1.050 Engineering Mechanics I

Lecture 27

Introduction: Energy bounds in linear elasticity

1.050 – Content overview

I. Dimensional analysis

1. On monsters, mice and mushrooms
2. Similarity relations: Important engineering tools

Lectures 1-3
Sept.

II. Stresses and strength

3. Stresses and equilibrium
4. Strength models (how to design structures, foundations.. against mechanical failure)

Lectures 4-15
Sept./Oct.

III. Deformation and strain

5. How strain gages work?
6. How to measure deformation in a 3D structure/material?

Lectures 16-19
Oct.

IV. Elasticity

7. Elasticity model – link stresses and deformation
8. Variational methods in elasticity

Lectures 20-31
Oct./Nov.

V. How things fail – and how to avoid it

9. Elastic instabilities
10. Plasticity (permanent deformation)
11. Fracture mechanics

Lectures 32-37
Dec.

1.050 – Content overview

I. Dimensional analysis

II. Stresses and strength

III. Deformation and strain

IV. Elasticity

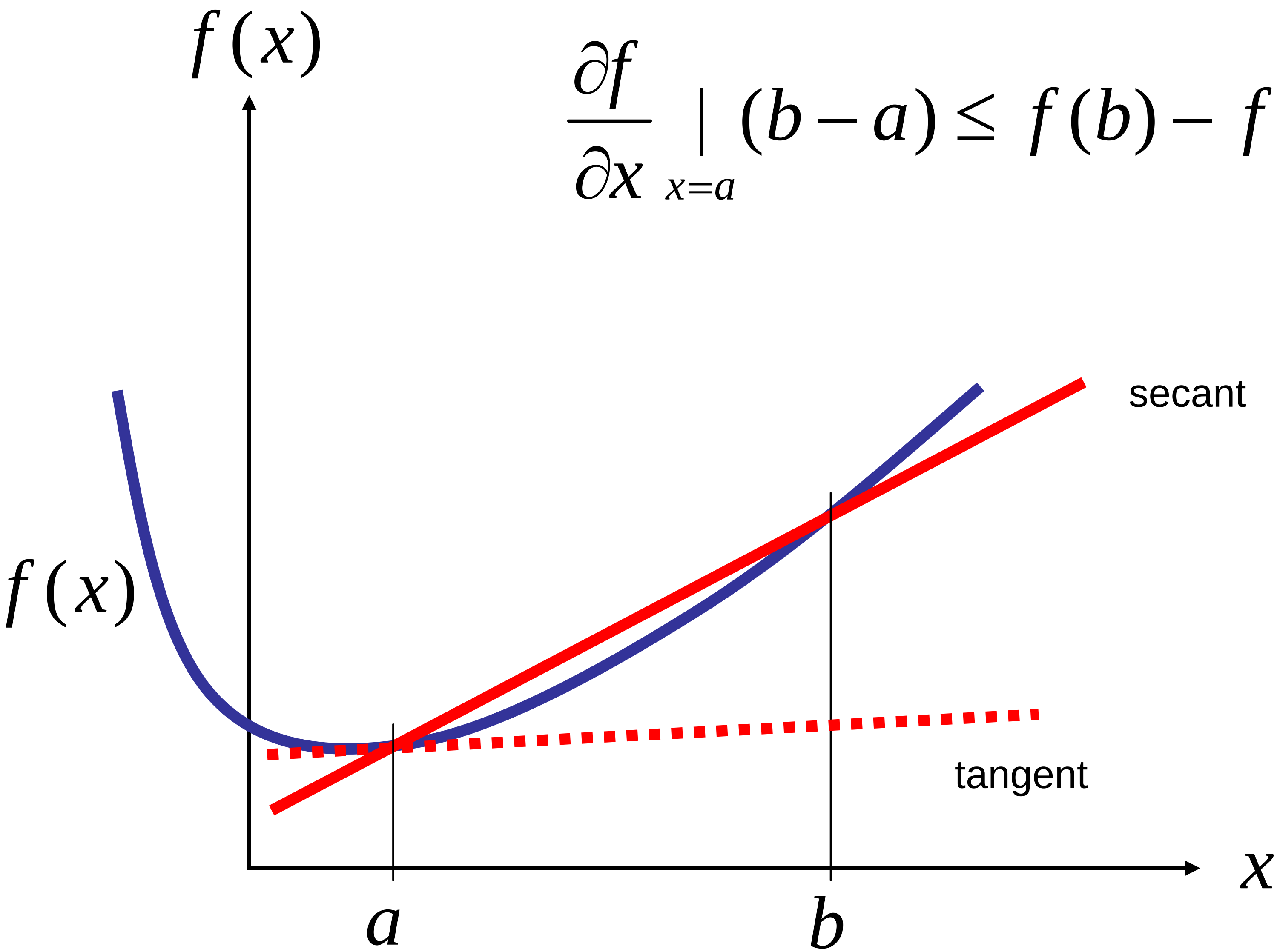
Lecture 20: Introduction to elasticity (thermodynamics)
Lecture 21: Generalization to 3D continuum elasticity
Lecture 22: Special case: isotropic elasticity
Lecture 23: Applications and examples
Lecture 24: Beam elasticity
Lecture 25: Applications and examples (beam elasticity)
Lecture 26: … cont'd and closure
Lecture 27: Introduction: Energy bounds in linear elasticity (1D system)
Lecture 28: Introduction: Energy bounds in linear elasticity (1D system), cont'd
…

