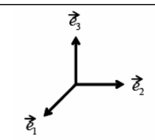
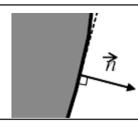
Variable	Definition	Notes & comments
$L_x L_y L_z M T$	$[q] = L_x^{\alpha_x} L_y^{\alpha_y} L_z^{\alpha_z} M^{\beta} T^{\gamma}$	Extended base dimension system
Π_i	**Theorem 1 (Pi-Theorem)** *Consider a relation between $N+1$ dimensional physical quantities, $q_0, q_1, ..., q_N$ of the form:* $q_0 = f(q_1, ..., q_N)$ (1.13) *Let k be the number of dimensionally independent variables. Let $\{q_1, ..., q_k\}$ be the complete, dimensionally independent subset of $\{q_1, ..., q_N\}$. The initial physical relation can be reduced to a dimensionless relation between $N-k+1$ similarity parameters $\Pi_0, \Pi_1, ..., \Pi_{N-k}$:* $\Pi_0 = \mathcal{F}(\Pi_1, ..., \Pi_{N-k})$ (1.14) *defined by:* $\Pi_i = \frac{q_i}{q_1^{a_1^i} q_2^{a_2^i} \cdots q_k^{a_k^i}}; \ i = 0, N-k$ (1.15) *where the exponents $a_1^i, ...a_k^i$ are determined from the dimension functions:* $[q_i] = [q_1]^{a_1^i} [q_2]^{a_2^i} \cdots [q_k]^{a_k^i}$ (1.16)	Pi-theorem (also definition of physical quantities,…)
Physical similarity	$\forall i = 0, N-k;\ \Pi_i^{(1)} = \Pi_i^{(2)}$	Physical similarity means that all Pi-parameters are equal
$\mathcal{N}_{Gal}$	$\mathcal{N}_{Gal} = \frac{hg\rho}{\sigma_0}$	Galileo-number (solid mechanics)
Re	$\mathrm{Re} = \frac{UD}{\nu}$	Reynolds number (fluid mechanics)

Lectures 1-3 and PS2

Important concepts include the extended base dimension system, distinction between units and dimensions, the formal Pi-theorem based procedure and the concept of physical similarity.

Applications include calculation of physical processes like atomic explosion, drag force on buildings etc.

Variable	Definition	Notes & comments
$\vec{x}$	$\vec{x} = x_1\vec{e}_1 + x_2\vec{e}_2 + x_3\vec{e}_3$	Position vector
$\vec{v}$	$\vec{v} = d\vec{x} / dt$	Velocity vector
$\vec{a}$	$\vec{a} = d\vec{v} / dt$	Acceleration vector
$\vec{p}$	$\vec{p} = m\vec{v} = m(v_1\vec{e}_1 + v_2\vec{e}_2 + v_3\vec{e}_3)$	Linear momentum
$\vec{e}_1, \vec{e}_2, \vec{e}_3$		Unit vectors that define coordinate system = basis
$\vec{n}$		Normal vector Always points outwards of domain considered
$\vec{x}_i \times \vec{p}_i$	$\vec{x}_i \times \vec{p}_i = \vec{x}_i \times m_i\vec{v}_i$	Angular momentum
$\vec{F}$	$\vec{F} = F_x\vec{e}_x + F_y\vec{e}_y + F_z\vec{e}_z$	Force vector (force that acts on a material point)

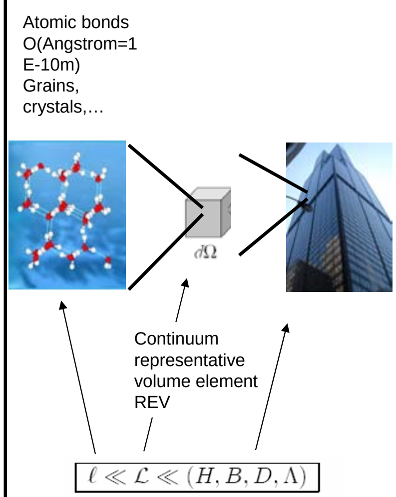
Covered in lecture 4 and PS1

Basic definitions of linear momentum, angular momentum, normal vector of domain boundaries

