1.050 Engineering Mechanics I

Lecture 33

How things fail – and how to avoid it

Elastic buckling

1

1.050 – Content overview

I. Dimensional analysis
1. On monsters, mice and mushrooms
2. Similarity relations: Important engineering tools

Lectures 1-3
Sept.

II. Stresses and strength
3. Stresses and equilibrium
4. Strength models (how to design structures, foundations.. against mechanical failure)

Lectures 4-15
Sept./Oct.

III. Deformation and strain
5. How strain gages work?
6. How to measure deformation in a 3D structure/material?

Lectures 16-19
Oct.

IV. Elasticity
7. Elasticity model – link stresses and deformation
8. Variational methods in elasticity

Lectures 20-32
Oct./Nov.

V. How things fail – and how to avoid it
9. Elastic instabilities
10. Plasticity (permanent deformation)
11. Fracture mechanics

Lectures 33-37
Dec.

2

1.050 – Content overview

I. Dimensional analysis

II. Stresses and strength

III. Deformation and strain

IV. Elasticity

V. How things fail – and how to avoid it
- **Lecture 33 (Mon): Buckling (loss of convexity)**
- **Lecture 34 (Wed): Fracture mechanics I (and surprise!)**
- **Lecture 35 (Fri): Fracture mechanics II**
- **Lecture 36 (Mon): Plastic yield**
- **Lecture 37 (Wed): Wrap-up plastic yield and closure**

3

Characterization of failure

- **Elasticity** = characterized by convexity (basis for energy methods)
- **Failure** = characterized by loss of convexity ("beyond elasticity")

4

Types of failure

- **Elastic buckling** (lecture 33)
  Purely elastic phenomenon, reach bifurcation point at which potential energy of system loses convexity (leads to sudden change of shape of structure); non-dissipative, fully elastic throughout
- **Fracture and cracking** (lecture 34, 35)
  "Brittle character", that is, stored elastic energy due to work done by external forces is suddenly released; energy dissipated into breaking of atomic bonds
- **Plastic collapse** (lectures 36, 37)
  "Ductile character", that is, ability of system to store energy supplied by external forces is exhausted leading to frictional dissipation

5

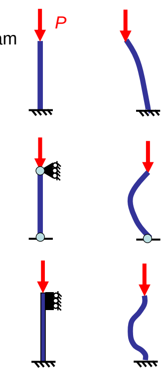
Other boundary conditions: Euler buckling

$$P < P_{crit} = \frac{\pi^2 EI}{(el)^2}$$

Clamped cantilever beam
$e = 2$

Single supported beam
$e = 1$

Double clamped cantilever beam
$e = \frac{1}{2}$



6

Summary: Elastic buckling

- **Loss of convexity** is characterized by bifurcation points at which the determinant of the solution matrix (2nd order beam theory) reaches zero: **No solution exists**
- **Approximation using potential energy method** leads to upper bound of the actual buckling load (use for complex boundary conditions, numerical approach..)
- General solution approach leads to **different critical buckling loads for different boundary conditions**

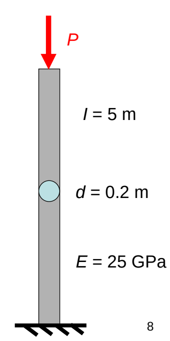
7

Example: Concrete column w/ circular cross-section

$$P < P_{crit} = \frac{\pi^2 EI}{(2l)^2}$$

$$EI = \frac{E\pi d^4}{64}$$ Circular cross-section

$$\boxed{P_{crit} = 194 \text{ kN}}$$



8