General solution procedure

Elasticity condition (no dissipation): $d\psi = \delta W$ reflecting that $dD = 0$
(this is the result from analyzing the TD as done in class)

- **Step 1**: Express $$d\psi(x_1, x_2, ..) = \frac{\partial \psi}{\partial x_1} dx_1 + \frac{\partial \psi}{\partial x_2} dx_2 + ... = \frac{\partial \psi}{\partial x_i} dx_i$$

- **Step 2**: Express $$\delta W(\xi_1, \xi_2, ..) = \frac{\partial F}{\partial \xi_1} d\xi_1 + \frac{\partial F}{\partial \xi_2} d\xi_2 + ... = \frac{\partial \psi}{\partial \xi_j} d\xi_j$$

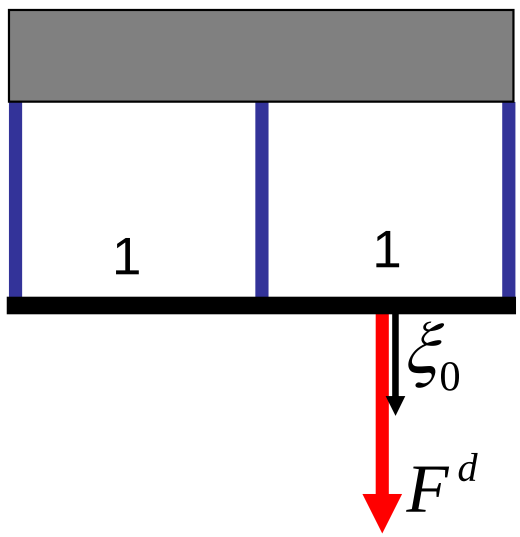
- **Step 3**: Solve equations $$\frac{\partial \psi}{\partial x_i} dx_i = \frac{\partial \psi}{\partial \xi_j} d\xi_j \qquad \forall dx_i, \forall d\xi_j$$

  Collect all terms dx_i and $d\xi_j$ and set the entire expression to zero.

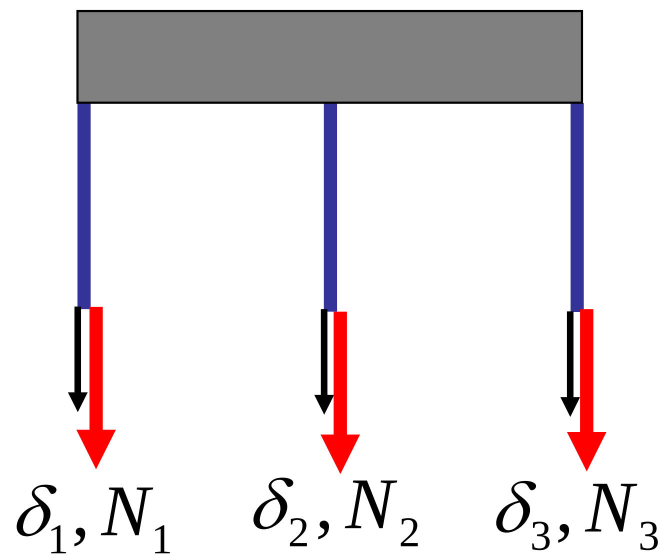
  In EQ, the expression must be satisfied for all displacement changes $dx_i, d\xi_j$

Example II: Truss structure (1)

Problem statement: Structure of three trusses with applied force F^d :