Variable	Definition	Notes & comments
	1. Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it. 2. The change of motion is proportional to the motive force impresses, and is made in the direction of the right line in which that force is impressed. 3. To every action there is always opposed an equal reaction: or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts.	**Newton's three laws**
	$d(\vec{p})/dt = d(m\vec{v})/dt \overset{\text{def}}{=} \vec{F}$	**Dynamic resultant theorem** Change of linear momentum is equal to sum of external forces
	$\frac{d}{dt}\sum_{i=1}^{N}(\vec{x}_i \times m_i\vec{v}_i) \overset{\text{def}}{=} \sum_{i=1}^{N}(\vec{x}_i \times \vec{F}_i^{ext}) = \sum_{i=1}^{N}\vec{M}_i^{ext}$	**Dynamic moment theorem** Change of the angular motion of a discrete system of i = 1,N particles is equal to the sum of the moments (or torque) generated by external forces
	Static: $\sum_{i=1}^{N}\vec{F}_i^{ext} \overset{def}{=} 0;\ \sum_{i=1}^{N}\vec{\mathcal{M}}_i^{ext} \overset{def}{=} 0$	Static EQ (solve truss problems)

Lecture 4: These laws and concepts form the basis of almost everything we'll do in 1.050.

The dynamic resultant theorem and dynamic moment theorem are important concepts that simplify for the static equilibrium. This can be used to solve truss problems, for instance.

<table>
<tr><th>Variable</th><th>Definition</th><th>Notes & comments</th></tr>
<tr><td>REV
$d\Omega$</td><td>

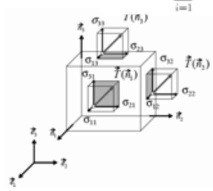
$\mathcal{L} \propto d\Omega^{1/3}$</td><td>REV=
Representative volume element
'd'=differential element

Must be:
(1) Greater than any in homogeneity (grains, molecules, atoms,..)
(2) Much smaller than size of the system</td></tr>
<tr><td>$\partial\Omega$</td><td>Surface of domain Ω</td><td>Note the difference between 'd' and '∂' operator</td></tr>
</table>

Skyscraper photograph courtesy of jochemberends on Flickr.

Lecture 5

The definition of REV is an essential concept of continuum mechanics: Separation of scales, i.e., the three relevant scales are separated sufficiently. There are three relevant scales in the continuum model. Note: The beam model adds another scale to the continuum problem – therefore the beam is a four scale continuum model.

Variable	Definition	Notes & comments
$\vec{T}(\vec{n},\vec{x}) = \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$	Force density that acts on a material plane with normal $\vec{n}$ at point $\vec{x}$ $\vec{T}(\vec{n}_j) = \sum_{i=1}^{3} \sigma_{ij} \vec{e}_i = \sigma_{ij} \vec{e}_i$ 	Stress vector (note: normal always points out of domain)
$[\sigma_{ij}]$	$[\sigma_{ij}] = \begin{array}{c\|ccc} & (\vec{T}_1) & (\vec{T}_2) & (\vec{T}_3) \\ \hline (\vec{e}_1) & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ (\vec{e}_2) & \sigma_{21} & \sigma_{22} & \sigma_{23} \\ (\vec{e}_3) & \sigma_{31} & \sigma_{32} & \sigma_{33} \end{array}$	Stress matrix
$\underline{\underline{\sigma}} = \sigma_{ij} \vec{e}_i \otimes \vec{e}_j$	$\vec{T}(\vec{n},\vec{x}) = \underline{\underline{\sigma}}(\vec{x}) \cdot \vec{n}$	Stress tensor
p		Pressure (normal force per area that compresses a medium)

Lecture 5, 6, 7

These concepts are very important. We started with the definition of the stress vector that describes the force density on a particular surface cut.

The stress tensor (introduced by assembling the stress matrix) provides the stress vector for an arbitrary plane (characterized by the normal vector). This requirement represents the definition of the stress tensor; by associating each entry with two vectors (this is a characteristic of a second order tensor).

The pressure is a scalar quantity; for a liquid the pressure and stress tensor are linked by a simple equation (see next slide).

