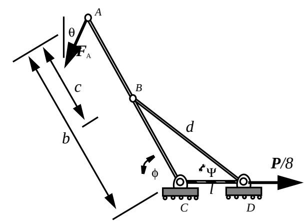
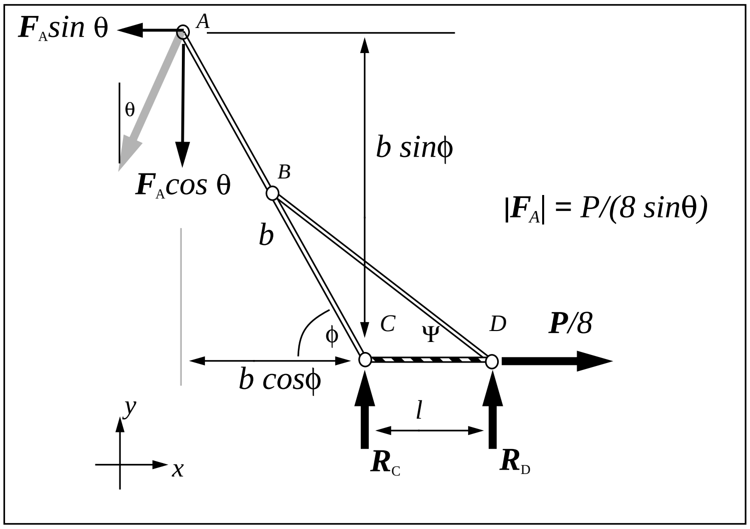



Static Equilibrium analysis of opening the umbrella.

We show one of the eight, rotationally symmetrically distributed, ribs. Hence the factor of 1/8.

**The Free Body Diagram** constructed to determine the internal forces acting between the shaft and the spring collar is shown below.




**The equilibrium equations** whose solution will give the reaction forces at C and D. Note that the resultant of the two horizontal components is zero. So we need only consider the resultant moment and the resultant of the vertical, *y*, components. We reference moments to the point C below.

$$\sum F_y = 0 \qquad -F_A \cdot cos\theta + R_C + R_D = 0$$

$$\sum M_{about\ C,\ ccw>0} = 0 \qquad (b\,cos\phi)(F_A cos\theta) + (b\,sin\phi)(F_A sin\theta) + l \cdot R_D = 0$$

Solving the 2nd for $R_D$, then, with this, the first for $R_c$ we obtain:

$$R_D = -\frac{P}{8\,sin\theta} \cdot \frac{b}{l}[cos\phi\,cos\theta + sin\phi\,sin\ \ ] = -\frac{P}{8\,sin\theta} \cdot \frac{b}{l}[cos(\phi - \theta)]$$

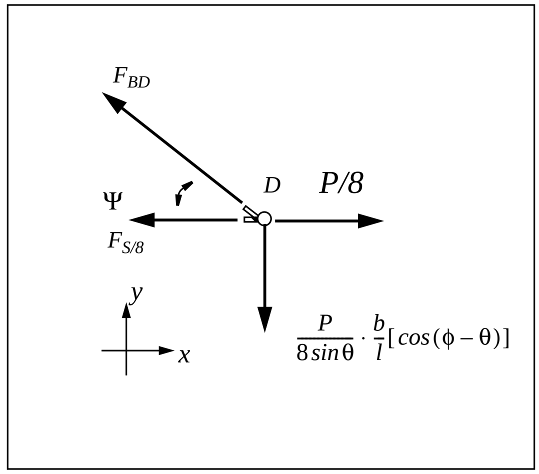
$$R_C = \frac{P}{8\,sin\theta}\left(cos\theta + \frac{b}{l}[cos(\phi - \theta)]\right)$$

The negative expression for the reaction force at *D* indicates that the actual force acts downward, pulling down on the pin at *D*. We are now in a position to obtain an expression for the internal force acting within member *CD*, the spring. We do this by isolating pin *D* as a particle and solve for that internal force and, at the same time, the force acting in the two force member *BD*. So overleaf...

We assume our unknown internal forces are pulling on the pin D, the members CD and BD are in tension. If one or the other or both come out negative, we know the one, or other, or both are in compression.

Note also that we have shown the vertical reaction force at the pin acting downward and written out its magnitude as determined above.

Note also that we show the force in the spring as 1/8th the total spring force $F_s$ since this one rib section accounts for but this fraction of the spring force assuming rotational symmetry.




We have available two scalar equilibrium equations to determine the two scalar unknowns - the magnitudes of the force in member *BD* and in the spring. Their directions are known since we model them as "two-force members".

$$\sum F_x = 0 \qquad -F_{s/8} - F_{BD} \cdot cos\Psi + P/8 = 0$$

$$\sum F_y = 0 \qquad -\frac{P}{8 sin\theta} \cdot \frac{b}{l}[cos(\phi - \theta)] + F_{BD} \cdot sin\Psi = 0$$

These solve to:

$$F_{BD} = \frac{P}{8 sin\theta sin\Psi} \cdot \frac{b}{l}[cos(\phi - \theta)]$$

$$F_s = P \cdot \left\{ 1 - \frac{cos\Psi}{sin\theta sin\Psi} \cdot \frac{b}{l}[cos(\phi - \theta)] \right\}$$

Some Observations:

- We can not say with certainty whether the spring force will be negative or positive. If the second term within the bracket is greater than 1.0, $F_s$ will be negative, indicating that the spring is in compression rather than in tension. This will be the case if theta, $\theta$, is small. We expect, know from experience, that this is the case.
- Apparently, though, the member *BD* is in tension since the cosine of the difference of the two angles will be positive for the range of values of $\theta$ and $\phi$ taken on as we close (or open) the umbrella. $F_{BD}$ will come out positive indicating it is pulling on the pin as we assumed.
- If our intent is to design this structural system or mechanism, i.e., choose a spring stiffness which will allow a person to close the umbrella without exerting undo force (=? pounds?), then we need to know more. In fact, we need to know how the angles $\Psi$ and $\phi$ change with $\theta$. This is a matter of geometry of displacement and rotations. (We also have to set an initial angle for the latter when the umbrella is fully deployed and choose the lengths *b, c* and even *l)*. This we will do in the next installment.