1.020 Ecology II: Engineering for Sustainability
Spring 2008

Lectures 08_20 Multiple Objectives, Pareto Optimality

Motivation/Objective
Develop a way to compare values of different resource uses. Consider tradeoff between using limited water for farm revenue vs. using water for preservation of the riparian ecosystem.

Approach
1. Introduce a riparian ecological abundance measure as a second objective in the resource allocation problem of Lectures 8_16 and 08_17. This simplified measure assumes abundance is a linear function of the flow downstream of the irrigation diversion. We would like to maximize both revenue and abundance but these objectives conflict.
2. Include abundance objective as a constraint in the resource allocation problem and use MATLAB to evaluate the tradeoff between revenue and diversity.
3. Display tradeoff as a Pareto frontier (set of Pareto optimal solutions).
4. Consider how tradeoff depends on technological inputs (e.g. water requirement, yield).

Concepts and Definitions Needed:
Multiobjective optimization problem:

$\underset{x}{Maximize}\, F_{rev}(x) =$ Revenue ($) — Objective function 1

$\underset{x}{Maximize}\, F_{abd}(x) =$ Ecological abundance (unitless 0-1) — Objective function 2

x = vector of decision variables — Decision variables

Such that following constraints hold for each resource:

Resource used (x) $\leq$ Resource available	Inequality constraints
Upper and lower bounds on x	Inequality constraints
Physical constraints (e.g. mass, energy balance)	Equality constraints

We seek Pareto optimal solutions (solutions where one objective can be improved only at the expense of the other). Other solutions are either infeasible or inferior.

For 2 crop example identify Pareto frontier by converting one objective (Objective 2) to a constraint: $F_{abd}(x) \geq F_{abdmin}$.
Pareto frontier is F_{rev} vs F_{abdmin} curve. Points along frontier correspond to particular solutions (x).
Modify optimization problem of Lecture 08_16 & 08_17 to include upstream water limit and abundance constraint:

$$Maximize\;\; F_{rev}(x) = \sum_{i=1}^{2} p_i Y_i x_i$$

$x = [x_1 \quad x_2 \quad D]$, x_i = Area crop i (ha), D = diversion to farm (MCM season^{-1})

$Y_i = Y_{i0} - d_i x_i \quad Y_{i0}$ = nominal yield (tonne ha^{-1} season^{-1})

d_i = yield reduction coef (tonne ha^{-2} season^{-1})

Constraints: (MCM = 10^6 m^3) , w_i = Water rqmt crop i (MCM ha^{-1} season^{-1})

Supply $D \le U$ = upstream flow (MCM season^{-1})

Water: $\sum_{i=1}^{2} w_i x_i \le D$ = water diverted to farm (MCM season^{-1})

Land: $\sum_{i=1}^{2} x_i \le L_{avail}$ = land available (ha)

Abundance: $\beta[U - D] \ge F_{abdmin} \rightarrow \beta D \le \beta U - F_{abdmin}$

Nonnegativity: $x_i \ge 0 \quad i = 1, 2, 3$

Input Arrays for MATLAB (quadprog):
Quadprog format:

$$\underset{x}{Minimize}\, F_{rev}(x) = \frac{1}{2} x^T H x + f^T x \qquad \text{Find decision variables } x \text{ that minimize } F_{rev}(x)$$

such that:

$Ax \le b$	Inequality constraints
$A_{eq} x = b_{eq}$	Equality constraints
$x_{lb} \le x \le x_{ub}$	Lower and upper bound constraints

For multiobjective problem (converted to minimization problem):

$$f = -[p_1 Y_{10} \quad p_2 Y_{20} \quad 0] \qquad H = 2\begin{bmatrix} p_1 d_1 & 0 & 0 \\ 0 & p_2 d_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ w_1 & w_2 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & \beta \end{bmatrix} \quad b = \begin{bmatrix} U \\ 0 \\ L_{avail} \\ \beta U - F_{abdmin} \end{bmatrix} \quad x_{lb} = [0\ 0\ 0] \quad A_{eq} = b_{eq} = x_{ub} = [] \quad \text{(unused)}$$

Make sure that H is a symmetric matrix.
Plot of F_{rev} vs F_{abdmin} gives Pareto frontier.

Tradeoff Results
Note dependence of tradeoff curve problem inputs (farm inputs, β, etc.)