Lecture 2 - summary

- Review: **Galileo problem**
- Pi-theorem (allows one to systematically approach the problem of expressing a physical situation in nondimensional variables)
  - By means of dimensional analysis reduce the complexity of a problem from *N+1* parameters to *N+1-k* parameters
  - Procedure:
    - Define physical problem (critical!) – define *N+1* parameters that characterize the problem
    - Set up exponent matrix – linear system; determine rank *k*
    - Choose *k* independent variables and express *N+1-k* other variables as functions of these (log representation, solve linear system) – yields nondimensional formulation
- Best invariants are not unique, some try and error – you can always recombine invariants as power functions of others.
- If $N = k$, jackpot – you have the solution (close to a multiplying constant)
- **Application**: Atomic bomb explosion

Theorem 1 (Pi-Theorem) *Consider a relation between $N+1$ dimensional physical quantities, $q_0, q_1, ..., q_N$ of the form:*

$$\boxed{q_0 = f\left(q_1, ..., q_N\right)} \tag{1.13}$$

Let k be the number of dimensionally independent variables. Let $\{q_1, ..., q_k\}$ be the complete, dimensionally independent subset of $\{q_1, ..., q_N\}$. The initial physical relation can be reduced to a dimensionless relation between $N-k+1$ similarity parameters $\Pi_0, \Pi_1, ..., \Pi_{N-k}$:

$$\boxed{\Pi_0 = \mathcal{F}\left(\Pi_1, ..., \Pi_{N-k}\right)} \tag{1.14}$$

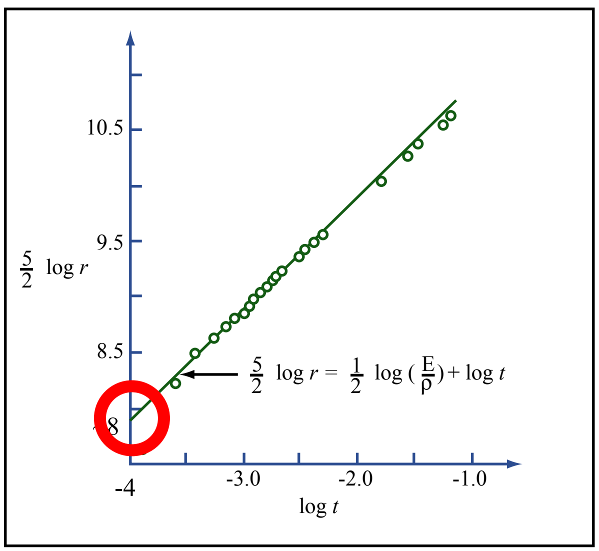
defined by:

$$\boxed{\Pi_i = \frac{q_i}{q_1^{a_1^i} q_2^{a_2^i} \cdots q_k^{a_k^i}};\ i = 0, N-k} \tag{1.15}$$

where the exponents $a_1^i, ...a_k^i$ are determined from the dimension functions:

$$\boxed{[q_i] = [q_1]^{a_1^i}\, [q_2]^{a_2^i} \cdots [q_k]^{a_k^i}} \tag{1.16}$$

Physical Similarity $\boxed{\forall i = 0, N-k;\ \Pi_i^{(1)} = \Pi_i^{(2)}}$



$$r = f(t, E, \rho)$$

D-analysis

$$\frac{5}{2}\log r = \frac{5}{2}\log(\text{const}) + \frac{1}{2}\log\left(\frac{E}{\rho}\right) + \log t$$

unknown

known

known

pp. 8-15 in manuscript

Figure by MIT OpenCourseWare, adapted from Taylor, G. I. "Formation of a Blast Wave by a Very Intense Explosion. II. The Atomic Explosion of 1945." Proceedings of the Royal Society *A* 201 (1950): 175-186.