

1.050: Energy Bounds in Elasticity (HW#11)

Due: Friday, December 7, 2007

MIT – 1.050 (Engineering Mechanics I)

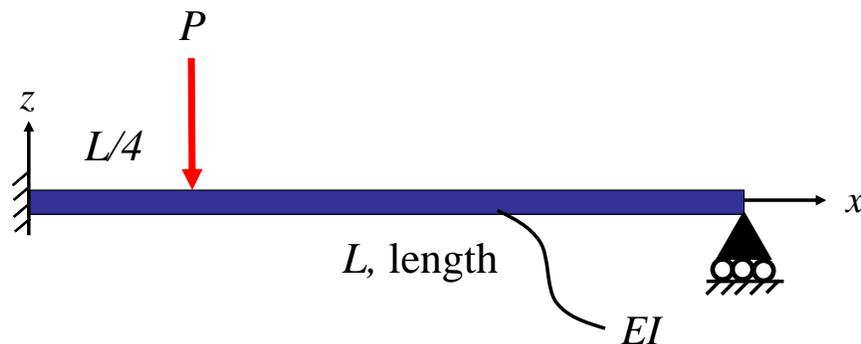
Fall 2007

Instructor: Markus J. BUEHLER

Team Building and Team Work: We strongly encourage you to form Homework teams of three students. Each team only submits one solution for correction. We expect true team work, i.e. one where everybody contributes equally to the result. This is testified by the team members signing at the end of the team copy a written declaration that "the undersigned have equally contributed to the homework". Ideally, each student will work first individually through the homework set. The team then meets and discusses questions, difficulties and solutions, and eventually, meets with the TA or the instructor. Important: Specify all resources you use for your solution.

The following set of exercises is designed to familiarize you with the use of energy bounds in linear elasticity. This problem set is focused on 3D methods and applications to beam structures and you will be able to apply the techniques you have learned in Lectures 30, 31 and 32. Please review the lecture notes and handouts carefully.

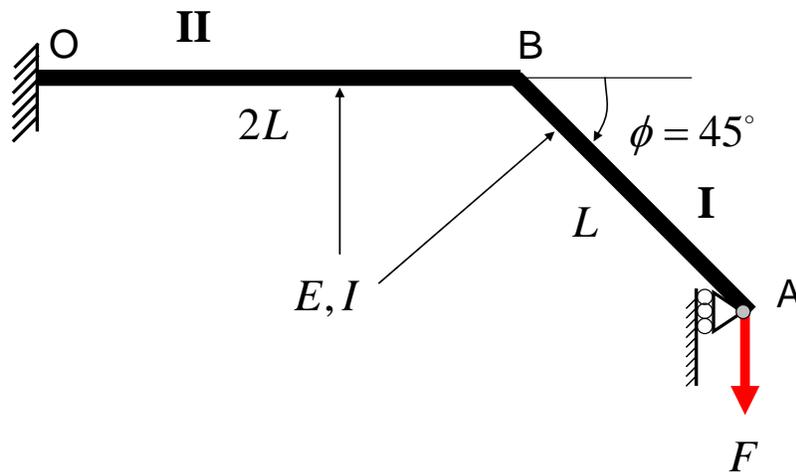
1. **Warm-up problem: a simple beam structure:** The sketch below shows a statically indeterminate beam of length L , clamped at $x = 0$, and simply supported at the other end. The beam is subjected to a concentrated load $\vec{F}^d = -P\vec{e}_z$ at $x = L/4$. In this exercise we are interested in calculating the elastic deflection of the beam (Hint: Review the class notes and handout for Lecture 32, and follow the proposed 7-step procedure).



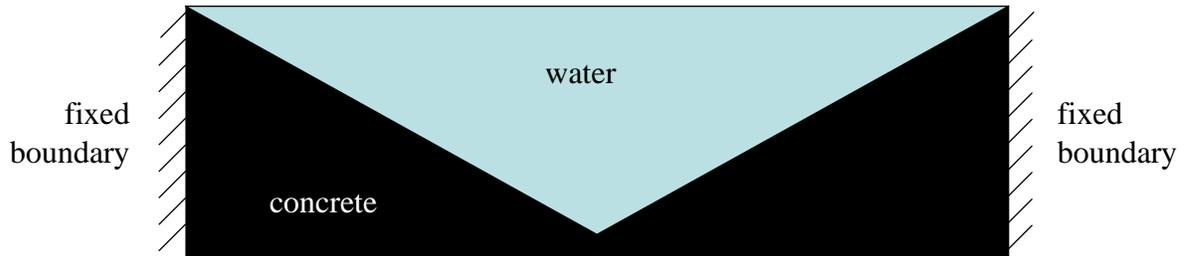
- What is the degree of indeterminacy of this structure?
- Find the target energy solution using Clapeyron's formulas.
- Determine the force and moment distribution as a function of reaction forces and moments.

