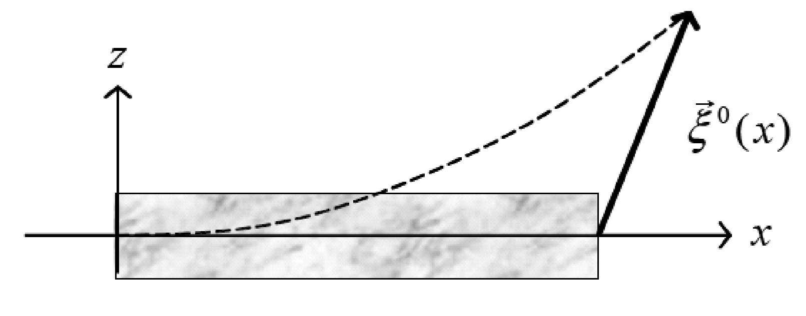
Lecture 19 - summary

Displacement vector

$$\vec{\xi} = \vec{\xi}^{\,0} + \vec{\xi}^{\,S}$$

Strain tensor

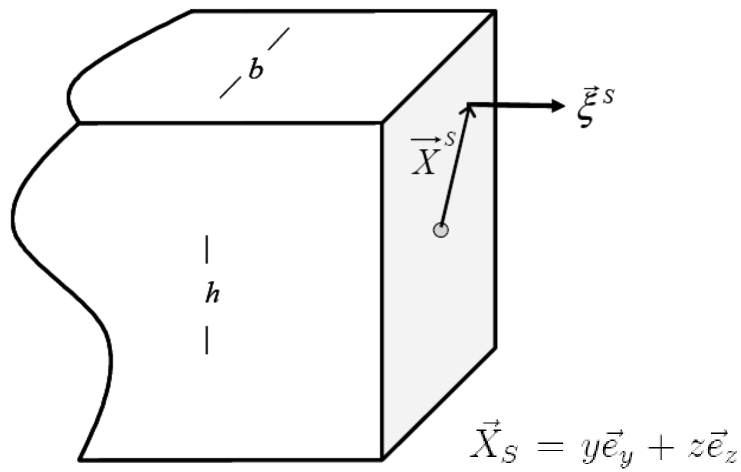
$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^{0} + \underline{\underline{\varepsilon}}^{S}$$



Decomposition into beam reference axis and section

From displacement in beam reference axis, $\vec{\xi}^{\,0}(x)$:

$$\epsilon^0_{xx} = \frac{\partial \xi^0_x}{\partial x};\ \epsilon^0_{xy} = \frac{1}{2}\frac{\partial \xi^0_y}{\partial x};\ \epsilon^0_{xz} = \frac{1}{2}\frac{\partial \xi^0_z}{\partial x}$$



Navier-Bernoulli assumption (N-B):
An initially plane beam section which is perpendicular to the beam reference axis remains plane throughout the beam deformation and perpendicular to the beam's axis in the deformed configuration.

1st consequence (section remains plane): $\vec{\xi}^{\,S} = \vec{\omega}(x) \times \vec{X}_S(y,z)$

2nd consequence: (remains perpendicular) $\omega_z = 2\epsilon^0_{xy} = \frac{\partial \xi^0_y}{\partial x};\ -\omega_y = 2\epsilon^0_{xz} = \frac{\partial \xi^0_z}{\partial x}$

Therefore: From knowledge of $\vec{\xi}$ can calculate displacements and strain tensor!

Total displacement and strain tensor coefficients:

$$\vec{\xi} = \vec{\xi}^{\,0} + \vec{\omega}(x) \times \vec{X}_S(y,z) \qquad \varepsilon_{xx} = \varepsilon^0_{xx} + \omega'_y z - \omega'_z y \qquad \varepsilon_{xy} = \varepsilon^0_{xy} - \frac{1}{2}\omega_z - \frac{1}{2}\omega'_x z \qquad \varepsilon_{xz} = \varepsilon^0_{xz} + \frac{1}{2}\omega_y + \frac{1}{2}\omega'_x y$$