Variable	Definition	Notes & comments
	$\int_{\partial\Omega} \mathbf{f} \cdot \vec{n}\, da = \int_{\Omega} \operatorname{div} \mathbf{f}\, d\Omega$	Divergence theorem (turn surface integral into a volume integral)
	in Ω: $\begin{cases} \vec{T}(\vec{n}) = \boldsymbol{\sigma} \cdot \vec{n} \\ \operatorname{div} \boldsymbol{\sigma} + \rho(\vec{g} - \vec{a}) = 0 \\ \sigma_{ij} = \sigma_{ji} \end{cases}$ on S: $\vec{T}(\vec{n}) + \vec{T}(-\vec{n}) = 0$ on $\partial\Omega$: $\vec{T}^d = \vec{T}(\vec{n})$	Differential equilibrium (solved by integration)
$\operatorname{div} \underline{\underline{\sigma}} + \rho(\vec{g} - \vec{a}) = 0$	$\frac{\partial\sigma_{11}}{\partial x_1} + \frac{\partial\sigma_{12}}{\partial x_2} + \frac{\partial\sigma_{13}}{\partial x_3} + \rho(g_1 - a_1) = 0$ $\frac{\partial\sigma_{21}}{\partial x_1} + \frac{\partial\sigma_{22}}{\partial x_2} + \frac{\partial\sigma_{23}}{\partial x_3} + \rho(g_2 - a_2) = 0$ $\frac{\partial\sigma_{31}}{\partial x_1} + \frac{\partial\sigma_{32}}{\partial x_2} + \frac{\partial\sigma_{33}}{\partial x_3} + \rho(g_3 - a_3) = 0$ In cartesian C.S.	Differential E.Q. written out for cartesian C.S.
	in Ω: $\begin{cases} \vec{T}(\vec{n}) = -p\vec{n} \\ \boldsymbol{\sigma} = -p\mathbf{1} \\ -\operatorname{grad} p + \rho\vec{g} = 0 \end{cases}$	EQ for liquid (no shear stress=material law)

Lecture 5, 6, 7

We expressed the dynamic resultant theorem for an arbitrary domain and transformed the resulting expression into a pure volume integral by applying the divergence theorem. This led to the differential EQ expression; each REV must satisfy this expression. The integration of this partial differential equation provides us with the solution of the stress tensor as a function of all spatial coordinates.

<table>
<tr><th>Variable</th><th>Definition</th><th>Notes & comments</th></tr>
<tr><td></td><td colspan="2">

basis $(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)$

$$\begin{aligned} \operatorname{div} \boldsymbol{\sigma} &= \nabla \cdot \boldsymbol{\sigma} \\ &= \left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} \right) \vec{e}_r \\ &\quad + \left(\frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + 2 \frac{\sigma_{r\theta}}{r} \right) \vec{e}_\theta \\ &\quad + \left(\frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{z\theta}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zr}}{r} \right) \vec{e}_z \end{aligned}$$

Divergence of stress tensor in cylindrical C.S.

</td></tr>
<tr><td></td><td colspan="2">

basis $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\varphi)$

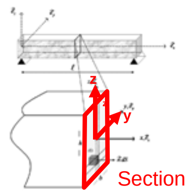
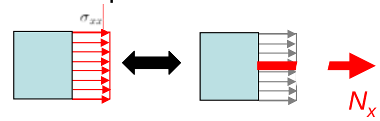
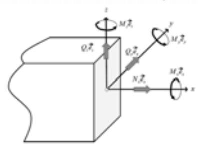
$$\begin{aligned} \operatorname{div} \boldsymbol{\sigma} &= \nabla \cdot \boldsymbol{\sigma} \\ &= \left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{1}{r} (2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\varphi\varphi} + \sigma_{r\theta} \cot\theta) \right) \vec{e}_r \\ &\quad + \left(\frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \sigma_{\theta\varphi}}{\partial \varphi} + \frac{1}{r} [(\sigma_{\theta\theta} - \sigma_{\varphi\varphi}) \cot\theta + 3\sigma_{r\theta}] \right) \vec{e}_\theta \\ &\quad + \left(\frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\varphi\theta}}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{1}{r} [3\sigma_{r\varphi} + 2 \cot\theta \, \sigma_{\theta\varphi}] \right) \vec{e}_\varphi \end{aligned}$$

Divergence of stress tensor in spherical C.S.

