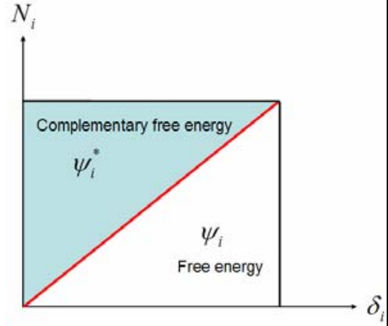
Clapeyron's formulas for 1D



For the **linear elastic** case:

$$\psi(\delta_i) = \psi^*(N_i)$$

We will now exploit this property (specific to linear elasticity!) to derive a set of equations called "Clapeyron's formulas":

Recall, total external work (lecture 27):

$$W^d = \vec{\xi} \cdot \vec{F}^d + \vec{\xi}^d \cdot \vec{R}$$

We split this up into: (1) work due to force BCs

$$W(\vec{\xi}) = \vec{\xi} \cdot \vec{F}^d$$

and (2) work due to displacement BCs

$$W^*(\vec{R}) = \vec{\xi}^d \cdot \vec{R}$$

Therefore

$$\psi(\delta_i) + \psi^*(N_i) = W^* + W$$

Using the fact that the complementary free energy and free energy are equal, we arrive at:

$$\psi(\delta_i) = \frac{1}{2}\left(W^* + W\right)$$

$$\psi^*(N_i) = \frac{1}{2}\left(W^* + W\right)$$

Now we calculate the potential and complementary energy:

$$\varepsilon_{\text{pot}} = \psi(\delta_i) - W \qquad \varepsilon_{\text{com}} = \psi^*(N_i) - W^*$$

By using the expressions for the free energy and complementary free energy…:

$$\varepsilon_{\text{pot}} = \frac{1}{2}\left(W^* - W\right) \qquad \varepsilon_{\text{com}} = \frac{1}{2}\left(W - W^*\right)$$

Summary – the following set of equations are called Clapeyron's formulas

$$\psi(\delta_i) = \frac{1}{2}\left(W^* + W\right)$$

$$\psi^*(N_i) = \frac{1}{2}\left(W^* + W\right)$$

$$\varepsilon_{\text{pot}} = \frac{1}{2}\left(W^* - W\right)$$

$$\varepsilon_{\text{com}} = \frac{1}{2}\left(W - W^*\right)$$

Significance: Can calculate free energy, complementary free energy, potential energy and complementary energy directly from **the boundary conditions (external work)**!