Lecture 6 - summary

Review: Continuum model

Three scales:

Structural scale (H,B,D..) >> REV >> molecular scale

The three scales are separated (“>>” operator)

$$\text{on } S: \boxed{\vec{T}(\vec{n}) + \vec{T}(-\vec{n}) = 0}$$

$$\text{on } \partial\Omega: \vec{T}^d = \vec{T}(\vec{n})$$

$$\text{in } \Omega: \begin{cases} \vec{T}(\vec{n}) = \boldsymbol{\sigma} \cdot \vec{n} \\ \text{div}\, \boldsymbol{\sigma} + \rho(\vec{g} - \vec{a}) = 0 \\ \sigma_{ij} = \sigma_{ji} \end{cases}$$

Application: Hydrostatics problem

Equilibrium condition

$$-\frac{\partial p}{\partial x} = 0; \quad -\frac{\partial p}{\partial y} = 0; \quad -\frac{\partial p}{\partial z} - \rho g = 0$$

Solution after integration:

$$p(z) = -\rho g z + C$$

Satisfying BCs leads to:

$$p(z) = \rho g (H - z)$$

Stress tensor and stress vector:

$$\boldsymbol{\sigma} = -\rho g (H - z) \mathbf{1}$$

$$\vec{T}(\vec{n}) = \boldsymbol{\sigma} \cdot \vec{n} = -\rho g (H - z) \vec{n}$$

In cartesian coordinates

$$\begin{aligned} \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} + \rho(g_1 - a_1) &= 0 \\ \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + \rho(g_2 - a_2) &= 0 \\ \frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + \rho(g_3 - a_3) &= 0 \end{aligned}$$