

1.050: Beam Elasticity (HW#9)

Due: November 14, 2007

MIT – 1.050 (Engineering Mechanics I)

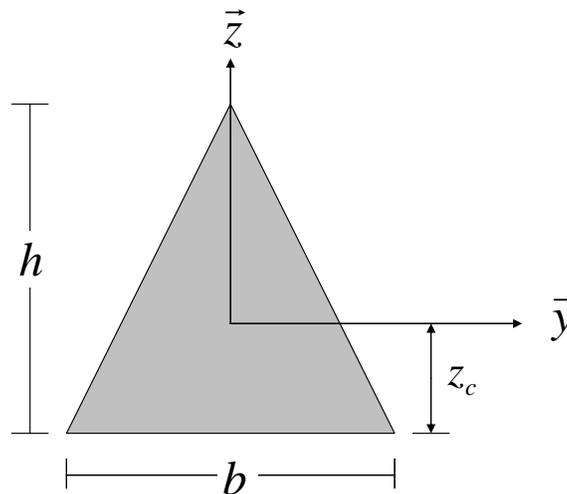
Fall 2007

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Team Building and Team Work: We strongly encourage you to form Homework teams of three students. Each team only submits one solution for correction. We expect true team work, i.e. one where everybody contributes equally to the result. This is testified by the team members signing at the end of the team copy a written declaration that "the undersigned have equally contributed to the homework". Ideally, each student will work first individually through the homework set. The team then meets and discusses questions, difficulties and solutions, and eventually, meets with the TA or the instructor. Important: Specify all resources you use for your solution.

The following set of exercises is designed to familiarize you with the use of the beam elasticity model. Where appropriate, display your responses graphically in form of free body force, moment, rotation, and deflection diagrams.

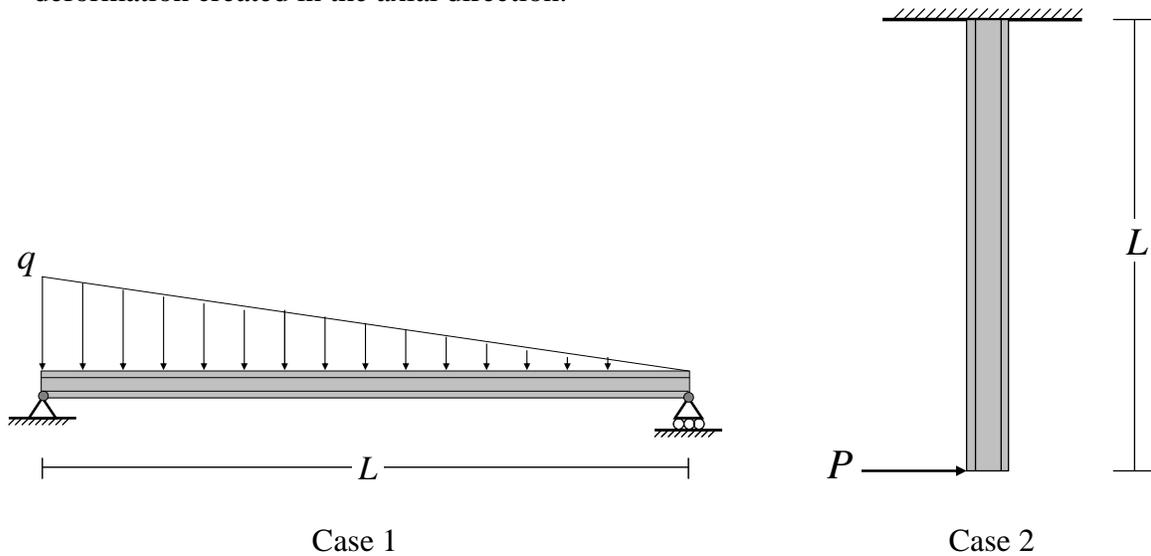
1. **Area Moments of Inertia:** For the given isosceles triangle cross-section, determine the following quantities:
 - a. The zero-order area moment, S (this represents the cross-sectional area).
 - b. The centroid of the cross-section in the z -axis, z_c .
 - c. The first-order area moment, S_z .
 - d. The second-order area moment, I_{zz} .



2. 2-D Beam Elasticity – Differential Approach: The sketch below shows two beam members subjected to different loads and boundary conditions.

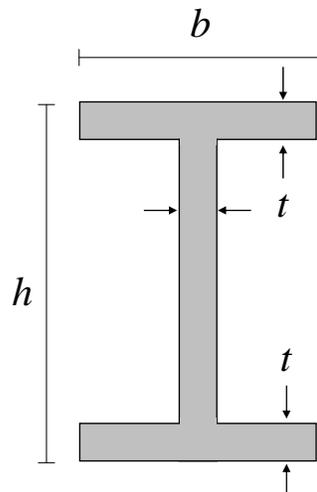
- a. For the two cases, calculate the shear force, bending moment, rotation, and deflection of the beam. Outline your process using the 4-step method for solving beam structure problems presented during 1.050 lectures. Display your responses graphically.

Note: For case 2, consider the self-weight of the hanging beam, as well as the deformation created in the axial direction.



- b. For both cases, the beam's cross-section and its dimensions are displayed on the sketch below ($t \ll b$ and $t \ll h$). Determine rigorously the position of the centroid, and the relevant area moment of inertia (refer to the example calculation for I as shown in lecture 25 and apply the method to this problem).

Sketch the stress distribution over the cross-section (in the z -direction), at the location x along the beam's axis where the maximum moment occurs.



