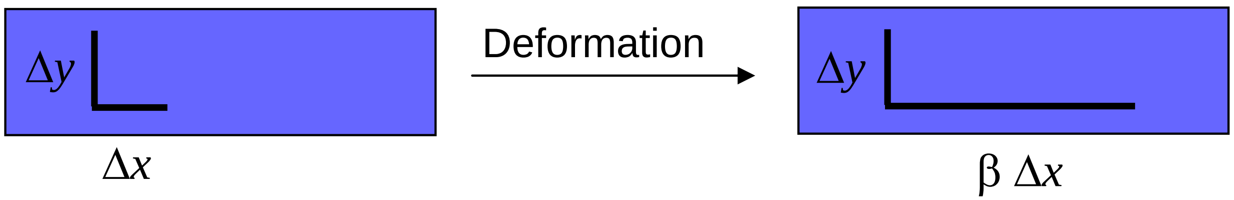
Lecture 17 - summary

Topic: How to describe deformation (cont'd from lecture 16)
Goal is to develop a mathematical language to describe deformation

Topics covered:

1.) Review and example – deformation gradient tensor (main tool)



Deformation gradient:

$$\left(F_{ij}\right)=\begin{bmatrix}\beta & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

2.) Applications to:

2.1 Volume change

$$J=\frac{d\Omega_d}{d\Omega_0}=\det \underline{\underline{F}}$$

J = Jacobian

2.2 Surface normal / surface area change

$$\vec{n}da=J\left(\underline{\underline{F}}^T\right)^{-1}\cdot\vec{N}dA$$

2.3 Length change

$$L_d^2-L_0^2=d\vec{X}\cdot\left(\underline{\underline{F}}^T\underline{\underline{F}}-\underline{\underline{1}}\right)\cdot d\vec{X}=d\vec{X}\cdot 2\underline{\underline{E}}\cdot d\vec{X}$$

$$\underline{\underline{E}}=\underline{\underline{F}}^T\underline{\underline{F}}-\underline{\underline{1}}$$

Strain tensor

$$\lambda_\alpha=\frac{\Delta L_\alpha}{L_{0,\alpha}}\sqrt{2E_{\alpha\alpha}+1}-1$$

relative length variation in the α-direction

2.4 Angle change

$$\sin\theta_{\alpha,\beta}=\frac{2E_{\alpha\beta}}{(1+\lambda_\alpha)(1+\lambda_\beta)}$$