</td></tr>
</table>

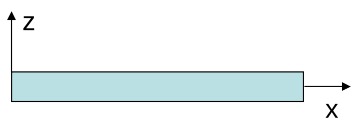
PS 4 (cylindrical C.S.)

This slide quickly summarizes the differential EQ expressions for different coordinate systems.

Variable	Definition	Notes & comments
	 *h,b* << l	Beam geometry
$\vec{F}_s = \begin{bmatrix} N_x \\ Q_y \\ Q_z \end{bmatrix}$	$N_x = \int_S \sigma_{xx}(y,z)\, dS$ $Q_y = \int_S \sigma_{yx}(y,z)\, dS$ $Q_z = \int_S \sigma_{zx}(y,z)\, dS$	Section quantities - forces 
$\vec{\mathcal{M}}^s = \begin{bmatrix} \mathcal{M}_x \\ \mathcal{M}_y \\ \mathcal{M}_z \end{bmatrix}$	$\begin{pmatrix} \mathcal{M}_x \\ \mathcal{M}_y \\ \mathcal{M}_z \end{pmatrix} = \int_S \begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times \begin{pmatrix} \sigma_{xx} \\ \sigma_{yx} \\ \sigma_{zx} \end{pmatrix} dS = \begin{pmatrix} \int_S [y\sigma_{zx} - z\sigma_{yx}]\, dS \\ \int_S z\sigma_{xx} dS \\ -\int_S y\sigma_{xx} dS \end{pmatrix}$ 	Section quantities - moments
$\underline{\underline{\sigma}}$	$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & & \text{sym} \\ \sigma_{yx} & \sim 0 & \\ \sigma_{zx} & \sim 0 & \sim 0 \end{pmatrix}$	Stress tensor beam geometry

Lecture 8

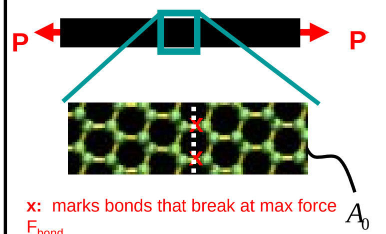
Introduction of the beam geometry. The beam is a 'special case' of the continuum theory. It introduces another scale: the beam section size (b,l) which are much smaller than the overall beam dimensions, but much larger than the size of the REV.

Variable	Definition	Notes & comments
	$\forall x \in [0,\ell]\,;\ \frac{d\vec{F}_S}{dx} + \vec{f}^{ext} = 0 \Rightarrow \begin{cases} \frac{dN_x}{dx} + f_x = 0 \\ \frac{dQ_y}{dx} + f_y = 0 \\ \frac{dQ_z}{dx} + f_z = 0 \end{cases}$ $\forall x \in [0,\ell]\,;\ \frac{d\vec{\mathcal{M}}_S}{dx} + \vec{e}_x \times \vec{F}_S = 0 : \begin{cases} \frac{d\mathcal{M}_x}{dx} = 0 \\ \frac{d\mathcal{M}_y}{dx} - Q_z = 0 \\ \frac{d\mathcal{M}_z}{dx} + Q_y = 0 \end{cases}$ +BCs	Beam EQ equations
	$\frac{dN_x}{dx} + f_x = 0 \quad \frac{dQ_z}{dx} + f_z = 0$ $\frac{d\mathcal{M}_y}{dx} - Q_z = 0$ +BCs 	2D planar beam EQ equations

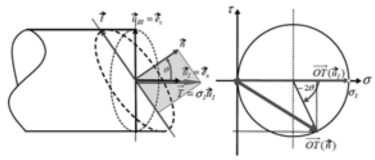
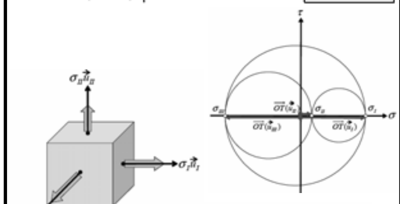
Lectures 8, 9

The beam EQ conditions enable us to solve for the distribution of moments and normal/shear forces.

The equations are simplified for a 2D beam geometry.

