Lecture 4 - summary

The Three Laws of Motion of Isaac Newton (1642 – 1727):

1. Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.
2. The change of motion is proportional to the motive force impresses, and is made in the direction of the right line in which that force is impressed.
3. To every action there is always opposed an equal reaction: or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts.

pp. 37-48 in manuscript

$$\vec{\wp} = m\vec{V} = m\left(V_1 \vec{e}_1 + V_2 \vec{e}_2 + V_3 \vec{e}_3\right)$$

Theorem 2 (Dynamic Resultant Theorem) *The change of the linear momentum is equal to the sum of external forces:*

$$\frac{d\vec{\wp}}{dt} = \frac{d}{dt}\left(m\vec{V}\right) \stackrel{def}{=} \vec{F}^{ext} \qquad (2.2)$$

The external force $\vec{F}^{ext} = \vec{F}_1 + \vec{F}_2 + \ldots$ *is a vector quantity.*

When the mass remains constant in time, that is, when the system is closed, the dynamic resultant theorem yields the inertia force definition $\vec{F}_{ext} = m\vec{a}$, where $\vec{a} = \frac{d}{dt}\vec{V}$ is the acceleration vector.

$$\vec{x}_i \times \vec{\wp}_i = \vec{x}_i \times m_i \vec{V}_i$$

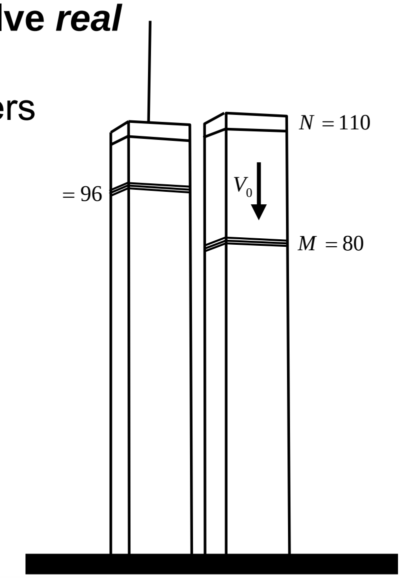
Theorem 3 (Dynamic Moment Theorem) *The change of the angular motion of a discrete system of* $i = 1, N$ *particles is equal to the sum of the moments (or torque) generated by external forces:*

$$\frac{d}{dt}\sum_{i=1}^{N}\left(\vec{x}_i \times m_i \vec{V}_i\right) \stackrel{def}{=} \sum_{i=1}^{N} \vec{x}_i \times \vec{F}_i^{ext} = \sum_{i=1}^{N} \vec{M}_i^{ext} \qquad (2.4)$$

The external moment $\vec{M}_i^{ext} = \vec{x}_i \times \vec{F}_i^{ext}$ *is a vector quantity.*

These laws can be used to solve *real* engineering problems

Example: Fall of the WTC towers on 9/11 2001



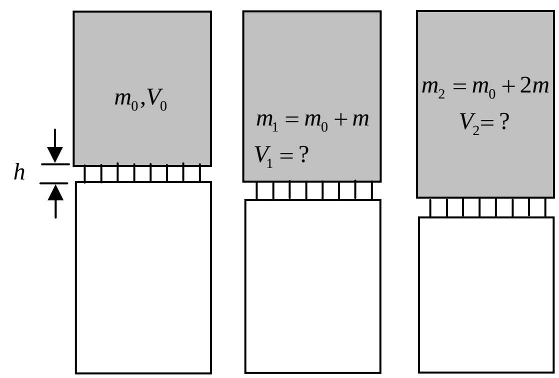
Photograph of towers removed due to copyright restrictions.

Test 2 hypotheses:

1) Free fall

$$\tau = \sqrt{\frac{2h}{g}}\left(\sqrt{1+M} - 1\right)$$

2) Kausel's model – discrete mass formulation



Instantaneous Conservation of linear momentum under changing mass

$$\delta \vec{p}_i = 0 \Rightarrow V_i = \frac{m_{i-1}}{m_i} V_{i-1}$$

$$\Delta t_i = \frac{V_i - V_{i-1}}{g}$$

	Free fall	Discrete mass model	Measurement
North Tower:	$\tau(M = 96) \approx 7.7$ s	10.9 s	≈ 10 s
South Tower:	$\tau(M = 80) \approx 7.0$ s	8.9 s	