be factored out¹ and we must then have

$$\lambda = \left(1 - \frac{d}{h}\right)/C$$

To answer the question posed, if $C = 1.2$ $\lambda = \frac{2E_s n A_s}{E_c b h} = 16.7 \frac{n A_s}{b h}$ and if we let $\rho = \frac{n A_s}{b h}$ be the *ratio of steel area to beam xsection area*,

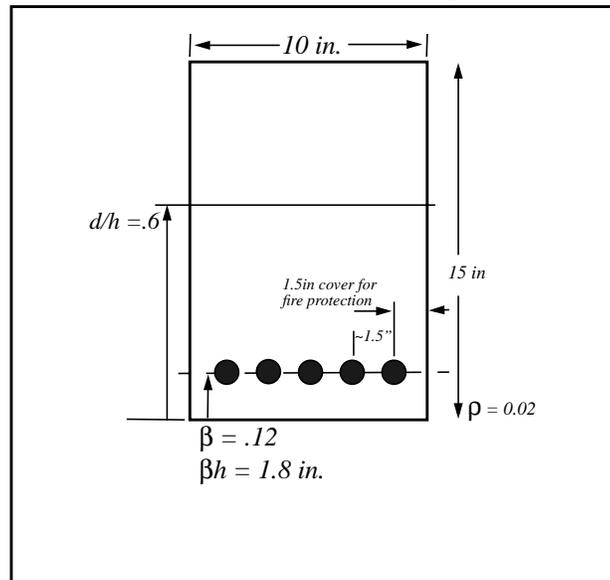
then $16.7\rho = \left(1 - \frac{d}{h}\right)/1.2$ or $\rho = \left(1 - \frac{d}{h}\right)/(20)$

We set out in a table, some realistic values for d/h , ρ , the area fraction of steel, and finally β .

d/h	ρ	β
0.55	0.022	0.01
0.575	0.021	0.065
0.6	0.020	0.12
0.625	0.018	0.175
0.65	0.017	0.23
0.675	0.016	0.285

We see the percent area of steel is about 2% in all cases. If we choose, arbitrarily, a beam cross section of say 10×15 inches, and take $d/h = 0.6$, the area of the steel will be $.020 \times 150 \text{ in}^2 = 3 \text{ in}^2$. If the rods have a diameter of 0.875 inches, (reinforcing bar #7)² each then has an area of 0.60 inches^2 , so we will need 5 rods.

With this xsection, the steel will fail in tension, the concrete in compression, at the same time.



1. Cody: This is why you always found the root of the quadratic to be $d/h = 1.0$.
2. .5 in diameter rods will not fit as well as these with diameter 0.875.