Variable	Definition	Notes & comments
	$\forall i;\ \vec{F}^{ext}(\vec{x}_i) + \vec{R}(\vec{x}_i) + \sum_j \vec{F}_S^j(\vec{x}_i) \overset{S.A.}{=} 0$ $\forall j;\ \vec{F}_S^j \in D_S \Leftrightarrow f\left(F_S^j = \vec{F}_S^j \cdot \vec{n}_j\right) \overset{S.C.}{\leq} 0$ $\forall j;\ f = \lvert F_S^j \rvert - (\sigma_0 A)^j \leq 0$	EQ for truss structures (S.A.) Strength criterion for truss structures (S.C.)
σ_0	$\sigma_0 = \frac{F_{\max}}{A_0}$	Tensile strength limit
	 x: marks bonds that break at max force F_{bond} $F_{\max} = F_{bond} N_{A_0}$ # bonds per area A_0 Strength per bond	Concept: Visualization of the 'strength' Number of atomic bonds per area constant due to fixed lattice parameter of crystal cell Therefore finite force per area that can be sustained

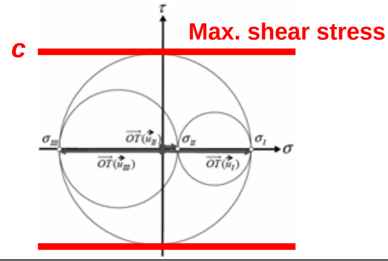
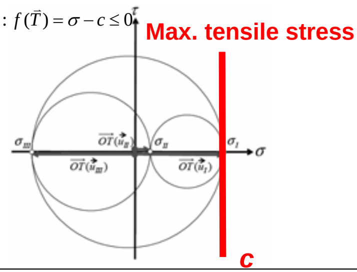
Lecture 10, PS 5 (strength calculation)

Variable	Definition	Notes & comments
$\vec{T}(\vec{x},\vec{n}) = \sigma\vec{n} + \tau\vec{t}$		Mohr plane (τ and σ) Mohr circle (Significance: Display 3D stress tensor in 2D)
σ, τ	$\sigma = \vec{n} \cdot \vec{T}(\vec{n}) = \vec{n} \cdot \boldsymbol{\sigma} \cdot \vec{n}$ $\tau = \vec{t} \cdot \vec{T}(\vec{n}) = \vec{t} \cdot \boldsymbol{\sigma} \cdot \vec{n}$	Basis in Mohr plane
$\sigma_I, \sigma_{II}, \sigma_{III}$ $\vec{u}_I, \vec{u}_{II}, \vec{u}_{III}$	$\boxed{\vec{T}^d = \boldsymbol{\sigma} \cdot \vec{u}_J = \sigma_J \vec{u}_J;\ J = I, II, III}$ $(\sigma_{ij}) = \begin{array}{c\|ccc} & \vec{T}(\vec{u}_I) & \vec{T}(\vec{u}_{II}) & \vec{T}(\vec{u}_{III}) \\ \hline (\vec{u}_I) & \sigma_I & 0 & 0 \\ (\vec{u}_{II}) & 0 & \sigma_{II} & 0 \\ (\vec{u}_{III}) & 0 & 0 & \sigma_{III} \end{array}$  $\sigma_I \overset{def}{\geq} \sigma_{II} \overset{def}{\geq} \sigma_{III}$	Principal stresses Principal stress directions Principal stresses and directions obtained through eigenvector analysis *Principal stresses =Eigenvalues* *Principal stress directions =Eigenvectors*

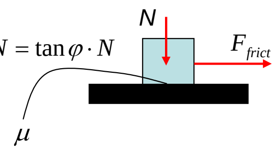
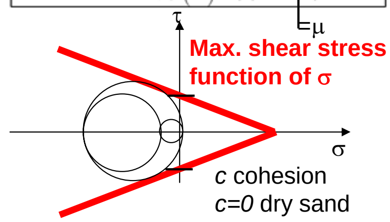
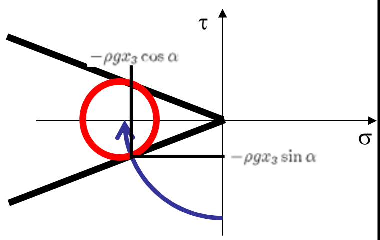
Lecture 11

