

1.050: Stresses and Equilibrium (HW #3)

Due: September 26, 2007

MIT – 1.050 (Engineering Mechanics I)

Fall 2007

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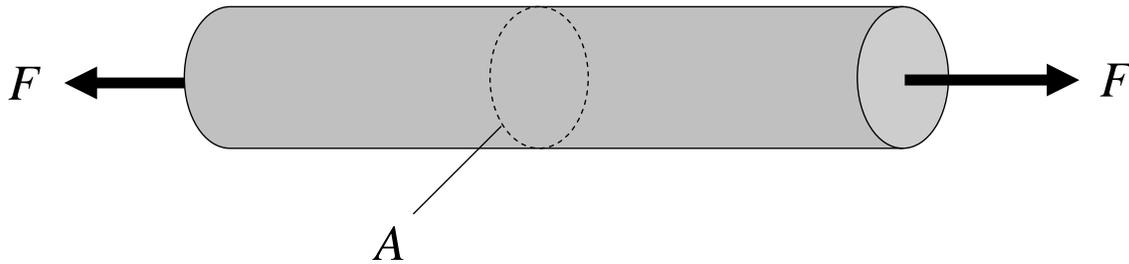
Team Building and Team Work: We strongly encourage you to form Homework teams of three students. Each team only submits one solution for correction. We expect true team work, i.e. one where everybody contributes equally to the result. This is testified by the team members signing at the end of the team copy a written declaration that "the undersigned have equally contributed to the homework". Ideally, each student will work first individually through the homework set. The team then meets and discusses questions, difficulties and solutions, and eventually, meets with TA or instructor. Important: Specify all resources you use for your solution.

The following set of exercises is designed to train you in the use of equilibrium conditions for discrete systems. For each exercise, show us how you come to your answer and result. We highly encourage you to make drawings (for instance of free body diagrams) where appropriate. Don't forget to add the coordinate system.

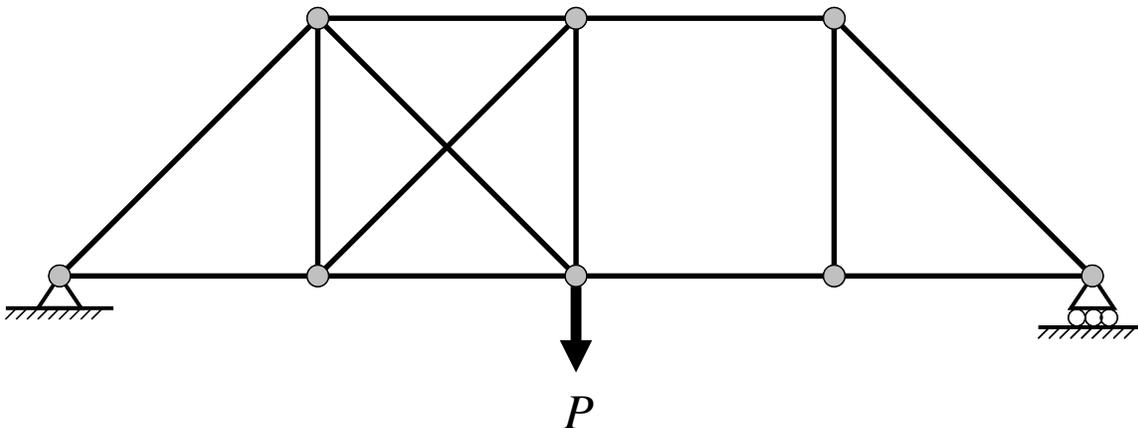
1. Stress tensor and REV.: We use Cartesian coordinates. Display an REV and draw the stress vectors it is submitted to, for the following cases:

$$\underline{\underline{\sigma}} = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & -3 \\ 1 & -3 & 0 \end{bmatrix} \quad ; \quad \underline{\underline{\sigma}} = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \underline{\underline{\sigma}} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad ; \quad \underline{\underline{\sigma}} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