V. How things fail – and how to avoid it

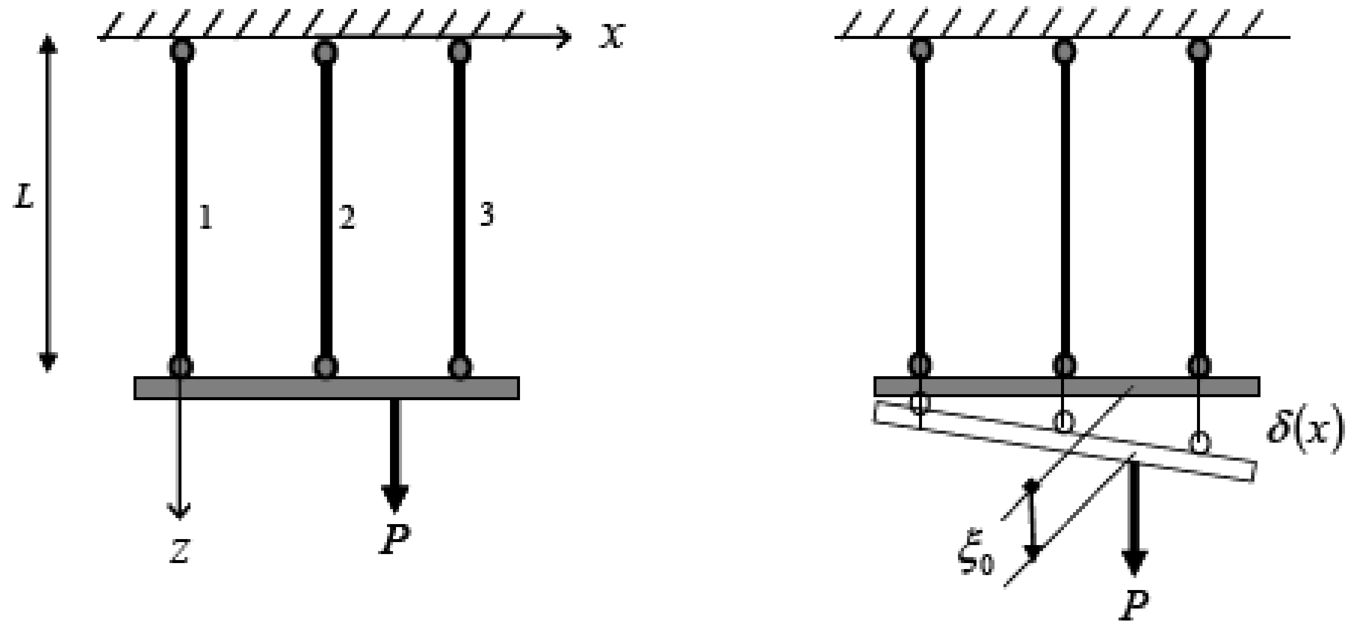
Lectures 32..37

Convexity of a function

$$\frac{\partial f}{\partial x}\Big|_{x=a}(b-a) \le f(b) - f(a)$$



Example system: 1D truss structure



We will use this example to illustrate all key concepts

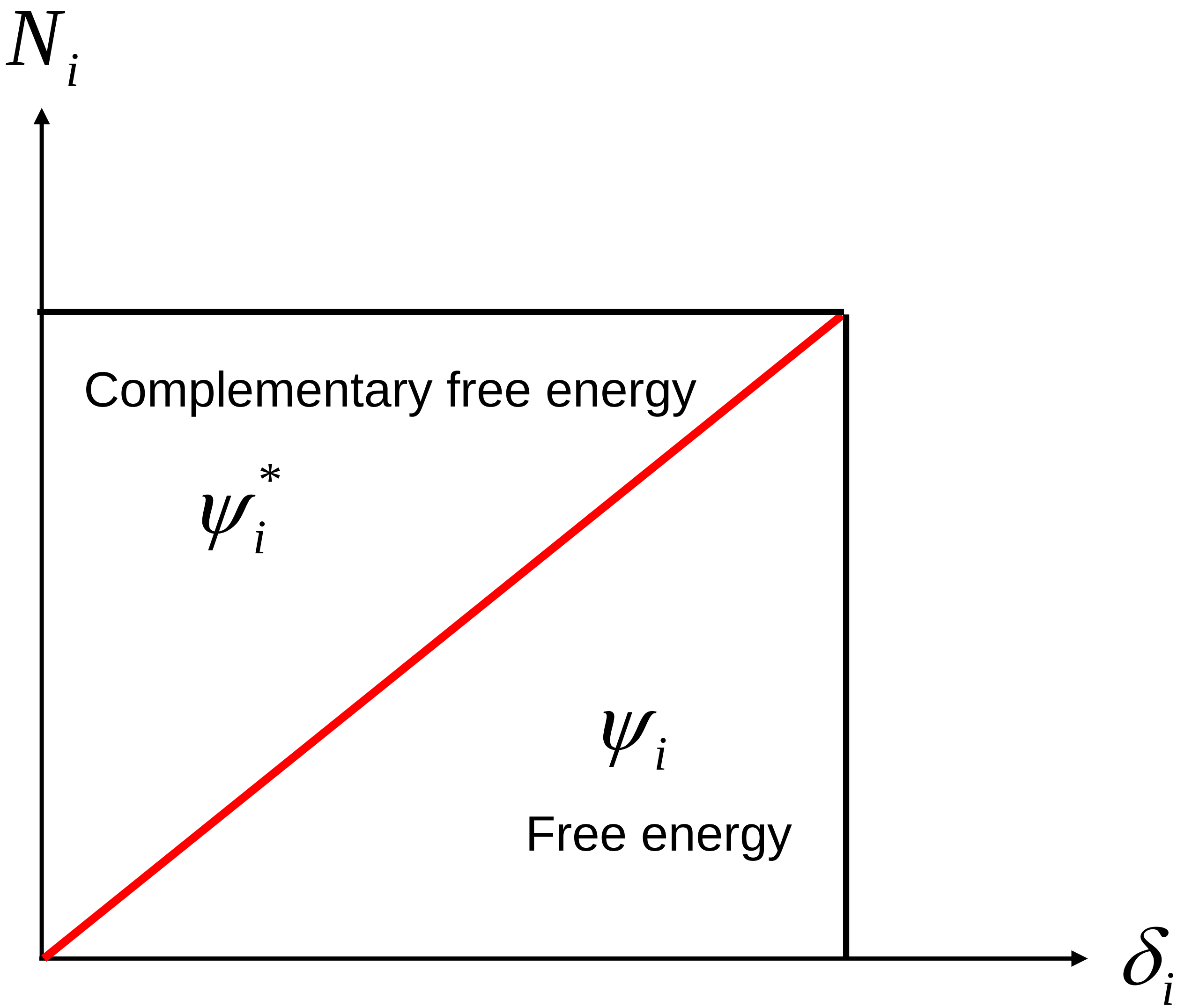
Total external work

$$W^d = \vec{\xi} \cdot \vec{F}^d + \vec{\xi}^d \cdot \vec{R}$$

Work done by prescribed forces Displacements unknown

Work done by prescribed displacements, force unknown

Total internal work



State equations

$$N_i = \frac{\partial \psi_i}{\partial \delta_i}$$

$$\delta_i = \frac{\partial \psi_i^*}{\partial N_i}$$

$$\sum_i \delta_i N_i = \psi_i^*(N_i) + \psi_i(\delta_i)$$

Combining it…

$$W^d = \vec{\xi} \cdot \vec{F}^d + \vec{\xi}^d \cdot \vec{R} \overset{!}{=} \psi + \psi^*$$

$$-\left(\psi^* - \vec{\xi}^d \cdot \vec{R}\right) \overset{!}{=} \psi - \vec{\xi} \cdot \vec{F}^d$$

Complementary energy $=: \varepsilon_{\text{com}}$

Complementary energy $=: \varepsilon_{\text{pot}}$

Solution to elasticity problem $-\varepsilon_{\text{com}} = \varepsilon_{\text{pot}}$

Quiz II – Monday Nov. 19

- Focus on material presented in lectures 16-26
- **Preparation:** Problem sets, old quizzes, lecture material
- Deformation and strain, isotropic elasticity, beam deformation (beam bending and beam stretching), forensic beam elasticity, sketch solution of beam problems, concept of superposition (frame structures)