Variable	Definition	Notes & comments
Two pillars of stress-strength approach	At any point, $\underline{\underline{\sigma}}$ must be: (1) Statically admissible (S.A.) **and** (2) Strength compatible (S.C.)	
	• Equilibrium conditions "only" specify statically admissible stress field, without worrying about if the stresses can actually be sustained by the material – **S.A.** *From EQ condition for a REV we can integrate up (upscale) to the structural scale* **Examples:** Many integrations in homework and in class; Hoover dam etc. • Strength compatibility adds the condition that in addition to S.A., the stress field must be compatible with the strength capacity of the material – **S.C.** *In other words, at no point in the domain can the stress vector exceed the strength capacity of the material* **Examples:** Sand pile, foundation etc. – Mohr circle	

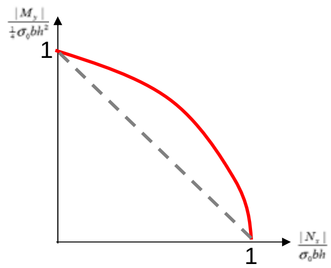
Lecture 10, 11, 12 (application to beams in lectures 13-15)

Variable	Definition	Notes & comments
D_k	$\forall \vec{x};\ \boldsymbol{\sigma}(\vec{x}) \in D_k \Leftrightarrow f(\vec{x}, \boldsymbol{\sigma}(\vec{x})) \overset{\text{S.C.}}{\leq} 0$	Strength domain (general definition) Equivalent to condition for S.C.
$D_{k,\text{Tresca}}$	Tresca: $\forall \vec{n};\ f(\vec{T}) = \lvert\tau\rvert - c \leq 0$ cohesion, $2c = \sigma_0$ 	Tresca criterion
$D_{k,\text{Tension-cutoff}}$	$\forall \vec{n}: f(\vec{T}) = \sigma - c \leq 0$ 	Tension cutoff criterion

Lecture 11

Variable	Definition	Notes & comments
F_{frict}	$F_{frict} = \mu N = \tan\varphi \cdot N$ 	Friction force Shear resistance increases with increasing normal force
$D_{k,\text{Mohr-Coulomb}}$	Mohr-Coulomb: $\forall \vec{n};\ f\left(\vec{T}\right) = \lvert\tau\rvert + \sigma\tan\varphi - c \leq 0$ 	Mohr-Coulomb
$\alpha_{\lim}$	 $\alpha_{\lim} = \varphi$	Angle of repose

Lecture 12 (Mohr-Coulomb criterion)
The definition of friction is included here for completeness

Variable	Definition	Notes & comments
D_S	$\forall x;\ \left(\vec{F}_S, \vec{\mathcal{M}}_S\right) \in D_S(x) \Leftrightarrow f\left(x, \vec{F}_S(x), \vec{\mathcal{M}}_S(x)\right) \leq 0$	Strength domain for beams
$\lvert \mathcal{M}_y \rvert_{\text{lim}} = M_0$	$\lvert \mathcal{M}_y \rvert_{\text{lim}} = M_0 = \frac{1}{4}\sigma_0 b h^2$ For rectangular cross-section *b,h*	Moment capacity for beams
$\lvert N_x \rvert_{\text{lim}} = N_0$	$\lvert N_x \rvert_{\text{lim}} = N_0 = bh\sigma_0$	Strength capacity for beams
$f(M_y, N_x) \leq 0$	$f(M_y, N_x) = \frac{\lvert M_y \rvert}{M_0} + \frac{\lvert N_x \rvert}{N_x} - 1 \leq 0$ $f(M_y, N_x) = \frac{\lvert M_y \rvert}{M_0} + \left(\frac{\lvert N_x \rvert}{N_x}\right)^2 - 1 \leq 0$ 	M-N interaction (linear) M-N interaction (actual); convexity

Lecture 13 and 14

Variable	Definition	Notes & comments
Safe strength domain	$\forall i;\ 0 \leq Q^{(i)} \leq \alpha_i Q^{(i)}_{\lim}$ $\sum_{i=1,n} \frac{Q^{(i)}}{Q^{(i)}_{\lim}} \leq \sum_{i=1,n} \alpha_i = 1$ $Q^{(i)}_{\lim}$: load bearing capacity of *i*-th load case	Linear combination is safe (convexity)

Lecture 15