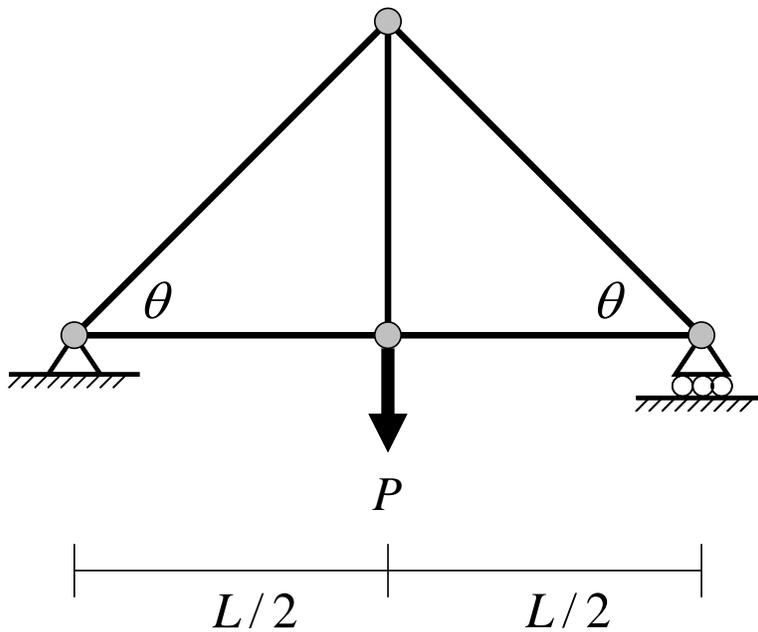
- 2. Stress state inside a truss member:** The figure below displays a typical truss member of cylindrical shape with cross section area A . The rod is subjected to a force F at its ends.
- The goal of this problem is to calculate the stress state inside truss member. Display your result in tensorial form; that is, write out the stress tensor in tensor notation. Also write out the stress vector for the normal that points in the axial direction.
 - Draw a typical REV from within the truss member and display the stress vector(s) it is submitted to.
 - Numerical example:* If the material strength is $\sigma_0 = 100$ MPa, and the cross-section area of the rod is $A = 5$ cm², calculate the maximum force that the rod can sustain.



- 3. Instability:** Show that the following structure is unstable. Modify it so that it becomes stable, while it remains statically determinate.



- 4. Truss structure as a discrete system:** For the truss system shown in the figure below,
- Is this truss system statically determinate?
 - Calculate the reactions at the structure's supports.
 - Calculate the force in all truss members, as a function of angle θ .
 - Given the information about material strength (σ_0 and cross-section area A_{bar}) for each truss member, calculate the angle θ for which the stress in the each member does not exceed the allowed material strength (Note: the material strength σ_0 is valid for both tension and compression behaviors).