- c. Let σ_o be the uniaxial strength of the material. For both cases 1 and 2, determine:
- The elastic limit load q_0^{el} , i.e. the limit load you would calculate at failure by assuming that the stress distribution in the beam is the elastic one.
 - The maximum load-bearing capacity, as we have done in previous beam-strength exercises. Consider a bending moment strength criterion for the beam.
- d. *Design Problem for Case 1*

We consider $q_0 = 20$ kN/m; $L = 5$ m. We also consider the section to be made of steel, which has a Young's modulus $E = 250$ GPa and a strength $\sigma_o = 220$ MPa. Check for the section dimensions (height $h = 20$ cm, flange width $b = 10$ cm and thickness $t = 1$ cm) that the following two design conditions are satisfied. If they are not, propose a design change (but do not recalculate):

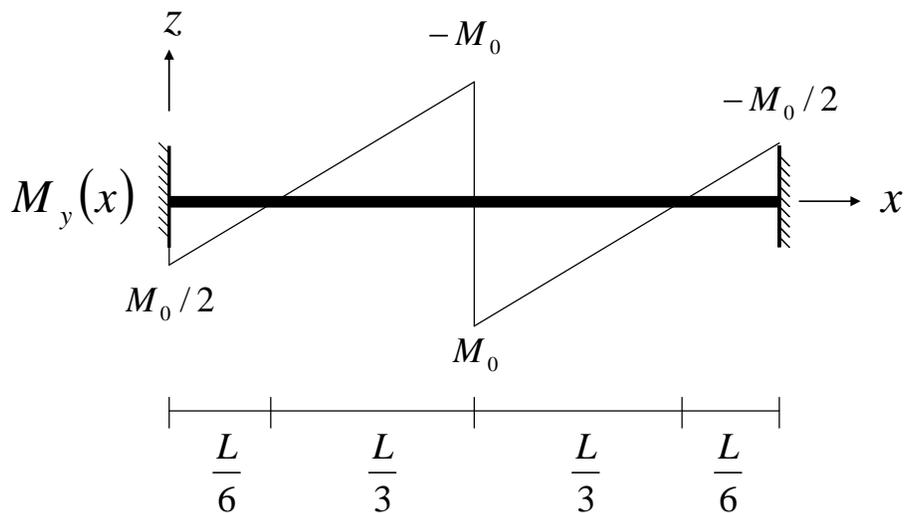
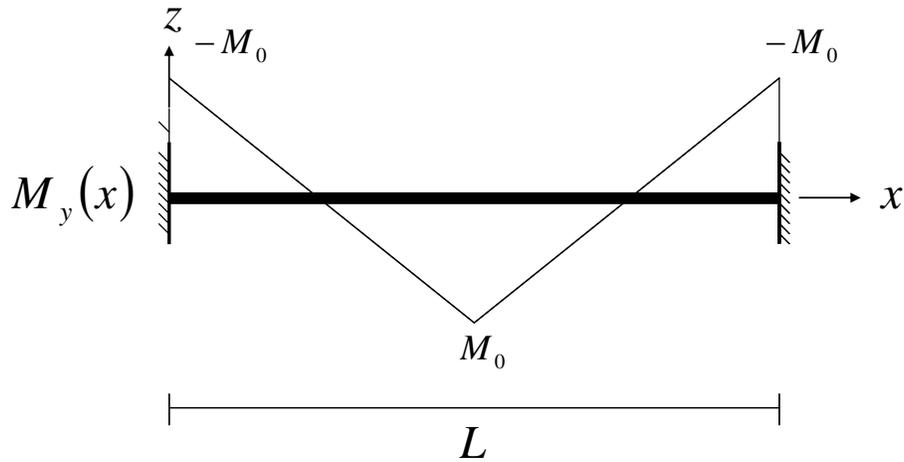
- The load must not exceed the elastic limit load.
- The maximum elastic deflection δ of the beam must not exceed $\delta/L = 1/500$.

3. Forensic Beam Elasticity: The sketches below show the elastic moment diagrams of a beam clamped on both sides for two different bending moment responses. For both cases:

- Determine the shear force distribution, and reconstruct the load (type and orientation) to which the beam is subjected.
- Determine the shape of the elastic rotation $\omega_y(x)$ and the deflection diagram $\xi_z(x)$.

Indicate where the maximum rotation and deflection occurs ($EI_{zz} = \text{const}$) (for this problem, numerical values are not important - the CLEAR SHAPE of the diagram is; including minima/maxima, deflection points, etc.).

- Compared to the elastic moment diagram displayed in the sketch, how does the moment diagram look (shape + values) when the beam reaches its load bearing capacity, $\forall x, |M_y(x)| \leq M_0$ (where M_0 stands for the maximum bending moment the section can support)?



4. Beam Structure Subjected to Hydrostatic Loading: A beam structure of length L and height H is cantilevered from a ceiling, holding a still body of water, of density ρ_w , as displayed in the sketch below. The width of the structure b , in the direction into the paper, is so large that allows treating this problem as two-dimensional, i.e. evaluate quantities depending on the width b in terms of ‘per unit width’.

- a. *Application of Hydrostatics Theory:* Determine the hydrostatic load distribution (have a look back to what we have seen in hydrostatics) and determine the resulting pressure distribution along the beam structure.

Hint: First write the stress vector for any plane (characterized by its normal vector) in the water domain, and then write the stress vector for the boundaries $B-C$ and between points A and B . The solution for this part forms the boundary conditions for Part b.

- b. *Beam Theory:* Determine the section forces and section moments that develop in the beam-type structure in response to the pressure loading determined in Part a of this exercise. Given that the beam structure has a Young's modulus E and a thickness t , determine the corresponding linear elastic rotation and deflections of the cantilever beam structure (Note: consider bending and axial deformation for the vertical beam $A-B$).

In particular, what are the horizontal and vertical displacements at point D ?

