1.020 Ecology II: Engineering for Sustainability
Spring 2008

Lectures 08_19 Derived Supply, Equilibrium, Groundwater Pumping

Motivation/Objective

Extend the derived demand model of Lectures 08_16 & 08_17 to include the cost of supplying water from a single groundwater well. Combine derived supply and demand to obtain an equilibrium solution.

Approach

1. Derive an expression for the marginal cost of supplying water from a single groundwater well. Identify factors affecting pumping cost.
2. Add the supply curve calculation to the MATLAB program of Lectures 08_16 & 08_17
3. Identify the equilibrium solution (intersection of supply and demand curves)
4. Consider how problem inputs (aquifer transmissivity, energy cost, etc.) affect solution.

Concepts and Definitions Needed:

Pumping cost at supply well depends on flow rate and depth to water:

$F_{cost} = p\dot{E}\Delta t = p\rho g\Delta t(z_g - h)Q$ (\$ season^{-1}) p = energy price (\$ joule^{-1})

$\dot{E}$ = power required to pump water (watts), Δt = length of irrigation season (sec season^{-1})

Q_i = Pumping rate at i (m^3 sec^{-1}), z_g = ground surface elev. (m) h = groundwater elev. (m)

Pumping at the well affects h. Derive $h(Q)$ using rate form of mass balance eq in radial coords:
Mass rate pumped from well = mass rate flowing through soil toward well

Mass balance: $\rho Q = \rho 2\pi\, rq$ (kg sec^{-1}) $q = T\dfrac{dh(r)}{dr}$ = radial flow per unit width (m^3 m^{-1}sec^{-1})

T =Soil transmissivity (m^2 sec^{-1})

Substitute q in mass balance to get differential eq. for h:

$$\frac{dh(Q)}{dr} = \frac{Q}{2\pi\, rT} \qquad \text{Solve for } h(Q): \qquad h(Q) = h_0 + \frac{Q}{2\pi T}\ln\left[\frac{r_w}{r_0}\right] \qquad r_0 >> r_w$$

r_w = well radius (m) r_0 = distance where $h(Q) = h_0$ = water level at well when $Q = 0$

Marginal cost of supplying water:

$$\mu(Q) = \frac{\partial F_{cost}}{\partial Q} = \frac{\partial}{\partial Q}\left[p\rho g\Delta t[z_g - h(Q)]Q\right] = p\rho g\Delta t\left[z_g - h_0 - \frac{Q}{\pi T}\ln\left(\frac{r_w}{r_0}\right)\right] \quad (\$\ \text{m}^{-3})$$

Include this expression in crop allocation optimization code from Lectures 08_16 & 08_17.
Plot of $\mu(Q)$ vs Q gives derived supply (a curve).
Intersection of supply $\mu(Q)$ and demand $\lambda(Q)$ defines equilibrium solution $\lfloor Q_{eq}, \lambda_{eq} \rfloor$.

Crop Allocation Results with Groundwater Supply Cost Included

Supply curve is linear in Q. Note dependence of equilibrium solution on problem inputs (price, transmissivity, unpumped depths to groundwater, etc.)