- d. Using the lower bound approach, compute the complementary energy for the beam.
- e. Minimize the complementary energy w.r.t. the hyperstatic force.
- f. Calculate the displacement at the point of the concentrated load application.

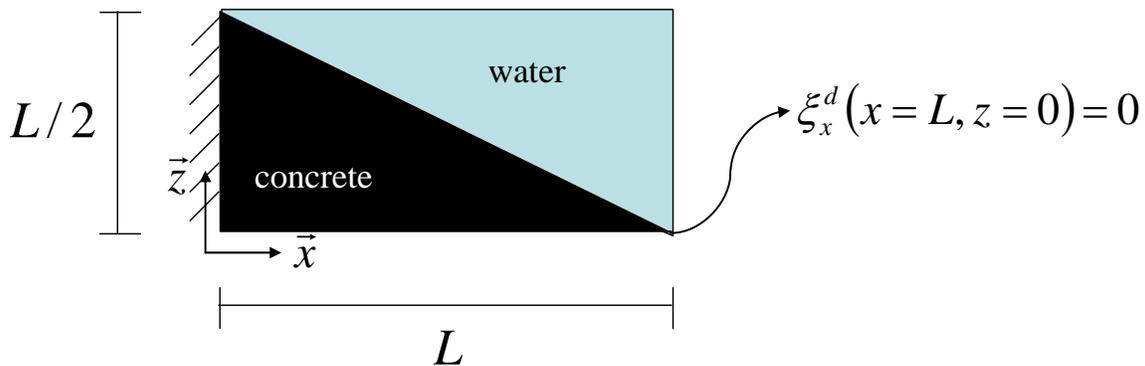
2. Beam deformation analysis – a statically indeterminate case: A frame structure as displayed below is loaded by a force F . The frame structure consists of two beam segments of length L (segment I) and length $2L$ (segment II), characterized by E (Young's modulus), I (area moment of inertia) and S (cross-sectional area). However, in contrast to our previous studies (this problem appeared in Quiz 2), a design engineer suggests to reduce the vertical displacement at the point of load application by the addition of a horizontal support at Point A. The goal of this exercise is to check the efficiency of that proposal using the energy approach. For simplicity, we consider beam bending only, i.e. $ES \rightarrow \infty$.



- a. By means of the complimentary energy approach, determine the reaction force at Point A. On this basis, determine and sketch the moment, shear force, and axial force along the beam's axis O-B-A. (Hint: for this problem, use the principle of superposition for determining the force and moment distributions that will be used for the complementary energy computations. Also, review the class notes and handout for Lecture 32, and follow the proposed 7-step procedure).
 - b. Using the results from Part a), determine the vertical displacement at the point of load application. How much (in %) does the addition of the horizontal support reduce (or increases) the vertical displacement.
 - c. Sketch the deflection curve along the beam's axis (do not need numerical values, only establish minima, maxima, point of inflection, etc.).
- 3. A water retaining structure – Upper Bound Energy Approach:** The sketch below shows a water retaining structure holding a body of water ($\rho_w = 1,000 \text{ kg/m}^3$). The structure is made of concrete of density $\rho_c = 3,000 \text{ kg/m}^3$, and its dimension in the y -direction is very large (thus, we will analyze all quantities per unit length in the y -direction). The focus of this exercise is the estimation of the elastic displacement at several locations along the wall bottom of the structure.



In this problem, we will use the upper bound energy approach. In order to simplify our analysis, we will use the following section of the structure and the given boundary conditions due to the symmetry of the problem:



We use the following form for the displacement field:

$$\vec{\xi}' = a \frac{x}{L} \left(1 - \frac{x}{L} \right) \vec{e}_x + b \frac{x}{L} \vec{e}_z$$

where a and b are the degrees of freedom.

- Show that the displacement field $\vec{\xi}'$ is kinematically admissible.
- Determine the strain tensor $\underline{\underline{\varepsilon}}'$.
- Using the displacement-based approach potential energy approach, determine a first approximation of the displacement field that minimizes the potential energy $\min_{a,b} E_{pot}(\vec{\xi}')$.
- Numerical application: Consider the elastic properties of concrete, Young's modulus $E_c = 20,000$ MPa, and Poisson's ratio $\nu_c = 0.2$. The length dimension of the structure is $L = 50$ m. Determine an estimate of the vertical and horizontal displacements at points $x = 0.25L$, $x = 0.50L$, $x = 0.75L$, $x = L$. Sketch the deformation at the bottom of the structure. Are the calculated values an upper or lower bound of the actual displacements for this structure?