Forces in each truss



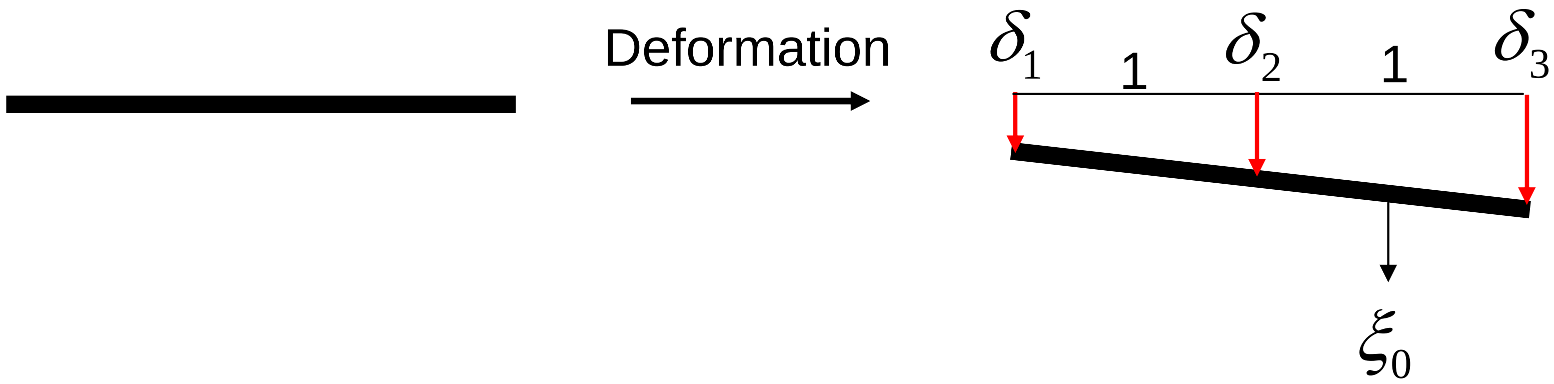
Distance *L*=1 between the trusses

$$\left.\begin{aligned} N_1 &= k\delta_1 \\ N_2 &= k\delta_2 \\ N_3 &= k\delta_3 \end{aligned}\right\}$$ Trusses behave like springs

Goal: Calculate displacements δ_i, ξ_0 for given F^d

Example II: Truss structure (2)

Rigid bar: If two displacements δ_1, δ_2 are specified can calculate the other displacements (kinematic constraint):



Therefore:

$$\delta_3 = \delta_1 + 2\frac{\delta_2 - \delta_1}{1} = 2\delta_2 - \delta_1$$

$$\xi_0 = \frac{3}{2}\delta_2 - \frac{1}{2}\delta_1$$

Example II: Truss structure (3)

Solution procedure:

Elasticity condition (no dissipation): $d\psi = \delta W$

- **Step 1:** $d\psi(\delta_1, \delta_2) = \frac{1}{2}k\left[(4\delta_1 - 4\delta_2)d\delta_1 + (-4\delta_1 + 10\delta_2)d\delta_2\right]$
- **Step 2**: $\delta W(\xi_1) = F^d\left[-\frac{1}{2}d\delta_1 + \frac{3}{2}d\delta_2\right]$
- **Step 3**: Solve equations $d\psi = \delta W \qquad \forall dx_i, \forall d\xi_j$

$$\frac{1}{2}k\left[(4\delta_1 - 4\delta_2)d\delta_1 + (-4\delta_1 + 10\delta_2)d\delta_2\right] = F^d\left[-\frac{1}{2}d\delta_1 + \frac{3}{2}d\delta_2\right]$$

$$\Bigg[\underbrace{\left(2k\delta_1 - 2k\delta_2 + \frac{1}{2}F^d\right)}_{\overset{!}{=}0}d\delta_1 + \underbrace{\left(-2k\delta_1 + 5k\delta_2 - \frac{3}{2}F^d\right)}_{\overset{!}{=}0}d\delta_2\Bigg] = 0$$

for elastic EQ

Example II: Truss structure (4)

This results in linear system of equations:

$$\underbrace{\begin{pmatrix} 2k & -2k \\ -2k & 5k \end{pmatrix}}_{M} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} -F^d/2 \\ 3F^d/2 \end{pmatrix}$$

Solve for the unknown variables $\delta_1, \delta_2, ..$

Note that (forming the inverse of a 2x2 matrix):

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

This can be used to calculate M^{-1} δ_1, δ_2

Example II: Truss structure (5)

This results in:

$$\begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \underbrace{\frac{1}{6k}\begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}}_{M^{-1}} \begin{pmatrix} -F^d/2 \\ 3F^d/2 \end{pmatrix} = \frac{F^d}{k}\begin{pmatrix} 1/12 \\ 1/3 \end{pmatrix}$$

Solve for the other unknown variables (utilize kinematic relationships and the spring equations):

$$\delta_3 = 7/12\frac{F^d}{k}$$

$$\xi_0 = 11/24\frac{F^d}{k}$$

$$\xrightarrow{N_i = k\delta_i}$$

$$N_1 = 1/12F^d$$

$$N_2 = 1/3F^d$$

$$N_3 = 7/12F^d$$