$$P = 220kN$$

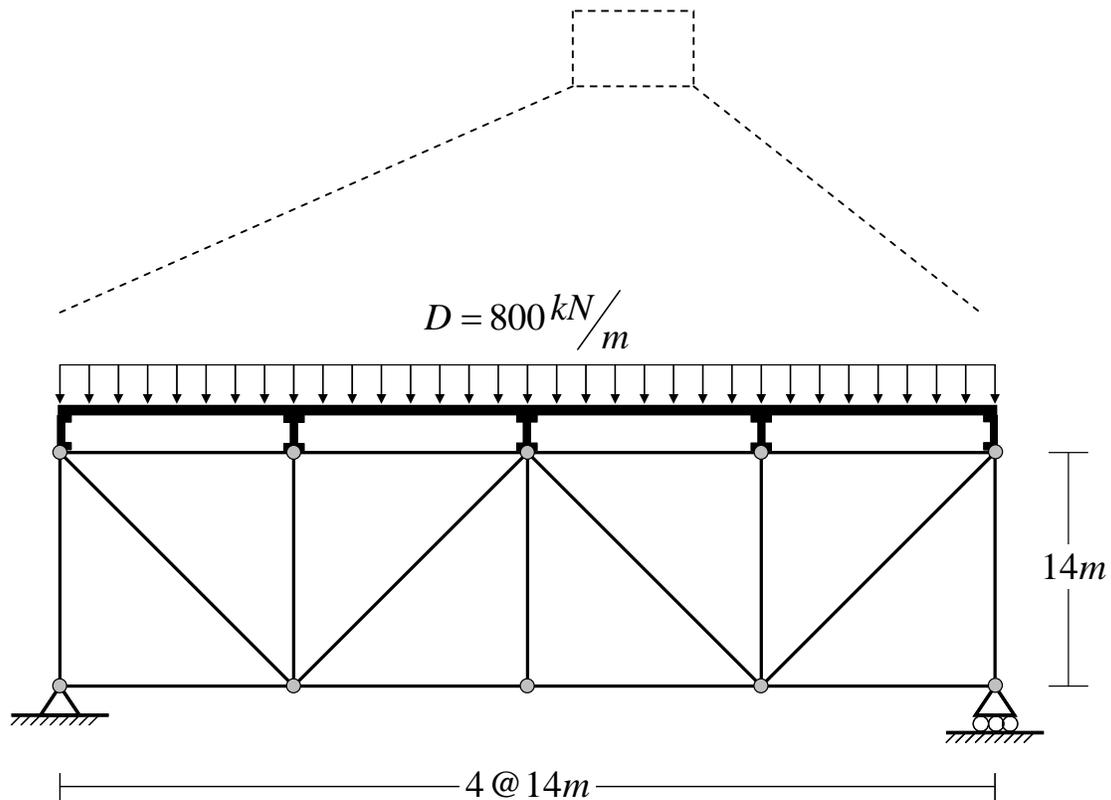
$$L = 2m$$

$$A_{bar} = 4cm^2$$

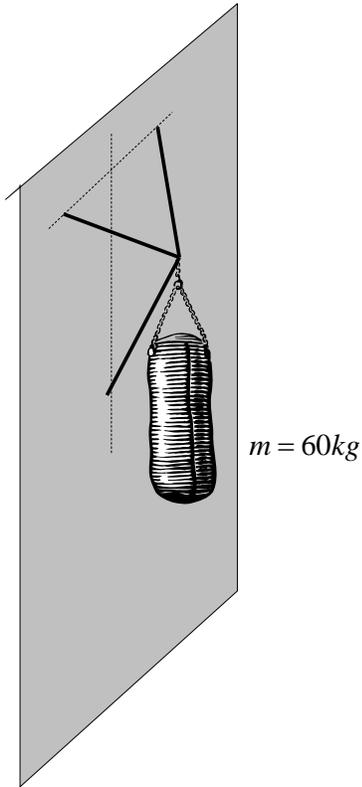
$$\sigma_0 = 500MPa$$

5. A truss system in a bridge deck: In recitation, we learned about the Akashi-Kaikyo suspension bridge in Japan. As shown in the photograph below, the bridge's deck rests on a truss system. While the actual truss system is a complex 3-D structure, we will analyze a 2-D section as displayed below. The 2-D truss system is subjected to a distributed load of $D = 800 \text{ kN/m}$, which represents the weight of the deck. The goal of this exercise is to calculate the force in all truss members. Think carefully about how to handle the distributed load in order to analyze the truss structure. Discuss what kind of implications your assumptions may have, and what kinds of phenomena may not be captured correctly.

Photograph removed due to copyright restrictions.



6. A 3-D truss structure:



You have probably observed at the gym a punching bag. In some cases, the bag is hanging from a truss structure that is connected to the wall, as depicted in the sketch to the left. The goal of this exercise is to determine the force in members AB, AC, and AD. Indicate if the members are subjected to tension or compression. In addition, compute all the reaction forces at nodes B, C, and D.

